Effects of STAR and TAR types nonlinearities on order selection criteria

Venus Khim-sen Liew[a] and Terence Tai-leung Chong[b,*]

[a] *Faculty of Economics and Management, Universiti Putra Malaysia, 43400 Serdang, Malaysia*
[b] *Department of Economics, The Chinese University of Hong Kong, Shatin, N. T., Hong Kong.*

Completed 15 February 2003. Revised 20 February 2003.

Abstract

This paper investigates via a simulation study the effects of nonlinearities on several commonly used order selection criteria. The most important finding of this study is that SIC, FPE, HQC and BIC perform considerably well in estimating the true autoregressive order, even in the presence of STAR or TAR nonlinearity. Thus we conclude that these criteria may be safely applied to determine the true order of STAR or TAR process.

Keywords: STAR process, TAR process, AR process, nonlinearities, order selection criteria

JEL classification: C22; C51

---

1. Introduction

There are ample of evidence on the presence of Smooth Transition Autoregressive (STAR) (Granger and Teräsvirta, 1993) and Threshold Transition (TAR) (Tong, 1990) types nonlinearities in economics time series (Henry et al., 2001; Taylor et al., 2001; van Dijk et al., 2002). The selection of autoregressive (AR) order p, normally based on certain selection criteria (Brockwell and David, 1996), is an important process in the modeling cycles of these nonlinear time series. The practicability of applying these criteria, which were originally proposed on the basis of linear frameworks, is of great interest as it has crucial implications on the eventually selected models. Başçi and Zaman (1988) and Liew and Chong (2003) have done simulation study to see the effects of nonnormal and ARCH errors on various order selection criteria. However, to the best of our knowledge, the effects of nonlinearities on these order selection criteria are yet to be determined. As such, the main objective of this study is to investigate, via simulation study, the effects of STAR and TAR types nonlinearities on the performance of some selected criteria in picking up the true autoregressive order p.

The most important finding of this study is that SIC, FPE, HQC and BIC perform considerably well in estimating the true autoregressive lag length, even in the presence of STAR or TAR nonlinearity.

2. STAR and TAR nonlinearities and order selection criteria

The STAR process of a series X_t ($t = 1, \ldots, T$) can be expressed as

[*] Corresponding author. Tel: +852-26-098-193;fax: +852-26-035-805.
E-mail address: chong2064@cuhk.edu.hk (T.T.-l. Chong)

$$X_t = (c + \sum_{i=1}^{m} \phi_i X_{t-i}) + (c^* + \sum_{i=1}^{n} \phi_i^* X_{t-i}) G + \varepsilon_t \quad (1)$$

where c and ϕ's are constant and autoregressive parameters to be estimated, with asterisk (*) stands for nonlinear parameters. G is a transition function of certain threshold values ((Granger and Teräsvirta, 1993), taking values in the range of (0.0, 1.0) by construction. ε_t are normally distributed random error terms with a zero mean and a finite variance σ^2.

STAR process allows a variable under investigation to adjust smoothly every moment within different regimes with the speed of adjustment depends on the size of the threshold values. Meanwhile, TAR and AR are a special cases of STAR process, whereby in the former case, transition function G takes discrete values of either zero or one and in the latter case, G consistently equals zero throughout the series.

Note that the linear autoregressive order m and nonlinear autoregressive order n in (1) are always unknown in real observations and have to be determined in advance. Practically, empirical researchers mostly assume *in priori* that $m = n = \hat{p}$, whereby $\hat{p}$ is determined by certain order certain selection criteria developed based on linear AR framework. We are interested to know the performance of these criteria in estimating the true order of the STAR or TAR.

To accomplish our task, we simulate STAR and TAR processes of $m = n = 2$. We generate ϕ_i and ϕ_i^*, i =1, 2 from a uniform distribution in the region (–0.25, 0.25) respectively. The choice of this region allows us to avoid undesired nonstationary process. Meanwhile c is generated from an arbitrary selected uniform distribution in the region (–100,100). For STAR process, we generate values of G from a uniform distribution in the region (0,1), whereas for TAR process, G takes the values of zeros and ones randomly. A more specific case of STAR process, namely the AR process is simulated by equating $G = 0$. We simulate data sets for various sample sizes: 25, 50, 100, 250, 500,1000, 2500, 5000, 10000 and 100000. For each combination of processes and sample sizes, we simulated 1000 independent series for the purpose of order estimation. The estimated order $\hat{p}$ can be any integer from 1 to 20.

