Signal Extraction in Continuous Time and the Generalized Hodrick-Prescott Filter

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Abstract

A widely used filter to extract a signal in a time series, in particular in the business cycle analysis, is the Hodrick-Prescott filter. The model that underlies the filter considers the data series as the sum of two unobserved component (signal and non signal) and a smoothing parameter which for quarterly series is set to a specified value. This paper proposes a generalization of the Hodrick-Prescott filter to a continuous time support, using the well-established relationship between cubic splines and state-space models. The spline formulation of the filter leads to a state space model with several practical advantages: first, the smoothing parameter can be either pre-specified or estimated as the other parameters in the model; second, the unobserved components can be modelled by the addition of particular ARIMA structures; lastly the model is capable of working in the presence of missing values or for irregular surveys. Monte Carlo experiments support these considerations.

Keywords: smoothing parameter, cubic spline, state-space model, irregular surveys.

1 Introduction

The Hodrick-Prescott (HP) filter is largely used for extracting a signal from a time series, in particular for the analysis of the real business cycle. The basic model from which the filter is derived considers simply the sum of two unobserved components: a signal g_t and a white noise c_t, generally interpreted respectively as the growth and the cyclical components. Besides the easy application, the interest in it is due to the various properties the filter possesses. For example, King and Rebelo (1993) show that the filter has a model-based interpretation, considering the series observed as generated by the sum of an IMA(2,0) stochastic trend and an orthogonal white-noise; as a result the HP filter solution is equivalent to find the minimum mean square error estimator of g_t and c_t; Harvey and Jaeger (1993) use the Kalman filter to obtain these estimators. Kaiser and Maravall (2001) note that the previous specifications for the growth component and the cycle imply an IMA(2,2) model for the overall series and obtain the HP filter as a Wiener-Kolmogorov filter, using its properties to improve its performance. Gómez (1999) shows that the HP filter is a particular case of the Butterworth family of filters.

The HP filter can be generalized to a continuous time support, using well-known results about the relationship between cubic splines and state-space models (Wahba, 1978, Wecker and Ansley, 1983, Koopman et al., 1999, Koopman and Harvey, 2003). This generalization provides several advantages:

- the model is more flexible with respect to the original HP model. For example, we can suppose an ARIMA structure for the component c_t, or a different specification for the signal. In this case the original HP filter is a particular case of our generalized model;

- the HP filter requires the choice of a smoothing parameter λ which "balances" the trade-off between the goodness of fit of the model to the observations and the degree of smoothness. HP (1997) suggest to fix the smoothing parameter equal 1600 for quarterly data; this result is obtained from empirical considerations about the U.S. quarterly GNP series (1950:Q1-1979:Q2) and eliminates the frequencies of 32 quarters or greater, but it has been adopted as the default value in many applications and in the computer routines. In our formulation the smoothing parameter is part of the data generating process (DGP) of the series, in the sense that it is present directly in the state-space representation and can be easily estimated;

- the observations are not necessarily equally spaced. This is perhaps the main advantage of our specification, because it includes the cases of missing observations and data recorded with different timing (for example, surveys with quarterly timing until the period t and with monthly timing from period $t+1$ onwards).

Our paper investigates these capabilities, proper of the continuous state-space models (see Harvey, 1989 ch. 9) in the particular context of signal extraction, evaluating the performance of the continuous time model with respect to the classical HP filter.

In section 2 the relationship between the HP representation and the cubic splines is described in some detail: this leads to the state-space specification of the Generalized HP (GHP) filter. Section 3 describes the uses of this model, stressing the three cases listed before. In section 4, a Monte Carlo analysis is performed to evaluate the performance of this model. Concluding remarks follow.

2 Hodrick-Prescott Filter and Cubic Splines

The filter proposed by Hodrick and Prescott (1997) has a long tradition as a method to extract the trend (or the cyclical) signal from a time series. They suppose that an observed time series y_t (generally considered by taking logarithms) is the sum of two unobserved components: a growth component g_t and a cyclical component c_t:

$$y_t = g_t + c_t, \qquad t = 1, \dots, T. \tag{1}$$

The purpose is to extract the trend component g_t and to obtain the cyclical component as a residual. We suppose that $c_t = y_t - g_t$ has zero mean in the long period. Assuming the sum of the squares of the second difference of g_t as a measure of its smoothness, a logical solution to this problem would be to solve the minimization problem:

$$\min_{\{g_t\}_{t=1}^{T}} \left[\sum_{t=1}^{T} (y_t - g_t)^2 \right]$$

subject to the constraint:

$$\sum_{t=1}^{T} \left(\nabla^2 g_t \right)^2 \leq \nu$$

where ∇^2 is the second order difference and ν is a known constant. This is equivalent to solve the following unconstrained programming problem:

$$\min_{\{g_t\}_{t=1}^{T}} \left[\sum_{t=1}^{T} (y_t - g_t)^2 + \lambda \sum_{t=1}^{T} \left(\nabla^2 g_t \right)^2 \right] \tag{2}$$

where λ is a positive known constant that controls the degree of smoothness of the series (the larger the value of λ, the smoother is the series obtained). We can call this parameter smoothing parameter.

Deriving (2) with respect to g_t after simple algebra (see Pedersen 1999, section 8), we obtain the growth filter:

$$G(B) = \frac{1}{\lambda (1 - B)^2 (1 - B^{-1})^2 + 1} \tag{3}$$

where B denotes the backward operator.

The specification of λ plays a crucial role in extracting the trend, but HP suggest to fix it to 1600 for quarterly series (see section 1).

We consider the problem in a continuous time support; more specifically we adopt the following signal-in-noise stochastic model:

$$y_t = g_t + c_t, \qquad t \in [\alpha, \omega]. \tag{4}$$

where g_t is generated by a Wiener process. Of course, in application to real time series data, there are just T observations not necessarily equally spaced; we stress this point saying that the observation y_t is recorded at time τ_t.

There is a correspondence between (4) and smoothing polynomial splines. The smoothing polynomial spline $g(\omega)$ of degree $2m-1$ satisfies this condition (Wecker and Ansley, 1983):

$$\min_{g(\omega)} \left\{ \sum_{i=1}^{n} [y_t - g_t]^2 + \lambda \int_a^\omega \left[g^{(m)}(u) \right]^2 du \right\} \tag{5}$$

among all functions whose first $m-1$ derivatives are continuous and the $m-th$ derivative square integrable, with λ arbitrary; $g^{(m)}$ denotes the $m-th$ derivative of the function g. It is immediate to note that (2) corresponds to the problem of minimization in (5) in a continuous time domain when $m=2$. In other terms, the extraction of the growth component in (1) for the discrete case is equivalent to the search of the optimal cubic polynomial spline in the problem (5) in the continuous case (Harvey and Jaeger, 1993). In these terms, the solution of (4)-(5) can be seen as a generalized HP filter.

Moreover, Wecker and Ansley (1983) show that (4)-(5) can be formulated as a dynamic linear system. We can represent the previous problem in a state-space form (see Carter and Kohn, 1997, Koopman et al., 1999, section 3.4, and Koopman and Harvey, 2003, section 5.2):

$$\begin{cases} y_t = \mathbf{f}'\alpha_t + c_t \\ \alpha_t = \mathbf{G}_t \alpha_{t-1} + k\mathbf{u}_t \end{cases} \tag{6}$$

where:

$$\alpha_t = \begin{bmatrix} g_t, & g_t^{(1)} \end{bmatrix}', \quad \mathbf{f} = \begin{bmatrix} 1, & 0 \end{bmatrix}', \quad \mathbf{G}_t = \begin{bmatrix} 1 & \delta_t \\ 0 & 1 \end{bmatrix},$$

$c_t \sim IIN(0, \sigma_c^2)$, and where $\mathbf{u}_t = [u_{1t}, u_{2t}]'$ are bivariate independent normally distributed variables with zero mean and variance matrix:

$$\mathbf{V}_t = \begin{bmatrix} \delta_t^3/3 & \delta_t^2/2 \\ \delta_t^2/2 & \delta_t \end{bmatrix}.$$

The precision parameter k is linked to the smoothing parameter λ by:

$$\lambda = \frac{\sigma_c^2}{k^2}.$$

The variable δ_t represents the time distance between two contiguous observations; formally $\delta_t = \tau_t - \tau_{t-1}$. Of course, when the observations are equally spaced, $\delta_t = 1$ for each t.

Filtering and smoothing (6) with the well-established techniques for dynamic models (Harvey, 1989, ch.9), we can obtain the unobserved signal g_t.

