Real exchange rate misalignment in Hungary: a long-memory regime-switching cointegration model

Gilles Dufrénot (ERUDITE and GREQAM)

Elisabeth Grimaud (GREQAM-CNRS)

Eugénie Latil (GREQAM-CNRS)

Valérie Mignon* (THEMA-CNRS)

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Abstract

This paper proposes an estimate of the Hungarian real exchange rate misalignments using a nonlinear cointegration approach. To capture the adjustment towards the long-run equilibrium, long-memory regime-switching models are estimated ($FI-STARMA$ and $FI-TARMA$ processes). This allows us to take into account two types of persistence: a permanent component due to the influence of real factors and a nonlinear component where persistence is associated with time-dependent effects. Our results suggest that the regime-switching is instantaneous since the $FI-TARMA$ process is adequate to describe the misalignment of the Hungarian currency.

JEL classification: C22, C51, F31, P34.

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*Corresponding author: Valérie Mignon, THEMA-CNRS, University of Paris X, 200 avenue de la République, 92001 Nanterre Cedex, France. Phone: 33 1 40 97 58 60. Email: Valerie.Mignon@u-paris10.fr

1 Introduction

Two dominant stylized facts have characterized the evolution of real exchange rates in European transition economies during the last decade. Firstly, real exchange rates have shown a sustained trending appreciation. Up until recently, such an appreciation was viewed as primarily due to a Balassa-Samuelson effect affecting the sector of tradables (see, for instance, De Broeck and Torsten (2001), Bostgen and Coricelli (2001), Arratibel, Rodriguez-Palenzuela and Thimann (2002), Egert (2002a, 2002b), Strahilov (2002)). However, whether the Balassa-Samuelson has been strong is today a subject of controversy. Indeed, other factors have been at play during the transition periods and may also have their importance in explaining part of the observed real appreciation. This is notably the case in the following transition countries: Hungary, Poland, Slovenia, Slovakia, the Baltic countries, the Czech republic and Croatia. The reform strategies conducted in these countries have been characterized by rapid adjustments illustrated by strong efforts on the strengthening of money and financial markets, the macroeconomic stabilization (with regard to inflation, public finance, foreign debt), the liberalization of micro-markets (with regard to prices, trade and entry). Globally, despite some occasional crises, these countries have experienced a gradual improvement of their economic fundamentals that may also account for the huge appreciation of their real exchange rates.

A second stylized fact, more recently documented, is the nonlinear adjustment of real exchange rates after an initial deviation (see, for instance, Taylor and Sarno (2001)). There are several intuitions that are suggestive of some form of nonlinearity in real exchange rates. Firstly, some authors relate the real exchange rates movements in the East Europe countries to the returns and show that the latter follow nonlinear dynamics (see Peel and Speight (1996)). Secondly, one cannot ignore the nominal aspects in the movements of real exchange rates. It has been evidenced that nonlinearities in the dynamics of nominal exchange rates capture features such as transaction costs, trading costs, contagion effects, or bubbles. This implies a time dependence property in the adjustment process after an initial deviation of the exchange rates. Asymmetric dynamics can also be observed when the adjustment process is regime-dependent, that is when the speed of adjustment towards the long-run equilibrium varies with the level and sign of the shocks affecting the macroeconomic fundamentals. Exchange rates models with nonlinear mean-reverting mechanisms have been successfully implemented to developed countries (recent studies include, among many others, Corrado, Miller and Zhang (2002), De Grauwe and Grimaldi (2002), Dufrénot and Mignon (2002), Holmes (2002), Chowdury, Sarno and Taylor (2003)). Applications to transition economies are rare. However, nonlinearities of the type described before may presumably characterize the dynamics of their exchange rates for several reasons. Firstly, transaction costs are strong determinants of the buy and sold decisions on emerging markets. Secondly, expectations of realignments in Eastern and Central Europe countries adjust more rapidly than the fundamentals, reflecting noisy trading, speculative attacks, rumors and contagion behaviors. Such features induce jumps, instability and highly volatile movements. It thus seems hard to believe that the residuals of a real exchange rate equation follow a linear process.

This paper uses a nonlinear cointegration model based on a fractionally integrated regime-switching specification in order to capture the above two properties in one East Europe country: Hungary. One way for approaching the strong appreciation of the real exchange rate is to think the currency as being affected by shocks on the economic fundamentals with very long transitory effects. We thus consider a long-memory model. Further, although there may exist a variety of nonlinear parametric forms to capture nonlinear mean-reverting effects of real exchange rates, the empirical works in the literature generally conclude in favor of smooth transition models. In this context, we propose a fractionally integrated smooth transition model ($FI-STARMA$). Moreover, in order to account for the possibility of an instantaneous regime-switching, a fractionally integrated threshold process is also studied ($FI-TARMA$). The important point here is that the long-memory and nonlinear properties of real exchange rates are modelled jointly. In the short-run, the exchange rates misalignments are dominated by nonlinear patterns. Such short-term misalignments can be viewed as originating for instance from the agents' behaviors on financial markets, thereby inducing rumors, contagion, speculative attacks, *etc.*, that are sources of nonlinearity. In the long-run, shocks on fundamentals dominate and the misalignments exhibit a long-memory property.

