

# **The P\* model as a general identity to analyze and forecast the behavior of the inflation rate in the economy of Puerto Rico**

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## **Abstract**

**In this work the P\* model is used to analyze and forecast the inflation rate in the economy of Puerto Rico. This model is based on two essential points: the first one is to identify the inflationary potential of an economic system through the estimation of the price level to which the inflation tends to adjust in the long run. The second, points that the price level will be adjust, in the long run, to the forecast of the model. Given the way in which the monetary sector in Puerto Rico its constituted, the model needs to complement with U.S.A. monetary variables , such as, monetary supply, to forecast the inflation. The results indicate a long run relationship between the monetary supply of United States (M1) and the price level, the real production and the island's preferential interest rate. The final model is a good representation of the generating process of information (GPI) and it could be used for forecasting purposes. The same predicts the development of inflation better than the two ARIMA models previously selected.**

## **I. Introduction**

The problem with inflation is one that affects all the countries in the world. As such, it has become a primary objective in monetary policy many countries, especially in Latin America. The adverse effects of the high inflation levels in investment and on the level of consumption become key essential problems today.

In spite of the before mentioned, a consensus on the main causes of inflation and ways to control it, forecast and how to analyse it in the decision making process does not exist. Recently, some works have risen (Hall and Mine, 1994; Hallman and Anderson, 1995; Orphanides and Porter, 1998; Galindo, 1997) which point out towards the P\* model as an indicator of the evolution of the magnitudes of price levels and the expected inflation. These models arise in face of the necessity of estimating a relationship between a monetary aggregate and the potential production as a long run indicator of the tendencies in prices. They have also been used to carry out forecasts on the behaviour of prices or as a basis to analyse the feasibility of economic policy as nominal anchors to control prices (Allen and Hall, 1991). The advantage of this model is its capacity to forecast prices based on simple rules on the behaviour of production and a monetary aggregate, under the assumption that the circulation velocity is constant or that at least it can gagged using the opportunity costs of money appropriately (that is to say the relevant interest rate) (Orphanides and Porter, 1998).

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This model seeks to identify the inflationary potential of an economic system given the estimation of the price level to which the inflation tends to adjust over the long run, since these are a function of a certain quantity of money in circulation (Galindo, 1997). The above-mentioned is based on that the circulation velocity of money and that the potential production correspond at equilibrium levels in the long run. Permanent changes in the circulation velocity of money or in the production potential generate permanent divergences in the real level of prices *vis a vis* the forecasted one.

It also indicates that the prices will adjust, in the long run, to those forecasted by the model. This way, the estimated value can be used as an anchor on prices and it becomes in a simple and quick indicator to identify the monetary conditions and its relationship with the inflation of a particular country.

In spite of the results hurtled in other countries, some authors (Orphanides and Porter, 1995; Arnold, 1995) point out that the effects of causality between prices and the monetary aggregate under study ought to be considered. Also, they question if the execution of the quantitative equation of money is given as a function of expectations, which can affect the level of transactions and hence, the circulation velocity of money.

For the case of Puerto Rico, the monetary supply is determined in an automatic way by the market forces and not by the government nor by any autonomous entity of the island.<sup>1</sup> Given their monetary dependence with the United States, the island does not have absolute control of the evolution of the inflation and could not establish anchors of prices by means of the monetary policy. Also, given that the existent relationship between the two economic systems, an effect of causality of the prices in Puerto Rico towards the US monetary supply does not exist.

However, in spite of the previous considerations, the P\* model could be utilized as a general identity to analyze the behavior of the inflation rate and their inertial effect based on the use of an error correction model. Besides, several works indicate that inflation, in Puerto Rico, is a monetary phenomenon. Toledo (2000) indicates that inflation, in Puerto Rico, is a monetary phenomenon in both short run and long run. It also found the existence of a cointegration relationship between the inflation in Puerto Rico and the monetary supply in the United States. Similar results were also demonstrated by Rodríguez (2002) by analyzing the economy of Puerto Rico by means of a reduced form of an IS-LM model.

The objective of this work is to analyse the application of this model for the economy of Puerto Rico. In the next section, a general presentation of the P\*, adapted to the economy of Puerto Rico will be generated. Then, the empirical evidence is presented and lastly, the main conclusions of study.

## **II. Theoretical Framework**

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<sup>1</sup> Also, the economic system of Puerto Rico, it is not affected significantly by international economic instability and external flow of capital because its capital markets it is highly integrated to that of United States. Although it is necessary to point out that a possibility exists for problems with the balance of payments, due to the non productive use of imported capital.