3 Characterizations and Fields of Application

The model (6) can be used directly to extract the signal from a time series, but, with some constraints or extensions, can represent also the classical HP model and more general cases. In this section we illustrate how to use the general model (6) in various contexts and with some particular specifications.

3.1 Classical HP Filter

The classical HP filter, with equally spaced observations, can be seen as a particular case of (6), constraining the first element of the vector α_t to be deterministic. In other terms, the model, in an extensive form, will be:

$$\begin{aligned} y_t &= g_t + c_t, \qquad (7) \\ g_t &= g_{t-1} + g_{t-1}^{(1)}, \\ g_t^{(1)} &= g_{t-1}^{(1)} + k u_t. \end{aligned}$$

Clearly, in this case the covariance matrix $\mathbf{V}$ collapses to the variance of u_t. To fix the smoothing parameter λ to a known value, we have to fix σ_c^2/k^2 (for example, 1600 for quarterly data). This constraint is an open problem and it has been considered perhaps the main weakness of the HP filter because the smoothing parameter has not an intuitive interpretation (Wynne and Koo, 1997). Furthermore, whereas there is a diffuse consensus of opinion (not properly justified) on the choice of 1600 for quarterly data, there is not a ”default” value for the smoothing parameter for annual or monthly data. The econometrics package E-views has 14400 for monthly data, but, for example, Dolado et al. (1993) use 4800, whereas Ravn and Uhlig (2002) 129600. For annual data Baxter and King (1999) propose the value of 10, whereas Dolado et al. (1993) 400, Backus and Kehoe (1992) 100 (these various values are reported in Maravall and del Río, 2001). Recently Maravall and del Río (2001) propose to choose the annual and monthly values of the smoothing parameter resulting from the aggregation of the quarterly filter associated with $\lambda = 1600$, solving simple equations derived from the relationship between the HP filter and the Butterworth filter. They obtain $\lambda = 7$ for annual data and $\lambda = 129119$ for monthly data. Anyway, they suppose a priori that the quarterly default value is “true”.

A more rigorous proposal was made by Pedersen (2001), who obtains the optimal smoothing parameter value minimizing a metric in the frequency domain that compares the cyclical component derived by HP and the “true” cyclical component obtained by an ideal filter.

From our point of view, all these approaches have the limit to consider the smoothing parameter extraneous to the the data generating process (DGP) of the observed series; in fact it is fixed (or calculated as in Pedersen, 2001, and Maravall and del Río, 2001) in a separate step with respect to the extraction of the components. But using the state-space representation (6) the smoothing parameter enters int the state equation, so it is part of the DGP and can be estimated as part of the overall inference.

3.2 Structural Models

The model (6) has the form of a structural time series model and it can be enriched to take into account for more flexible specifications. As noted by Pollock (2001), the simple model underlying the HP filter is not adequate to represent most of the data generating process of time series. A more appropriate representation would be one that supposes separate ARIMA processes to generate each of the unobserved components (see, for example, Gómez and Maravall, 2001). For example, a stationary ARMA structure can be hypothesized for the dynamics of the cyclical component. In this case it would be sufficient to modify the state vector, maintaining the general structure. For example considering an AR(1) dynamics for c_t, we have:

$$\begin{cases} y_t = \mathbf{f}'\alpha_t \\ \alpha_t = \mathbf{G}_t\alpha_{t-1} + k\mathbf{u}_t \end{cases}$$

where

$$\alpha_{\mathbf{t}} = \begin{bmatrix} g_t, & g_t^{(1)}, & c_t \end{bmatrix}', \quad \mathbf{f} = \begin{bmatrix} 1, & 0, & 1 \end{bmatrix}',$$

$$\mathbf{G}_t = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \phi_1 \end{bmatrix}, \quad \mathbf{u}_t = [u_{1t}, u_{2t}, u_{3t}]' \sim N(\mathbf{0}, \mathbf{V}_t),$$

$$\mathbf{V}_t = \begin{bmatrix} \delta_t^3/3 & \delta_t^2/2 & 0 \\ \delta_t^2/2 & \delta_t & 0 \\ 0 & 0 & \sigma_c^2/k^2 \end{bmatrix}$$

and ϕ_1 represents the autoregressive coefficient. The methodology explained in this paper is easily extended to state space models with more parameters. Another possibility would be to add a trigonometric function to represent the cycle (see Harvey and Jaeger, 1993).