The plan of the paper is as follows. In section 2, we present the data, the real exchange rate model and the econometric methodology. Section 3 gives some results concerning tests of fractional cointegration and (non)linearity. Section 4 reports estimations relating to two models which combine the long-memory and nonlinearity properties: $FI-STARMA$ and $FI-TARMA$ models. Section 5 contains some developments on the nonlinear persistence of exchange rate misalignments. Section 6 concludes.

2 The data, the long-run model and the econometric methodology

2.1 The data

We consider seasonally adjusted monthly series for Hungary over the 1992:1 to 2000:7 period. The data consist of the following variables.

- *$REER$: the real effective exchange rate.* Nominal exchange rate data are taken from the Central Bank of Hungary. The real exchange rate is computed using a consumer price index (CPI)-based end of period rate[1]. The effective exchange rate is calculated by considering a weighted average of the exchange rate against the US Dollar and the Deutsche Mark. The weights are based on the structure of the foreign trade.

- *DEF: the ratio of the budget deficit over GDP.* Data on the budget deficit are taken from the Hungary national statistics and are measured in local currency.

- *NFA: the ratio of net foreign assets over GDP.* Data are obtained from the Central Bank of Hungary and calculated in terms of nominal and local currency.

- *$RIRD$: the real interest rate differential* (domestic minus foreign interest rate). We use 365-day Treasury bill rates for Hungary and the US.

- *$RELPRICE$: the prices of non-traded goods over those of traded goods.* The latter are used to capture Balassa-Samuelson effects. We use the prices of services (services are supposed to be the least likely to be traded) and the prices of durable consumer goods (the latter are good proxies of non tradable goods).

2.2 The theoretical framework: a BEER model

Our long-run equation is a behavioral equilibrium exchange rate model (BEER) *a la* Clark and MacDonald (1999, 2000).

$$\begin{aligned} REER_t = \alpha_0 + \alpha_1 DEF_t + \alpha_2 NFA_t + \alpha_3 RIRD_t + \alpha_4 RELPRICE_t \\ + \beta_1 DUM1_t + \beta_2 DUM2_t + \beta_3 DUM3_t + \varepsilon_t, \end{aligned} \quad (1)$$

where ε_t is an error term. $DUM1$, $DUM2$ and $DUM3$ are dummy variables that capture the influence of some macroeconomic events on the real exchange rate.

$DUM1$ is defined as

$$DUM1_t = \begin{cases} 1, & \text{if } t \in [1995{:}3,\ 1995{:}12] \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

[1] We computed a consumer price index as the weighted average of services and durable consumer good prices, so as to take into account the prices of both traded and non traded goods.

Indeed, in March 1995 the Government launched a set of austerity policies to restore the internal and external balances (a devaluation of the currency of 9%, the adoption of a crawling-peg system, a restrictive monetary policy aimed at reducing inflation, a more stringent fiscal policy).

$DUM2$ is defined as

$$DUM2_t = \begin{cases} 1, & \text{if } t \in [1994{:}2,\ 1994{:}3] \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

This corresponds to the financial crisis on the US bond market, which had an influence on the interest rate differential.

$DUM3$ is defined as

$$DUM3_t = \begin{cases} 1, & \text{if } t \in [1998{:}8,\ 1998{:}12] \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

This dummy captures the contagion effect of the Russian financial crisis. The Russian financial systemic crisis induced a financial vulnerability on most East Europe stock and bond markets, leading to a decline of the net foreign assets.

2.3 The econometric methodology

In regard to the arguments exposed in the introduction, our aim is to apply a fractionally integrated smooth transition model ($FI-STARMA$) to the error term ε_t in order to capture *simultaneously* the persistence and nonlinear properties of the exchange rate misalignment. In this view, we briefly describe the modelling strategy used in the next sections.

1. Equation (1) is estimated and the resulting residuals $\{\widehat{\varepsilon}_t\}$ are used to test for fractional cointegration.
2. If the fractional cointegration hypothesis is accepted, then an autoregressive fractionally integrated moving average model ($ARFIMA(p, d, q)$) is fitted to the $\{\widehat{\varepsilon}_t\}$ series and selected using usual criteria (information criteria, measures of forecast accuracy,...). Let $\{\widehat{v}_t\}$ be the series of the residuals estimated from the $ARFIMA$ model.
3. The $\{\widehat{v}_t\}$ series is used to test for linearity. In the present case, we test the null hypothesis of a "linear" $ARFIMA$ model against the alternative hypothesis of a $FI-STARMA$ model.
4. If the null is rejected, the parameters of the $FI-STARMA$ model are estimated. Misspecification tests are then applied to residuals.
5. For purpose of comparison, one can also fit other models to the $\{\widehat{v}_t\}$ series. For instance, a $FI-TARMA$ model can be estimated to see whether the nonlinear dynamics is more appropriately described by an instantaneous switching.
6. An interesting question is to characterize the persistence of exchange rate misalignments in presence of nonlinear dynamics, when both properties are combined in the same model. In this paper, we use nonlinear measures of persistence in time series such as nonlinear autocorrelation functions and correlation based on entropy measures.