The lag length selection criterion to be evaluated include (1) Akaike information criterion, $\text{AIC}_p = \ln(\hat{\sigma}_p^2) + 2p/T$; where $\hat{\sigma}_p^2 = (T-p-1)^{-1} \sum_{t=p}^{T} \hat{\varepsilon}_t^2$; (2) Akaike's information corrected criterion, $\text{AICC}_p = T[\ln(\hat{\sigma}_p^2)] + T[1-(p-2)/T]^{-1}[1+(p/T)]$; (3) Schwarz information criterion, $\text{SIC}_p = \ln(\hat{\sigma}_p^2) + [p \ln(T)]/T$; (4) Hannan-Quinn criterion, $\text{HQC}_p = \ln(\hat{\sigma}_p^2) + 2T^{-1} p \ln[\ln(T)]$; (5) the final prediction error, $\text{FPE}_p = \hat{\sigma}_p^2 (T-p)^{-1}(T+p)$ and (6) Bayesian information criterion, $\text{BIC}_p = (T-p) \ln[(T-p)^{-1} T \sigma_p^2] + T[1 + \ln(\sqrt{2\pi})] + p \ln[p^{-1}(\sum_{t=1}^{T} X_t^2 - T\hat{\sigma}_p^2)]$. Interested readers are referred to Brockwell and Davis (1996) and Başçi and Zaman (1998) and the references therein for more details.

The probability of estimating the true lag length by each of these criteria is determined. We also investigate the distribution of the estimated order, so as to gain more insights on the behavior of various criteria.

3. Simulation results

The probability of various criteria in correctly estimating the true order of the STAR, TAR and AR process are tabulated in Table 1. Generally, Table 1 shows that SIC, FPE, HQC and BIC (but not AICC and AIC, which show identical results everywhere) perform considerably well in estimating the true autoregressive lag length, in all cases. Table 1 also revealed that the performance of order selection criteria involved generally improves as the sample size increases; this is not a surprising result. Perhaps, most importantly, Table 1 shows that there is no distinct difference in the performance of these criteria in correctly estimating the true lag order p for all processes. For instance, the probability score of BIC in correctly estimating the true order of are 90.2, 89.6 and 88.8, in that order, for AR, TAR and STAR processes with T = 100000. A major implication of this finding is that these criteria, which are originally proposed for the estimation of linear AR process, works equally well in TAR or STAR types nonlinear process. Note that in similar studies, Liew and Chong (2003) concluded that these criteria perform well even in the presence of heteroscedastic errors, whereas Başçi and Zaman (1998) contented that these criteria are affected by kurtosis. Thus, the current finding lends us empirical support on the use of conventional order selection criteria in estimating the true order of nonlinear autoregressive processes.