The P\* model is based on the feasibility of using a monetary aggregate with unitary elasticity with respect to prices as an indicator of their tendency in the long run. This can be found in the work of Rodríguez (2002) when using the monetary supply (M1) of the United States.<sup>2</sup> In this case, the P\* model arises from the quantitative equation of money (Galindo, 1997; Orphanides and Porter, 1998) , but using the United States monetary supply:

$$M_t V_t = P_t Y_t \quad (1)$$

where:

$M_t$  = the monetary supply of the United States;

$V_t$  = the money circulation velocity in Puerto Rico;

$P_t$  = the level of prices of Puerto Rico;

$Y_t$  = the national production of Puerto Rico.

The equilibrium level of prices (P\*) can be obtained from (1):

$$P^*_t = M_t V_t / Y_t \quad (2)$$

Applying logarithms, the estimation of (2) its general form becomes:

$$p_t = \beta_1 m_t + \beta_2 y_t + \beta_3 v_t + \varepsilon_t \quad (3)$$

In this equation, it is expected that:  $\beta_1 = 1$ ,  $\beta_2 = -1$  and  $\beta_3 = 1$ . It also presents a simple rule to forecast the behaviour of prices, assuming that the velocity of money is constant. Therefore this makes it possible to estimate equation (3) with other variables and, if the obtained parameters match the expected, then the future value of the prices is calculated. The equation (3) can be rewritten as:

$$p^*_t = \beta_1 m_t + \beta_2 y_t + \beta_3 z_t + \varepsilon_t \quad (4)$$

In this equation  $z_t$  is a vector that includes the necessary variables to approach the velocity of circulation. The available evidence for the case of Puerto Rico suggests the existence of a long run stable relationship between the monetary supply of the United States and the real production, the level of prices and the prime rate of Puerto Rico (Rodríguez, 2002). In this case the prime rate is presented as an approximation approach of the circulation velocity of money:

$$p_t = \beta_1 m_t + \beta_2 y_t + \beta_3 r_t + \varepsilon_t \quad (5)$$

in the one which:  $\beta_1 = 1$  and  $\beta_2 = -1$  and  $\beta_3 = 1$  ( Hall and Mine, 1994; Bordes, Girardin and. Marimoutou , 1993).

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<sup>2</sup> Although that the variable M1, is a saving and transactions instrument, it shows a great structural stability with the variables of study.

The equation (5) can be estimated through the use of the Johansen procedure (1988). This allows for the analysis of the relationships of causation and exogeneity of the series, eliminating the bias in the estimators. If there no bias in the parameters and the estimators are efficient, this relation could be considered as an error correction model that captures the short run fluctuations and the dynamics of long run adjustment before any unanticipated shocks.

This produces a  $P^*$ , which can be compared with the observed value, whereby the gap between them is the cointegration error obtained by the Johansen procedure (1988). So that the short run dynamics of this relationship can be represented in an error correction model:<sup>3</sup>

$$\Delta p_t = \alpha_{11}(p - p^*)_{t-1} + \alpha_{12}\Delta p_{t-1} + \alpha_{13}m_{t-1} + \alpha_{14}\Delta y_{t-1} + \alpha_{15}\Delta r_{t-1} + \xi_t \quad (6)$$

This means that the  $P^*$  model takes advantage of the common tendencies between the series by means of the Johansen procedure to forecast the price level. The lagged difference terms capture the effects of the series, for example; the transportation costs, short term changes in production and in the financial sector and, other peculiarities of the economy of the island, which affect prices in the short run. The model can also include the effects between the gaps of the real production and the potential production, which can be associated to a change in prices as a direct result of pressures over the installed capacity or to the differential between the level unemployment and potential production (Kuttner, 1989; Galindo, 1997). That is to say that, the changes in the level prices, can have real effects, which can be transmitted by several periods. In this case, it can be considered the following model:

$$\Delta p_t = \alpha_{11}(p - p^*)_{t-1} + \alpha_{12}(y - y^*)_{t-1} + \alpha_{13}p_{t-1} + \alpha_{14}\Delta m_{t-1} + \alpha_{15}\Delta y_{t-1} + \alpha_{16}\Delta r_{t-1} + \mu_t \quad (7)$$