3 Testing for fractional cointegration and nonlinearity

3.1 Evidence of strong persistence in the misalignment dynamics

We first apply unit root tests to the individual series using augmented Dickey-Fuller and Phillips-Perron approaches[2]. All the variables are found to be $I(1)$, except the interest rate differential

[2] The results are not reported, but are available upon request.

Table 1: Unit root and fractional cointegration tests on residuals

ADF	Lo	GPH (t_d)
-3.51	0.80	-4.95

t_d is the t-statistic of d.

which is $I(0)$. To test for fractional cointegration, we use a two-step procedure. We first estimate the long-run relationship (1) and secondly test whether the estimated residuals follow a fractionally integrated process using Dittmann (2000)'s simulated critical values.

The OLS estimate of the long-run relationship is given by:

$$\begin{aligned} REER_t = &-0.031 - 0.41\ DEF_t + 0.128\ NFA_t + 0.103\ RIRD_t - 0.306\ RELPRICE_t \\ &+ 0.05\ DUM1_t - 0.018\ DUM2_t + 0.026\ DUM3_t, \end{aligned} \tag{5}$$

Prima facie, the regression results seems to be unsatisfactory since many coefficients are wrongly signed. However, in order to interpret these results, it is necessary to test if the estimated residuals have the good statistical properties. Table 1 reports the results of standard augmented Dickey-Fuller (ADF) cointegration test and two fractional cointegration tests: the Lo (1991)'s test and the Geweke and Porter-Hudak (1983)'s test (GPH). Fractional cointegration tests applied to residual series allows us to test the null hypothesis of a unit root ($d' = 1$) against the alternative of fractional integration ($d' < 1$). This is equivalent to a test of the null $d = 0$ against $d < 0$, with $d = d' - 1$ where d' is the fractional difference parameter of the series in levels and d the fractional difference parameter of the series in first differences. The Lo test is sensitive to long-memory and invariant to a general class of short-term memory processes. Since the limiting distribution of the statistic is known (see Lo (1991)), it is possible to test the null hypothesis of short-term memory against the alternative of the long-memory (fractional integration) of the error term. The GPH method exploits the behavior of the spectral density around zero. Geweke and Porter-Hudak (1983) use near zero frequencies and perform a univariate regression of the log-periodogram on the log of the frequencies. The slope estimate is an estimate of the difference parameter d.

According to the results in table 1, the standard ADF cointegration test on the residuals yields to the conclusion that they are $I(1)$. Conversely, using critical values tabulated by Dittmann (2000), fractional cointegration tests lead to conclude that the real exchange rate is fractionally cointegrated with its determinants.

As indicated in the introduction, there are some economic reasons to believe that the exchange rates in Eastern Europe countries have adjusted nonlinearly to their long-run equilibrium values during the era of transition. The rejection of the usual cointegration hypothesis comes from the inappropriateness of linearity assumption that is done when testing the (non)stationarity of $\{\widehat{\varepsilon}_t\}$. The fractional cointegration tests are less distorted in the presence of unspecified nonlinearities.

We then fit an $ARFIMA(p, d, q)$ model to the $\{\Delta\widehat{\varepsilon}_t\}$ series using a two-step approach: we use the GPH method to obtain an estimate $\widehat{d}$ of the parameter d, filter the series $\{\Delta\widehat{\varepsilon}_t\}$ using Diebold and Rudebush (1989)'s approach and finally fit an $ARMA$ model to the filtered series. The best $ARMA$ model is chosen according to information criteria and Theil forecast criteria. We retain an $ARFIMA(3, \widehat{d}, 2)$ with $\widehat{d} = -0.51$. We therefore conclude that the residuals $\{\widehat{\varepsilon}_t\}$ of the long-run model are strongly persistent with an estimate of the long-memory coefficient equal to 0.49. Let $\{\widehat{v}_t\}$ be the series of residuals estimated from the $ARFIMA(3, \widehat{d}, 2)$ model. Our purpose is now to test whether the $\{\widehat{v}_t\}$ series contains nonlinear components.

3.2 Linearity test

According to previous findings in the literature relating to the nonlinear adjustment of exchange rates, we first search for nonlinearities of a $STAR$ type. $STAR$ models have been successfully applied to study the nonlinear persistent mean-reverting dynamics of exchange rates in both developed and transition economies (see MacDonald (1997), Michael, Nobay and Peel (1997), Baum, Barkoulas and Caglayan (2001), Taylor, Peel and Sarno (2001), Taylor and Sarno (2001)).