3.3 Missing Observations

For observed time series with missing data, the HP filter can be applied only if an estimation method to fill the missing observations is used. This it would not be necessary in our approach. In fact, the introduction in the model of the variable δ_t allows for the presence of missing data. Specifically, the first equation of the state vector in model (6) can be written:

$$g_t = g_{t-1} + \delta_t g_{t-1}^{(1)} + u_{1t}.$$

For example, if the observation at time i is missing, being available those at time $i-1$ and $i+1$, the first state equation at time $i+1$ will be:

$$g_{i+1} = g_{i-1} + 2g_{i-1}^{(1)} + u_{1i+1}.$$

If also the observation at time $i+1$ is missing, whereas the one at time $i+2$ is available, the first state equation at time $i+2$ would be:

$$g_{i+2} = g_{i-1} + 3g_{i-1}^{(1)} + u_{1i+2}.$$

and so on.

3.4 Irregular Surveys

A similar situation arises when a time series is recorded with a frequency and, from a certain time, with a different frequency. The use of the variable δ_t in model (6) provides again the possibility to take into account this situation. For example, let us suppose that the variable y_t is recorded quarterly until time i and monthly from time $i+1$. In this case, the variable δ_t would be defined:

$$\delta_t = \begin{cases} 3 & \text{if } t \leq i \\ 1 & \text{if } t > i \end{cases}$$

Similarly, if the variable y_t is recorded annually until the time i and quarterly, from time $i+1$, the variable δ_t would be defined:

$$\delta_t = \begin{cases} 4 & \text{if } t \leq i \\ 1 & \text{if } t > i \end{cases}$$

The extension to other irregular surveys is straightforward.

4 Monte Carlo Evaluation

To evaluate our approach in the various fields of applications, we perform several Monte Carlo simulations, under the hypothesis that the data are generated by the model (1). We recall that King and Rebelo (1993) show that the data can be seen as the sum of an IMA(2,0) model (component g_t) and a white noise (component c_t). So, we can generate separately the two components by the models:

$$\begin{aligned} \nabla^2 g_t &= \varepsilon_t^{(g)}, \qquad \varepsilon_t^{(g)} \sim NID(0, k^2) \\ c_t &= \varepsilon_t^{(c)}, \qquad \varepsilon_t^{(c)} \sim NID(0, \sigma_c^2) \end{aligned}$$

and $\lambda = \sigma_c^2/k^2$. Referring the considerations of Hodrick and Prescott (1997) about the variances of the components, we can fix the value of k^2 to 1/64 and obtain the value of σ_c^2 in correspondence of different values of λ. In particular, we choose the values of three different smoothing parameters, compatible with three cycles of reference (the values are taken by Table 5 of Maravall and del Río, 2001):

1) $\lambda = 1$ for annual series, $\lambda = 179$ for quarterly series, $\lambda = 14400$ for monthly series, which correspond cycles of length 5.7 years;

2) $\lambda = 7$ for annual series, $\lambda = 1600$ for quarterly series, $\lambda = 129119$ for monthly series, which corresponds cycles of length 9.9 years;

3) $\lambda = 25$ for annual series, $\lambda = 6199$ for quarterly series, $\lambda = 501208$ for monthly series, which corresponds cycles of length 13.9 years.

We generate annual series of length 20, 30, 40 years, quarterly series of 10, 20, 30 years, monthly series of 5, 10, 20 years, using the various λ specifications; so we have 27 different sets of simulations and for each one we generate 1000 series. We perform the following experiment: from each series we extract the trend with GHP using the model (6), with the correct HP filter and with the HP filter fixing λ to the default value (we choose the most frequently used values, that are 10 for annual series, 1600 for quarterly series, 14400 for monthly series). The use of the true λ in the HP filter is clearly a theoretical situation, because the researcher does not know a priori it; this is a useful benchmark to compare the GHP and the HP filter with fixed λ. The results are compared with the true trend using RMSE and Theil index; the first one would indicate how distant from the true signal are the estimated signals, whereas the second would stress the ability of the methods to track turning points in the series.

In Table 1 the means and the standard errors of the indices calculated on these simulations are showed, with the maximum likelihood estimated parameters in the GHP procedure. In general, the performance of the GHP is similar to that of the HP with true smoothing parameter and better of the HP with the default λ. Only in the cases of annual data with $\lambda = 7$ and 25 the classical HP filter seems to have a better behaviour, but the default value is near to the true values. In the other cases the performance of GHP increases with the number of observations. The GHP filter shows a good ability to capture the turning points of the series.