In the case of Puerto Rico, the equilibrium price level is given by (5) and in the above-mentioned function when substituting in (7), yields:

$$\Delta p_t = \alpha_{11}[p - (\beta_1 m + \beta_2 y + \beta_3 r)]_{t-1} + \alpha_{12}[y - (\beta_1 m + \beta_2 y + \beta_3 r)]_{t-1} + \alpha_{13}p_{t-1} + \alpha_{14}\Delta m_{t-1} + \alpha_{15}\Delta y_{t-1} + \alpha_{16}\Delta r_{t-1} + v_t \quad (8)$$

The dynamics of inflation is modeled assuming that any the increments in the monetary supply of the United States generate increments in the price index of Puerto Rico. Nevertheless the presence of idle capacity leaves:

$$(p - p^*)_t = (v - v^*)_t + (y - y^*)_t \quad (9)$$

This equation indicates that the deviations in the observed price level and forecasted price level should be compensated with fluctuations in the velocity of money and the

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<sup>3</sup> Since only annual data for production exists, an annual periodicity will be used. If prices need to be forecasted monthly or quarterly, the utilized variable (parameter) can be replaced by any measure which is a good indicator of the short run economic activity.

total production (Galindo, 1997). Therefore, a cointegration error (in the event of this existing relationship) and the reduced form of the dynamic equation of prices needs to be estimated, using an error correction model.

Inflation ( $gp_t$ ) can be modeled considering that it tends to its equilibrium value. For that, the growth rate of inflation ( $ggp_t$ ) can be estimated in its simpler form (Galindo, 1997):

$$ggp_t = \phi_{ij}(p - p^*)_{t-1} \quad (10)$$

This equation indicates that the rate of growth of inflation accelerates when  $p^* > p$  and vice versa. In this case, the general form of the model of price gap can be solved :

$$ggp_t = \delta_{11}(p - p^*)_{t-1} + \sum_{i=1} \delta_{12}(i)\Delta ggp_{t-i-1} + \varphi_{1t} \quad (11)$$

The advantage in the case of Puerto Rico is that all the variables in (11) are  $I(0)$ . This model can be expressed in an equivalent form for inflation (Hallman, Porter and Small, 1991; Orphanides and Porter, 1998):

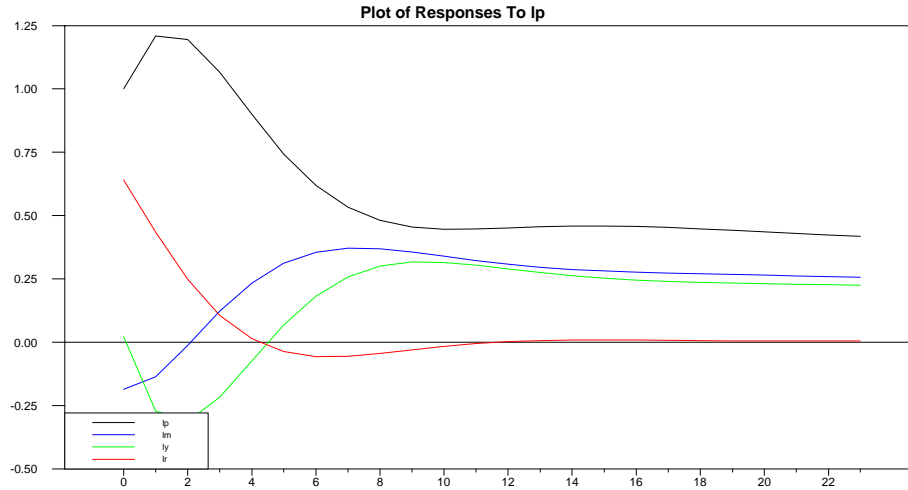
$$gp_t = \gamma_{11}(p - p^*)_{t-1} + \sum_{i=1} \gamma_{12}(i)\Delta gp_{t-i-1} + \sum_{i=1} \gamma_{13}(i)\Delta z_{t-i-1} + \zeta_{1t} \quad (12)$$

### **III. Empirical Evidence**

The series to be used are of annual periodicity, dating from 1964 to 1997. The consumer price index (li), the real production level (ly), the prime rate (lr) and the monetary supply (M1) of the United States (lm), will be the variables for use in the P\* model. All the variables are logarithmic.

With this model the only equation under scrutiny is prices, since it is the only one that will be used for forecasting purposes. From this the cointegration error will be obtained and another model will be generated to forecast the inflation as it appears in the equation (12). To prove the efficiency of the model, it will be compared with an ARMA(1,1) and an ARMA(2,1). This will be carried out comparing the root mean squared error from each model.