We assume that the residuals $\{\widehat{\upsilon}_t\}$ of the $ARFIMA$ model follow a $STARMA(p,q)$ process:

$$\widehat{\upsilon}_t = \left[\phi_0^1 + \sum_{i=1}^{p} \phi_i^1 \widehat{\upsilon}_{t-i}\right] F(z_{t-d},\theta) + \left[\phi_0^2 + \sum_{i=1}^{p} \phi_i^2 \widehat{\upsilon}_{t-i}\right] [1 - F(z_{t-d},\theta)] + \left[\sum_{j=1}^{q} \beta_j w_{t-j}\right] + w_t, \tag{6}$$

where w_t is an iid process. z_{t-d} is a transition variable that involves a regime-switching dynamics in the adjustment process and F is either a logistic or an exponential function:

$$F(z_{t-d},\theta) = \{1 + \exp\left[-\gamma(z_{t-d} - c)\right]\}^{-1}, \quad \gamma > 0, \quad \theta = (\gamma, c), \tag{7}$$

$$F(z_{t-d},\theta) = 1 - \exp\left[-\gamma(z_{t-d} - c)^2\right], \quad \gamma > 0, \quad \theta = (\gamma, c). \tag{8}$$

γ is a transition parameter that controls the length of transition between the regimes. c is a threshold value. Here, the candidates for the transition variable will be lagged values of $\Delta\widehat{\varepsilon}_t$. In order to avoid the estimation of too many parameters (regarding the small number of observations), we suppose that only the AR coefficients are subject to regime switching.

The above formulation is equivalent to assume that the $\{\Delta\widehat{\varepsilon}_t\}$ series follows a fractionally integrated $STARMA(p,d,q)$ model ($FI - STARMA(p,d,q)$):

$$\begin{aligned}\Delta\widehat{\varepsilon}_t \quad = \quad & \left[a_0^1 + \sum_{i=1}^{p} a_i^1 (1-L)^d \Delta\widehat{\varepsilon}_{t-i}\right] F(z_{t-d},\theta) + \left[a_0^2 + \sum_{i=1}^{p} a_i^2 (1-L)^d \Delta\widehat{\varepsilon}_{t-i}\right] [1 - F(z_{t-d},\theta)] \\ & + \left[\sum_{j=1}^{q} \beta_j w_{t-j}\right] + w_t \end{aligned} \tag{9}$$

The $FI - STARMA$ model provides a useful way to capture both the persistent and nonlinear properties of the misalignments. According to (9), the dynamics of $\Delta\widehat{\varepsilon}_t$ is dominated in the long-run by a persistent adjustment (which degree is given by the parameter d) and in the short-run by a nonlinear behavior (reflected by the $STARMA$ model). This formulation is analogous to the $FI - STAR$ model introduced by van Dijk, Franses and Paap (2002), the only difference being the moving average coefficients that we add here.

The linearity test concerns the short-term dynamics and amounts to see whether the coefficients differ across the two extreme regimes. The testing methodology is thus very close to the approach used when testing for linearity against usual $STAR$ alternatives[3]. For an extensive presentation of the test when the alternative is a $FI - STAR$ model, the reader is referred to van Dijk, Franses and Paap (2002). Here, we simply describe the main steps:

1. One estimates an $ARFIMA$ model on $\{\Delta\widehat{\varepsilon}_t\}$. Let $\{\widehat{\upsilon}_t\}$ be the corresponding residual series and $SS0$ the sum of squared residuals.

2. One regresses $\{\widehat{\upsilon}_t\}$ on the following auxiliary regressors:

    - the explanatory variables in the $ARFIMA$ model: $(1-L)^d \Delta\widehat{\varepsilon}_{t-i}$ $(i = 1,2,3)$, $\widehat{\upsilon}_{t-j}$ $(j = 1,2)$;
    - the cross-products $\left[(1-L)^d \Delta\widehat{\varepsilon}_{t-i}\right] z_{t-d}^k$, $i = 1,2,3$, $k = 1,2,3,4$;

[3] For an overview of the different linearity tests used in the $STAR$ framework, the reader can consult van Dijk, Franses and Teräsvirta (2002).

Table 2: Linearity tests against a $FI-STARMA$ alternative ($p-$values)

z_{t-d}	LM1	LM2	LM3	LM4
$\Delta\widehat{\varepsilon}_{t-1}$	0.094	0.135	0.169	0.269
$\Delta\widehat{\varepsilon}_{t-2}$	0.307	0.377	0.135	0.164
$\Delta\widehat{\varepsilon}_{t-3}$	0.174	0.01	0.025	0.044
$\Delta\widehat{\varepsilon}_{t-4}$	0.377	0.454	0.353	0.209
$\Delta\widehat{\varepsilon}_{t-5}$	0.045	0.108	0.08	0.017
$\Delta\widehat{\varepsilon}_{t-6}$	0.378	0.107	0.01	0.0015
$\Delta\widehat{\varepsilon}_{t-7}$	0.192	0.153	0.021	0.032
$\Delta\widehat{\varepsilon}_{t-8}$	0.202	0.245	0.279	0.396

- the gradient value of the likelihood with respect to d under the null hypothesis $-\sum_{j=1}^{t-1}(1/j)\widehat{\upsilon}_{t-j}$.

The sum of squared residuals of the regression is noted $SS1$.