Another Monte Carlo experiment looks at the performance of the GHP filter for irregular surveys. For this purpose we use the simulated quarterly series of 10 years and then we consider as annual the initial i years of the series ($i = 1, 2, 3, 4, 5$); in other terms we drop the second, the third and the fourth observation of the initial i years and then estimate the trend with the model (6), using the appropriate specification for the variable δ_t; then we estimate the trend with the HP filter with the true λ and with the default value using only the second part of the series (that with quarterly data) and compare the results, using only the common second part of the series. The same experiment is performed using the monthly simulated series, and considering quarterly the first i years ($i = 1, ..., 5$). The results are showed in Table 2. It is interesting to note that the performance of the GHP filter is always better then the case of HP filter with default values (a part the case of $\lambda = 6199$ with only an annual data) and its performance becomes better then the HP filter with the true smoothing parameter, increasing the number of irregular observations.

5 Final Remarks

In this paper we have developed a new approach for the extraction of unobserved signals in time series, which generalizes the Hodrick-Prescott filter, using the well-known results for cubic spline models, recently diffused in the time series literature by Koopman et al. (1999) and Koopman and Harvey (2003). The advantages of this methodology are its flexibility, its general form which avoids the specification of structural models for the signals, the possibility to estimate the smoothing parameter, the possibility to work with missing values and irregular surveys.

The estimation of the models can be performed with classical maximum likelihood estimation, using the state-space representation. The extraction of the signals could be improved using the methodology of Carter and Kohn

(1997), in which an efficient MCMC algorithm for estimating the unobserved components was developed. This last one works in a Bayesian framework and possesses several additional advantages: robustness in presence of a limited number of data, automatic calculation of confidence intervals for the signals, implicit estimation of missing data. In addition, the use of a Bayesian approach for estimation allows the inclusion of priors on the smoothing parameters. We have applied this methodology in real cases with good results, but it implies cumbersome calculations and its evaluation with Monte Carlo experiments would be prohibitive. Potentially, the Carter and Kohn approach could extend the procedure considering mixtures and non Normal distributions.

Our Monte Carlo experiments suggest that the GHP approach approximates the true signal and performs generally better then the classical HP filter with the default values. In particular, in the presence of irregular surveys, the GHP filter improves the performance of the HP filter, using the full information available.

Table 1: Results of Monte Carlo experiment for several smoothing parameters λ and time series lengths

	GHP				HP true		HP default	
	σ_c	k	RMSE	THEIL	RMSE	THEIL	RMSE	THEIL
annual series (λ default=10)								
λ generator=1								
20 years	0.119	0.119	0.085	0.030	0.080	0.028	0.105	0.036
	0.036	0.050	0.020	0.034	0.016	0.031	0.024	0.038
30 years	0.121	0.122	0.082	0.018	0.079	0.018	0.105	0.023
	0.026	0.037	0.015	0.020	0.014	0.019	0.021	0.025
40 years	0.122	0.121	0.081	0.013	0.079	0.013	0.104	0.017
	0.022	0.029	0.013	0.017	0.013	0.016	0.017	0.020
λ generator=7								
20 years	0.319	0.115	0.177	0.062	0.167	0.059	0.168	0.059
	0.070	0.071	0.049	0.067	0.043	0.066	0.043	0.066
30 years	0.324	0.118	0.172	0.038	0.164	0.036	0.165	0.037
	0.055	0.050	0.041	0.044	0.037	0.040	0.037	0.041
40 years	0.327	0.119	0.169	0.027	0.163	0.027	0.164	0.027
	0.047	0.041	0.035	0.035	0.033	0.034	0.033	0.034
λ generator=25								
20 years	0.605	0.110	0.291	0.102	0.274	0.097	0.281	0.100
	0.118	0.087	0.089	0.112	0.082	0.108	0.083	0.113
30 years	0.613	0.115	0.281	0.063	0.267	0.059	0.274	0.061
	0.095	0.062	0.074	0.074	0.068	0.068	0.070	0.069
40 years	0.620	0.116	0.276	0.045	0.265	0.043	0.273	0.044
	0.081	0.051	0.066	0.066	0.061	0.057	0.062	0.059
quarterly series (λ default=1600)								
λ generator=179								
10 years	1.647	0.115	0.602	0.046	0.570	0.044	0.706	0.053
	0.202	0.074	0.175	0.065	0.163	0.061	0.218	0.067
20 years	1.661	0.121	0.558	0.024	0.545	0.023	0.696	0.029
	0.144	0.042	0.115	0.031	0.110	0.030	0.160	0.037
30 years	1.665	0.122	0.542	0.015	0.536	0.014	0.696	0.019
	0.114	0.032	0.091	0.017	0.090	0.017	0.133	0.021
λ generator=1600								
10 years	4.911	0.111	1.450	0.109	1.356	0.104		
	0.581	0.118	0.520	0.153	0.495	0.147		
20 years	4.963	0.117	1.333	0.057	1.272	0.054		
	0.414	0.063	0.355	0.074	0.331	0.070		
30 years	4.982	0.117	1.274	0.034	1.242	0.033		
	0.333	0.044	0.285	0.041	0.276	0.039		