Table 1
Probability of correctly estimating the true order of various processes

		Sample sizes, T									
Processes	Criteria	25	50	100	250	500	1000	2500	5000	10000	100000
AR	AICC	17.4	18.2	17.8	19.5	19.5	17.8	20.5	20.1	22.2	22.2
	AIC	17.4	18.2	17.8	19.5	19.5	17.8	20.5	20.1	22.2	22.2
	SIC	62.8	66.1	65.8	68.0	69.2	72.2	77.6	82.6	82.1	86.4
	FPE	67.5	69.0	67.7	72.2	73.3	75.4	74.6	79.2	78.5	84.0
	HQC	69.0	71.5	70.3	73.1	75.4	76.9	80.2	85.8	84.5	88.6
	BIC	70.7	73.1	71.5	75.2	76.6	78.8	81.4	86.4	85.8	90.2
TAR	AICC	19.0	18.6	18.0	17.4	18.7	17.2	18.5	17.6	19.7	21.4
	AIC	19.0	18.6	18.0	17.4	18.7	17.2	18.5	17.6	19.7	21.4
	SIC	64.9	66.3	64.3	65.7	67.2	73.2	78.9	81.0	84.0	85.8
	FPE	70.8	72.9	69.5	72.2	68.1	72.7	77.9	78.9	81.2	81.6
	HQC	70.6	72.4	69.2	71.4	72.2	78.1	82.0	83.9	87.9	88.2
	BIC	73.6	75.8	72.4	74.9	73.6	79.2	83.8	86.2	88.9	89.6
STAR	AICC	19.7	17.4	17.4	18.2	19.5	18.0	18.3	20.3	20.1	21.0
	AIC	19.7	17.4	17.4	18.2	19.5	18.0	18.3	20.3	20.1	21.0
	SIC	64.5	65.8	65.4	65.6	69.2	73.8	80.0	82.4	83.5	85.0
	FPE	68.0	69.6	68.7	67.5	74.6	72.6	78.7	79.4	80.0	81.0
	HQC	70.6	72.1	70.1	70.0	74.2	76.8	82.8	85.4	86.7	87.4
	BIC	72.2	73.7	70.8	72.3	78.0	79.2	84.2	87.0	88.3	88.8

An analysis on the distribution of estimated order for various criteria revealed that AICC and AIC has severely over-estimated the true order in all process. For example, for T=25, (Table 2) the last finding is true even if the sample size is practically large enough. In contrast, SIC, FPE, HQC and BIC are found to have under-estimated the true order with a probability of around one-third.

Table 2
The distribution of estimated order for various criteria

Criteria	Estimated order																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
AICC	40	197	64	50	36	45	43	44	35	33	45	28	32	35	34	38	30	46	54	71
AIC	40	197	64	50	36	45	43	44	35	33	45	28	32	35	34	38	30	46	54	71
SIC	351	645	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
FPE	222	682	55	24	7	6	1	2	1	0	0	0	0	0	0	0	0	0	0	0
HQC	290	686	20	3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
BIC	225	713	32	37	2	1	0	1	0	0	0	0	0	0	1	0	1	0	4	13

Notes: STAR process, T=25, B=1000. Results for other processes and sample sizes provide similar information and hence are not shown.

The most interesting fact uncovered by Table 2 is that the mode of the distribution is exactly the true order. This gives us the intuitive that it would be possible for one to correctly estimate the order of a given series with a probability of one via bootstrapping, that is, the order can be determined from the mode of the resulted bootstrap distribution of estimated order. The authors currently undertake a research along this direction.

To sum up, this study finds that conventional AR order selection criteria may be safely applied the in determining the true order of STAR or TAR process.

References

Başçi, S., Zaman, A., 1998. Effects of skewness and kurtosis on model selection criteria. Economic Letters 59, 17–22.

Brockwell, P. J., Davis, R. A., 1996. Introduction to Time Series and Forecasting. Springer, New York.

Engle, R. F., 1982. Autoregressive conditional heteroscedasticity with estimates of the variance of UK inflation. Econometrica 50, 987–1008.

Granger, C. W. J. and Teräsvirta, T. 1993. Modelling Nonlinear Economic Relationships. Oxford University Press, Oxford.

Henry, O. T., Olekalns, N. and Summers, P. M., 2001. Exchange rate instability: a Threshold Autoregressive approach. The Economic Record, 77, 160–166.

Liew, V. K. S., and Chong, T. T. L., 2003. Effects of ARCH errors on autoregressive lag length selection criteria. Preprint.

Tong, H., 1990. Non-linear Time Series. Oxford University Press, Oxford.

Taylor, M.P., Peel, D.A., Sarno, L., 2001. Nonlinear mean-reversion in real exchange rates: towards a solution to the purchasing power parity puzzles. International Economic Review, 42, 1015–1042.

van Dijk, D., Teräsvirta, T., Franses, P.H., 2002. Smooth transition autoregressive models: a survey of recent developments. Econometric Reviews, 21, 1–47.