3. One computes the following $F-$version of the LM statistic:

$$LM_k = \frac{(SS0 - SS1)/(nd0 - nd1)}{SS1/(nd1)} \tag{10}$$

where $nd0$ and $nd1$ are the number of degrees of freedom of the regressions. Thus, four statistics are computed. Given that the test is based on Taylor expansions of the transition function, significant statistics for odd values of k indicate the presence of nonlinearities of a logistic type, while significant statistics for even values of k mean that the dynamics is of an exponential type.

Table 2 contains the $p-$values of LM statistics for our data. One can remark that the two lowest $p-$values under 5% correspond to $z_{t-d} = \Delta\widehat{\varepsilon}_{t-6}$. Thus, we now proceed to the estimation of a $FI-STARMA$ model with this transition variable.

4 Estimation of FI-STARMA and FI-TARMA models

In order to estimate $FI-STARMA$ models, we have two possibilities. We can apply a maximum likelihood approach directly to equation (9). In this case, the long-memory parameter d and the other parameters are estimated simultaneously. Adapting Sowell (1992)'s exact maximum likelihood estimator to the series $\{\Delta\varepsilon_t\}$ yields problems of local maxima while also being time-consuming. To reduce the dimensionality of the maximum likelihood estimation problem, we choose to use a two-step estimator. Firstly, d is estimated by the GPH method and the series $\{\Delta\widehat{\varepsilon}_t\}$ is filtered to get $\{\widehat{\upsilon}_t\}$ (this corresponds to the first step of the two-step estimation of the $ARFIMA$ modelling). Secondly, we apply a maximum likelihood estimator to the filtered series $\{\widehat{\upsilon}_t\}$ in order to get estimators of the $STARMA$ parameters. We estimate many models with different initial values for the parameters (notably for γ and c). We finally retain the model whose residuals have good properties with regard to several misspecification tests: the lowest residual variance, the Godfrey-Breusch test, the Jarque-Bera test and the Keenan test[4].

The first step gives us a value of the long-memory parameter equal to -0.51 for $\{\Delta\widehat{\varepsilon}_t\}$ (see the preceding section). The second step yields the following best logistic-$STARMA$ estimation

[4]Note that many other misspecification tests have been proposed for $STAR$ specifications, see for instance Eitrheim and Teräsvirta (1996) and Lundbergh and Teräsvirta (1998).

for the filtered series (only the significant parameters are reported)[5]:

$$\widehat{\upsilon}_t = \left[\underset{(9.73)}{1.19}\,\widehat{\upsilon}_{t-1} - \underset{(-4.61)}{0.80}\,\widehat{\upsilon}_{t-2} + \underset{(4.63)}{0.525}\widehat{\upsilon}_{t-3}\right]\widehat{F}(\Delta\widehat{\varepsilon}_{t-6}) - \underset{(-7.36)}{1.103}\,\widehat{\upsilon}_{t-2}\left(1-\widehat{F}(\Delta\widehat{\varepsilon}_{t-6})\right) - \underset{(-18.01)}{1.144}\,w_{t-1} + \underset{(23.54)}{1.13}\,w_{t-2} \quad (11)$$

$$\widehat{F}(\Delta\widehat{\varepsilon}_{t-6}) = \left\{1+\exp\left[-188(\Delta\widehat{\varepsilon}_{t-6} - (\underset{(-8.32)}{-0.016}))\right]\right\}^{-1} \quad (12)$$

$\sigma_w^2 = \underset{(5.67)}{121 \times 10^{-4}}$, $JB = \underset{(0.268)}{2.62}$, $Keenan(1) = \underset{(0.731)}{0.118}$, $Keenan(4) = \underset{(0.68)}{0.17}$, $GB(1) = \underset{(0.068)}{5.36}$, $GB(4) = \underset{(0.116)}{7.38}$

In (11) and (12), the numbers reported between brackets are the $t-$ratios. For the diagnostic statistics on the estimated residuals, the numbers in parentheses are the $p-$values of the tests. σ_w^2 is the residual variance, JB refers to the Jarque-Bera normality test, $Keenan(i)$ is the Keenan linearity test with i lags and $GB(j)$ is the Godfrey-Breusch test with j lags.

These results show that the $FI-STARMA$ model is adequate in the sense that the residuals are uncorrelated and contain no remaining nonlinearities. The extreme regimes are characterized by very different short-term dynamics with only one significant parameter in the second regime. Figure 1 displays the exchange rate misalignment dynamics resulting from the $FI-STARMA$ model. The figure suggests short-term movements around an appreciating smooth trend, especially since 1995 (recall that a decreasing movement of the exchange rate means an appreciation of the currency). The slow appreciating tendency is the consequence of an austerity policy adopted after March 1995 with some efforts to restore internal and external balances. Further, since 1995, the Central bank's main goal has been to sustain disinflation and this has contributed to an appreciation of the real effective exchange rate in a range between 1.5% and 3%. Besides, it is seen that the margin of fluctuations decrease, especially during the last years, thereby corroborating the fact that the observed exchange rate was considered as sustainable and thus less sensitive to shocks.