Table 1 (continued)

	GHP				hp true		hp default	
	σ_c	k	rmse	THEIL	rmse	THEIL	rmse	THEIL
λ generator=6199								
10 years	9.656	0.120	2.552	0.193	2.360	0.180	2.469	0.190
	1.140	0.177	1.045	0.282	0.962	0.262	0.986	0.280
20 years	9.762	0.112	2.302	0.097	2.174	0.092	2.294	0.098
	0.806	0.084	0.700	0.126	0.656	0.120	0.657	0.129
30 years	9.801	0.113	2.185	0.059	2.108	0.056	2.217	0.060
	0.648	0.056	0.576	0.074	0.549	0.067	0.554	0.071
monthly series (λ default=14400)								
λ generator=14400								
5 years	14.793	0.105	3.325	0.075	3.133	0.071		
	1.412	0.141	1.257	0.128	1.148	0.125		
10 years	14.863	0.110	3.040	0.043	2.910	0.042		
	0.976	0.070	0.858	0.060	0.803	0.060		
20 years	14.973	0.120	2.877	0.023	2.811	0.022		
	0.697	0.043	0.589	0.032	0.561	0.030		
λ generator=129119								
5 years	44.232	0.159	8.731	0.198	7.986	0.179	8.579	0.197
	4.214	0.333	3.955	0.356	3.522	0.314	3.519	0.355
10 years	44.459	0.101	7.392	0.105	6.972	0.101	7.804	0.113
	2.892	0.113	2.656	0.150	2.450	0.150	2.460	0.168
20 years	44.837	0.113	6.871	0.053	6.586	0.051	7.474	0.058
	2.075	0.063	1.778	0.071	1.693	0.073	1.734	0.083
λ generator=501208								
5 years	87.124	0.272	16.646	0.380	15.040	0.339	16.747	0.384
	8.293	0.642	8.044	0.698	7.157	0.603	6.965	0.694
10 years	87.542	0.107	12.909	0.185	12.137	0.177	15.182	0.220
	5.682	0.169	5.248	0.276	4.875	0.272	4.883	0.328
20 years	88.305	0.107	11.817	0.089	11.176	0.085	14.536	0.113
	4.073	0.084	3.560	0.123	3.377	0.126	3.444	0.161

Table 2: Results of Monte Carlo experiment for several smoothing parameters λ and irregural time series