As one can observe, in the impulse-response function of this equation, prices react, in a transitory form, to unanticipated shocks of the variables in the model. This effect spreads over several months. The variance decomposition indicates that the real shocks of the real production are those that have the greatest impact over the forecast variance error of the price level. The real production affects the evolution of the series over the whole forecast horizon by approximately fifty percent. But, this does not eliminate the relative importance of the monetary supply of the United States, the same price index and the preferential interest rate, which also affects the forecast variance error of the price level by approximately a 15, 20 and 9 percent, over a 10-year period. That is to say that price movements are entirely endogenous.



**Table I**  
**Variance decomposition tests for the price equation**

Step	Est. error	$lp_t$	$ly_t$	$lm_t$	$lr_t$
1	0.016369586	100.00000	0.00000	0.00000	0.00000
2	0.028032094	83.97635	9.97321	3.75543	2.29501
3	0.039189461	67.86848	22.31344	7.88901	1.92908
4	0.049479836	55.00383	32.76520	11.01958	1.21139
5	0.058783561	45.24709	40.45778	13.05443	1.24071
6	0.067063509	38.05654	45.66811	14.23217	2.04319
7	0.074331298	32.83830	49.03009	14.84108	3.29052
8	0.080651923	29.06397	51.16090	15.11458	4.66054
9	0.086136848	26.31674	52.52809	15.20971	5.94547
10	0.090923652	24.29066	53.44154	15.22013	7.04768
11	0.095153036	22.76941	54.09009	15.19657	7.94394
12	0.098951400	21.60218	54.58271	15.16407	8.65104
13	0.102421673	20.68403	54.98037	15.13364	9.20195
14	0.105641330	19.94193	55.31678	15.10914	9.63214
15	0.108665014	19.32532	55.61062	15.09105	9.97301
16	0.111529265	18.79957	55.87234	15.07845	10.24964
17	0.114257588	18.34135	56.10796	15.06997	10.48072
18	0.116864878	17.93513	56.32117	15.06428	10.67942
19	0.119360811	17.57060	56.51447	15.06032	10.85461
20	0.121752205	17.24079	56.68986	15.05730	11.01205

These results are of great importance, nevertheless that the order of integration of the prime rate differs from the rest of the variables. In this case, several multiple solutions may arise over the long-run, since linear combinations of series I(1) would generate I(0) series (Rodríguez, 2002).

**Table II**  
**Order of integration of the series by means of the Dickey-Fuller (ADF)<sup>a</sup> and Phillips-Perron (PP) tests**

Variable	ADF	PP
$p_t$	-1.754	-1.055
$Dp_t$	-2.862*	-2.300*
$m_t$	-1.136	-0.878
$Dm_t$	-3.235*	-2.818*
$y_t$	-0.971	-1.475
$Dy_t$	-5.369*	-5.105*
$R_t$	-2.887*	-2.299*

\*Indicates 95 percent of significance

The above-mentioned finding is corroborated with the Johansen test (1988) that indicates the existence of two cointegration vectors. This suggests the long-run existence of several solutions, in addition to the price equation (Rodríguez, 2002).

**Table III**  
**Cointegration tests for  $lp_t, ly_t, lm_t, lr_t$**

Eigenvalue	$H_0: \text{rank} = p$	$-T \ln(1 - \lambda_{p+1})^a$	95%	$-T \sum \ln(1 - \lambda_{p+1})^b$	95%
0.959	$p = 0$	105.87	15.00	152.85**	36.58
0.707	$p \leq 1$	40.47	11.23	46.98*	21.58
0.164	$p \leq 2$	5.91	7.37	6.51	10.35
0.018	$p \leq 3$	0.6	2.98	0.6	2.98

a/  $-T \ln(1 - \lambda_{p+1})$  = maximum characteristic root test;

b/  $-T \sum \ln(1 - \lambda_{p+1})$  = trace test;

It doesn't include intercept neither tendency.

By normalizing the first cointegration vector of the model, the restated equation is:

$$li = -0.310 * ly + 1.037 * lm + 0.106 * lr \quad (13)$$

The signs are those expected according to economic theory. In this case, the parameters indicates the relationships that the economic agents use to maintain the prices in the trajectory towards equilibrium. Henceforth, this means that, price level dynamics in the short run, can be interpreted in an error correction model.

The test for weak exogeneity for each of the variables in the price equation of the VAR model is rejected, with the exception of the preferential interest rate. But, it is important to consider that the alpha values for the equation are very close to zero. This means that, in spite of the weak exogeneity results, the variables under consideration contain relevant information to explain the behavior of the system. In this case, it could be

argued that the weak exogeneity is rejected as a result of the relation between the interest rate and the monetary aggregate and price index (Rodríguez, 2002).