One important point here is the following. If we omit the long-memory modelling and test the linearity hypothesis directly on the residuals of the long-run equation (using the usual $STAR$ tests), the linearity assumption is strongly accepted, thereby suggesting that the misalignment dynamics contains no nonlinear component. In table 3, the statistics $\overline{LM1}$, $\overline{LM2}$, $\overline{LM3}$, $\overline{LM4}$ refer to the Lagrange multiplier statistics of the usual linearity test against a $STAR$ alternative. The test is applied to the series $\{\Delta\widehat{\varepsilon}_t\}$. As is seen, all $p-$values lie above 5%. One way to explain these results is as follows. In presence of a strong long-memory component (as in our case here), the linear persistence dominates the nonlinear dynamics. In this case, testing the linearity hypothesis on a series which is not filtered from its long-memory component will always lead to reject the nonlinearity hypothesis, although the series may contain nonlinear components. In other words, it is impossible to make the $STARMA$ dynamics apparent, by omitting the long-memory component of the misalignment.

It is worth noting that procedures exist in the literature that allow to separate the permanent component of a time series — here the long-memory component — from its transitory component — here the $STARMA$ components —. For instance, Clarida and Taylor (2001) generalize the standard Beveridge-Nelson approach to the nonlinear case. Our problem here is not to separate these two components, but rather to characterize the persistent dynamics of the misalignment when the long-memory and nonlinear components are combined (this topic is examined in detail in the next section).

The estimated $FI-STARMA$ model produces another interesting feature. This model indeed behaves like a $FI-TARMA$ model, given the high value of the parameter γ, which suggests

[5] We tried many exponential-$STARMA$ specifications, unsuccessfully.

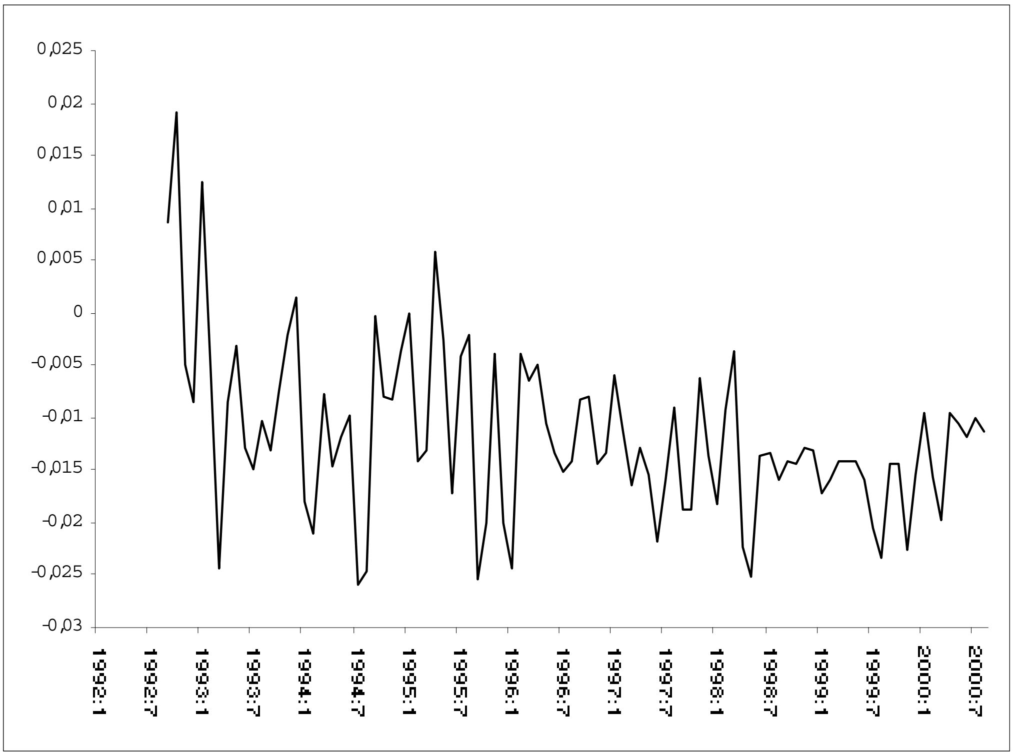

Figure 1: FI-STARMA model. Misalignment dynamics.

Table 3: Linearity tests against a $STAR$ alternative on the residuals of the long-run equation ($p-$values)

z_{t-d}	$\overline{LM1}$	$\overline{LM2}$	$\overline{LM3}$	$\overline{LM4}$
$\Delta\widehat{\varepsilon}_{t-1}$	0.897	0.944	0.906	0.968
$\Delta\widehat{\varepsilon}_{t-2}$	0.995	0.954	0.790	0.812
$\Delta\widehat{\varepsilon}_{t-3}$	0.949	0.491	0.683	0.686
$\Delta\widehat{\varepsilon}_{t-4}$	0.998	0.999	0.982	0.943
$\Delta\widehat{\varepsilon}_{t-5}$	0.924	0.952	0.947	0.980
$\Delta\widehat{\varepsilon}_{t-6}$	0.942	0.385	0.296	0.928
$\Delta\widehat{\varepsilon}_{t-7}$	0.582	0.300	0.410	0.999
$\Delta\widehat{\varepsilon}_{t-8}$	0.780	0.781	0.806	0.882