	GHP				hp true		hp default	
	σ_c	k	RMSE	THEIL	RMSE	THEIL	RMSE	THEIL
annual for t years and then quarterly (λ default = 1600)								
λ generator=179								
t=1	1.648	0.113	0.588	0.045	0.575	0.044	0.709	0.052
	0.210	0.074	0.183	0.066	0.175	0.064	0.232	0.070
t=2	1.645	0.113	0.592	0.046	0.581	0.045	0.711	0.053
	0.222	0.076	0.195	0.073	0.188	0.074	0.240	0.081
t=3	1.645	0.114	0.596	0.047	0.588	0.046	0.706	0.054
	0.235	0.079	0.208	0.083	0.195	0.084	0.246	0.090
t=4	1.641	0.113	0.600	0.049	0.596	0.047	0.693	0.054
	0.248	0.082	0.220	0.103	0.208	0.099	0.246	0.101
t=5	1.636	0.113	0.608	0.051	0.602	0.049	0.666	0.054
	0.265	0.087	0.239	0.121	0.231	0.113	0.251	0.119
λ generator=1600								
t=1	4.906	0.111	1.432	0.108	1.377	0.104		
	0.607	0.128	0.544	0.159	0.541	0.156		
t=2	4.896	0.110	1.422	0.107	1.402	0.108		
	0.640	0.127	0.580	0.173	0.573	0.187		
t=3	4.896	0.112	1.426	0.110	1.430	0.111		
	0.676	0.130	0.620	0.203	0.594	0.212		
t=4	4.887	0.109	1.437	0.115	1.471	0.117		
	0.709	0.129	0.660	0.247	0.642	0.245		
t=5	4.870	0.110	1.474	0.120	1.519	0.124		
	0.745	0.141	0.705	0.286	0.716	0.308		
λ generator=6199								
t=1	9.645	0.123	2.535	0.191	2.417	0.183	2.515	0.191
	1.184	0.198	1.077	0.296	1.052	0.283	1.070	0.296
t=2	9.619	0.128	2.520	0.191	2.478	0.192	2.570	0.200
	1.252	0.203	1.156	0.325	1.137	0.345	1.141	0.357
t=3	9.625	0.128	2.518	0.195	2.567	0.202	2.635	0.207
	1.320	0.207	1.231	0.376	1.204	0.400	1.193	0.408
t=4	9.602	0.126	2.539	0.203	2.703	0.217	2.736	0.220
	1.389	0.205	1.293	0.454	1.313	0.462	1.305	0.472
t=5	9.569	0.129	2.615	0.214	2.855	0.234	2.869	0.235
	1.449	0.233	1.382	0.530	1.454	0.596	1.448	0.597

Table 2 (continued)

	GHP				hp true		hp default	
	σ_c	k	RMSE	THEIL	RMSE	THEIL	RMSE	THEIL
quarterly for t years and then monthly (λ default=14400)								
λ generator=14400								
t=1	14.868	0.111	2.956	0.042	2.936	0.042		
	1.013	0.074	0.872	0.062	0.854	0.063		
t=2	14.869	0.110	2.948	0.041	2.957	0.041		
	1.043	0.073	0.940	0.063	0.922	0.063		
t=3	14.883	0.108	2.956	0.041	3.009	0.042		
	1.103	0.074	1.005	0.061	1.008	0.067		
t=4	14.875	0.106	2.971	0.040	3.026	0.042		
	1.168	0.074	1.065	0.062	1.053	0.067		
t=5	14.874	0.105	2.995	0.041	3.078	0.042		
	1.227	0.076	1.144	0.069	1.157	0.072		
λ generator=129119								
t=1	44.472	0.101	7.231	0.104	7.076	0.102	7.884	0.114
	2.981	0.118	2.649	0.156	2.557	0.159	2.628	0.176
t=2	44.473	0.100	7.159	0.101	7.223	0.102	7.972	0.112
	3.079	0.118	2.807	0.160	2.784	0.164	2.819	0.179
t=3	44.496	0.099	7.139	0.099	7.366	0.103	8.122	0.114
	3.247	0.119	2.997	0.159	3.049	0.168	3.071	0.191
t=4	44.455	0.099	7.198	0.098	7.493	0.103	8.190	0.113
	3.427	0.122	3.192	0.160	3.257	0.168	3.201	0.188
t=5	44.446	0.098	7.291	0.101	7.779	0.108	8.402	0.116
	3.593	0.120	3.438	0.180	3.560	0.198	3.482	0.207
λ generator=501218								
t=1	87.560	0.110	12.724	0.184	12.378	0.181	15.339	0.221
	5.849	0.182	5.294	0.293	5.103	0.291	5.217	0.344
t=2	87.561	0.111	12.625	0.179	12.800	0.182	15.522	0.219
	6.033	0.179	5.554	0.302	5.586	0.302	5.586	0.351
t=3	87.596	0.112	12.588	0.176	13.158	0.184	15.806	0.222
	6.370	0.181	5.849	0.299	6.176	0.311	6.082	0.375
t=4	87.518	0.117	12.668	0.176	13.703	0.189	15.947	0.221
	6.754	0.184	6.213	0.304	6.639	0.319	6.346	0.369
t=5	87.508	0.115	12.871	0.181	14.588	0.204	16.391	0.227
	7.087	0.185	6.668	0.344	7.218	0.384	6.882	0.406

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