**Table IV**  
**Weak exogeneity tests for the price equation**

$\chi^2(1)$	$lp_t$	$lm_t$	$ly_t$	$lr_t$
3.84	45.67	27.61	43.80	0.17

But, by the alpha parameters being so low, and if some of the considered variables are excluded, invalid statistical inferences can be obtained and relevant information is lost when obtaining an appropriate estimation through the information generating process (Ericsson and Irons, 1994; Rodríguez, 2002). When rejecting the weak exogeneity hypothesis, the representation of the model in an error correction form should include more than one cointegration vector for each equation henceforth, a similar number of correction of errors in each equation (Ericsson and Irons, 1994; Rodríguez, 2002).

**Table V**  
**Alpha Parameters for the price equation**

$dlp_t$	$dln_t$	$dly_t$	$dln_t$
0.125	0.41	0.062	0.000

The tests of strong exogeneity, synthesized in the Table VI, indicate the presence of a feedback effect between the level of prices and the real production level. This indicates that unexpected changes in the level of real production are directly transferred to the price level. The surprise effect in prices, will affect the economic agents expectations, and therefore, real production (Hoover, 1981).

**Table VI**  
**Granger Causality Tests**

Lag	$lp_t \rightarrow ly_t$
1	2.822*
Lag	$ly_t \rightarrow lp_t$
1	4.116*
Lag	$lp_t \rightarrow lr_t$
1	0.003
Lag	$lr_t \rightarrow lp_t$
1	13.569*

The obtained results when analyzing the price model signals that an approximation of the generating process of information can be obtained to forecast inflation stemming from  $P^*$ . The final model, presented in Table VII, does not have autocorrelation problems [LM  $\chi^2(4)$ : 9.880], heterocedasticity [ARCH  $\chi^2(2)$ :2.507] and the errors are normally distributed [JB $\chi^2(2)$ :0.341]. It also presented a better forecast for inflation than the two ARMA models selected, as per the values of the root mean squared error, presented in Table VIII.

**Table VII  
Inflation Model**

Dependent Variable: LI  
 Method: Semingly Unrelated equations  
 Date: 04/09/02 Time: 08:47  
 Sample(adjusted): 1966 1997  
 Included observations: 32 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.535658	0.311386	1.720240	0.0973
ECO(-1)	-3.541897	0.905738	3.910509	0.0006
DI	0.286898	0.267726	1.071612	0.2937
DY	-11.04772	2.881736	-3.833704	0.0007
DM	-1.603489	2.045034	-0.784089	0.4401
DR	0.795845	0.462270	1.721604	0.0970
R-squared	0.583206	Mean dependent var		1.531061
Adjusted R-squared	0.503053	S.D. dependent var		0.501789
S.E. of regression	0.353733	Akaike info criterion		0.926814
Sum squared resid	3.253309	Schwarz criterion		1.201639
Log likelihood	-8.829023	F-statistic		7.276183
Prob(F-statistic)	0.000223			

**Table VIII  
Root Mean Squared Error of the Selected Models (RMSE)**

Model	RMSE
P*	0.318
ARMA(1,1)	0.483
ARMA(2,1)	0.496

#### **IV. Conclusions**

The analysis of the present and future evolution of prices is essential in the decisions making process of public policy in Puerto Rico. These can receive influences of the monetary and real sectors. Currently, there are very few works that analyze the evolution of the prices in the short and long run. In this work, the P\* model is presented as a feasible solution to analyze the present and future behavior of the prices and the inflation.

According to the obtained results, you can surmise that the price level in Puerto Rico is a monetary variable that is influenced by the monetary policy of the United States. It also receives influence from the real production and by the volume and velocity of the transactions carried out on the Island. For the determination of the prices, it is necessary to consider the generation of the economic agents expectations. These conjectures are fundamental in the decisions making process in Puerto Rico.

The VAR model captures the empiric regularities adequately on the evolution of prices. This settles by means of the Johansen procedure that two cointegration vectors exist. In this case, the long run equation of prices is estimated. The signs are those expected according to economic theory and the statistical tests yielded satisfactory results. This indicates that the long run evolution of the price level can be obtained by means of calculating of the cointegration error of the MCE of the level of prices, which will be used to estimate future movements in inflation.

Given these results, this model is a good approach for the generating process of information (GPI) of the future evolution of annual inflation and is good as tool to forecast future inflation compared with other models.

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