Table 4: Tsay test on $\{\widehat{\upsilon}_t\}$ ($p-$values)

z_{t-d}	$\Delta\widehat{\varepsilon}_{t-1}$	$\Delta\widehat{\varepsilon}_{t-2}$	$\Delta\widehat{\varepsilon}_{t-3}$	$\Delta\widehat{\varepsilon}_{t-4}$	$\Delta\widehat{\varepsilon}_{t-5}$	$\Delta\widehat{\varepsilon}_{t-6}$	$\Delta\widehat{\varepsilon}_{t-7}$	$\Delta\widehat{\varepsilon}_{t-8}$
$p-$value	0.328	0.312	0.0375	0.7095	0.5285	0.917	0.620	0.087

instantaneous regime-switching. To see whether such a process is adequate for our data, we apply Tsay (1989)'s test based on arranged autoregressions on the filtered series $\widehat{\upsilon}_t = (1-L)^{-0.51}\Delta\widehat{\varepsilon}_t$. The results are shown in table 4.

The smallest $p-$value in table 4 corresponds to the transition variable $z_{t-d} = \Delta\widehat{\varepsilon}_{t-3}$. The best estimated $FI-TARMA$ model gives the following results:

$$\begin{aligned}\widehat{\upsilon}_t &= \left[\underset{(-7.95)}{-0.565}\widehat{\upsilon}_{t-1} \underset{(-9.67)}{-0.702}\,\widehat{\upsilon}_{t-2} + \underset{(3.93)}{0.364}\widehat{\upsilon}_{t-3}\right]\widehat{F}(\Delta\widehat{\varepsilon}_{t-3}) - \left[\underset{(2.23)}{0.003} \underset{(9.30)}{+0.956}\,\widehat{\upsilon}_{t-2}\right]\left(1-\widehat{F}(\Delta\widehat{\varepsilon}_{t-3})\right) \\ &\quad \underset{(47.60)}{+0.741}\,w_{t-1} \underset{(16.97)}{+1.25}\,w_{t-2} + \underset{(6.17)}{0.133}w_{t-3} \end{aligned} \tag{13}$$

$$\widehat{F}(\Delta\widehat{\varepsilon}_{t-3}) = \begin{cases} 1, & \text{if } \ \Delta\widehat{\varepsilon}_{t-3} \leq 0 \\ 0, & \text{otherwise} \end{cases} \tag{14}$$

$\sigma_w^2 = \underset{(6.38)}{0.107} \times 10^{-4}$, $JB = \underset{(0.367)}{2.004}$, $Keenan(1) = \underset{(0.74)}{0.111}$, $Keenan(4) = \underset{(0.926)}{0.008}$, $GB(1) = \underset{(0.003)}{9.004}$, $GB(4) = \underset{(0.02)}{11.95}$

As before, $t-$ratios are given in parentheses for (13) and $p-$values are between brackets for the diagnostic statistics. As is seen, we obtain a smallest residual variance σ_w^2 for the $FI-TARMA$ model, which suggests that this model would be more adequate to describe the misalignment of the Hungarian currency.

5 Characterizing the nonlinear persistence of the misalignment

As a consequence of the combination of long-memory and $STARMA$ (or $TARMA$) specifications, this section addresses the important issue of the characterization of the persistent dynamics (movements of the exchange rate around a sustained appreciation trending) in presence of a nonlinear behavior. Several measures can be used, which are summarized in Dufrénot and Mignon (2002). Here, we use two indicators: a nonlinear autocorrelation function and an entropy-based persistence measure.

5.1 Nonlinear autocorrelation function

Several measures of nonlinear autocorrelation functions have been proposed in the empirical literature (see, among others, Cox (1991), Granger and Teräsvirta (1992)). Here, we use Cox (1991)'s indicator. The basic idea is to study the memory of a time series by exploiting the information contained in the moments and cumulants of orders higher than the second. Cox suggests the following nonlinearity index for a time series $\{X_t\}$:

$$NL(\tau) = \left[\frac{\rho_{21}(\tau)}{\rho_{11}(\tau)} - \rho_3\right]^2 \ / \ \left[\rho_4 + 2 - \rho_3^2\right], \tag{15}$$

where

$$\rho_r = c_r \ / \ c_2^{r/2}, \quad \text{with } c_r = E\left[X_t\right]^r, \tag{16}$$

Table 5: Linear and nonlinear autocorrelation functions

	Standard ACF		*Nonlinear ACF*	
Lag	*FI-STARMA*	*FI-TARMA*	*FI-STARMA*	*FI-TARMA*
1	0.25*	0.61*	-0.10	-0.52*
2	-0.15	0.07	0.24*	-0.05
3	0.31*	-0.28*	-0.19	0.30*
4	0.24*	-0.34*	-0.15	0.35*
5	-0.18	-0.15	0.21*	0.16
6	0.06	0.12	-0.06	-0.13
7	0.22*	0.19	-0.17	-0.17
8	-0.08	0.11	0.12	-0.03
9	-0.13	0.03	0.14	0.04
10	0.14	0.01	-0.17	-0.01
20	0.09	-0.23*	-0.11	0.38*
30	0.09	0.17	-0.03	-0.28*

*: significant coefficient at the 5% significance level.

$$\rho_{11} = corr(X_t, X_{t\pm\tau}), \tag{17}$$

$$\rho_{21} = corr(X_t^2, X_{t\pm\tau}), \tag{18}$$

Equation (15) includes the third- and fourth-order marginal cumulants, which are both important to characterize the nonlinear components of a time series. A comparison of the standard autocorrelation function and this nonlinear autocorrelation function has been made for $ARFIMA$ models and long-range nonlinear processes (see Dufrénot and Mignon (2002)). Using Monte Carlo simulations, the authors show that for $ARFIMA$ models with a highly significant value of the parameter d (around 0.45), the standard autocorrelation function exhibits a slow decrease as the lag τ is augmented, while the nonlinear autocorrelation function has no significant coefficients. Further, long-memory nonlinear models have a long-range dependence that is detected by both the usual and the nonlinear autocorrelation function.

From the estimated $FI{-}STARMA$ model, we deduce a series of misalignment of the Hungarian Forint and use it to compute both the linear and nonlinear autocorrelation functions (see table 5). We also report results for the $FI-TARMA$ model. It is seen that both models exhibit some dependence structure. This is particularly true for the $FI-TARMA$ process which shows a long-memory dependence structure, since we have significant coefficients for high lags.

5.2 Long-range dependence based on entropy measures

Another way to identify the degree of persistence in the memory of the estimated $FI-STARMA$ model is to use indicators based on entropy measures. Such kind of indicators have been proposed in order to obtain the so-called "shadow" autocorrelation functions (see, for instance, Granger and Lin (1994), Escribano and Aparicio (1997)).

Let us consider a time series $\{X_t\}$ and the mutual information function defined as

$$I(X_t, X_{t\pm\tau}) = H_t(X_t) + H_\tau(X_{t\pm\tau}) - H(X_t, X_{t\pm\tau}), \tag{19}$$

where $H(\cdot)$ is the Shannon entropy, that is

$$H_j(X_j) = -\int f_j(X_j) \ln [f_j(X_j)]\, dX_j, \quad j = t, t \pm \tau, \tag{20}$$

and

$$H(X_t, X_{t\pm\tau}) = -\int\int f(X_t, X_{t\pm\tau}) \ln [f(X_t, X_{t\pm\tau})]\, dX_t dX_{t\pm\tau} \tag{21}$$

Table 6: Entropy-based autocorrelation functions

Lag	*FI-STARMA*	*FI-TARMA*
1	0.77*	0.95*
2	0.84*	0.68*
3	0.70*	0.86*
4	0.51*	0.39
5	0.80*	0.86*
6	0.63*	0.72*
7	0.78*	0.85*
8	0.32	0.64*
9	0.39	0.76*
10	0.51*	0.18
20	0.62*	0.18
30	0.66*	0.77*

*: significant coefficient at the 5% significance level.

$f_t(X_t)$ and $f_{t\pm\tau}(X_{t+\tau})$ are the marginal densities of $\{X_t\}$ and $\{X_{t\pm\tau}\}$ and $f(X_t, X_{t\pm\tau})$ is their joint density. The Shannon entropy is used to determine the fraction of information in $X_{t\pm\tau}$ that helps reducing the uncertainty inherent to X_t. A strong dependence implies that $I(X_t, X_{t\pm\tau})$ tends to infinity when $\tau \rightarrow +\infty$. In finite samples, long-range dependence implies that the mutual information function is statistically significant for high values of τ. Table 6 reports the entropy-based correlation function, which correctly detects the long-memory property of the $FI-STARMA$ and $FI-TARMA$ model. Indeed, a great number of coefficient are significantly different from zero, meaning that the speed of adjustment towards the fundamental value of the Hungarian real exchange rate might be very slow.

6 Conclusion

This paper has proposed an estimate of the misalignment of the Hungarian real exchange rate during the 1992-2000 period. To account for the nonlinear adjustment of the real exchange rate towards its fundamentals, a nonlinear cointegration approach has been retained. More specifically, in order to capture simultaneously the persistence and nonlinear properties of the exchange rate misalignment, long-memory regime-switching models have been proposed. Two models have been estimated: a long-memory smooth transition model ($FI-STARMA$) and a long-memory model with instantaneous switching ($FI-TARMA$). According to estimation results, the $FI-TARMA$ process appeared to be more adequate to describe the misalignment of the Hungarian currency. These results illustrate that two types of persistence have to be taken into account in order to explain the misalignment of the Hungarian real exchange rate: a permanent component due to the influence of real factors and a nonlinear component, where persistence is associated with time-dependent effects.

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