Multiple-Output Production With Undesirable Outputs: An Application to Nitrogen Surplus in Agriculture

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Many production processes yield both good outputs and undesirable ones (*e.g.* pollutants). In this paper, we develop a generalization of a stochastic frontier model which is appropriate for such technologies. We discuss efficiency analysis and, in particular, define technical and environmental efficiency in the context of our model. Methods for carrying out Bayesian inference are developed and applied to a panel data set of Dutch dairy farms, where excess nitrogen production constitutes an important environmental problem.

Keywords: Dairy farm, Efficiency, Environment, Longitudinal data, Stochastic frontier

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1 Nitrogen Pollution and Dairy Farming

The environmental problems posed by excess nitrogen produced in the farming sector form a major concern of the European Union (EU), and have led to the 1992 Common Agricultural Policy reform and the Nitrates Directive. The share of agriculture in the total runoff of nitrogen discharge is estimated to be as large as 60% (EEA 1995). Animal density within the EU is highest in the Netherlands, which also has the highest measured nitrogen surplus per unit area (see Brouwer, Hellegers, Hoogeveen and Luesink 1999). The environmental impact is substantial. Nitrates pollute the surface waters and leach into groundwater aquifers, contaminating the drinking water supply. In addition, the evaporation of ammonia is a major contributor to acid rain. The main external nitrogen inputs to dairy farms are the application of chemical fertilizers used in roughage production (mainly grass and green maize) and the purchase of external roughage and concentrate used in animal production. Part of this input is absorbed in the production of marketable outputs (milk, meat, livestock and roughage sold), but most of it (over 75% in the dataset used here, according to Reinhard, Lovell and Thijssen 1999) is released into the environment as nitrogen surplus. The nitrogen surplus largely stems from the application of excess fertilizer and manure produced by the livestock (leading to nitrogen exchange with the soil and ammonia evaporation from land), and also contains ammonia emission from stables and manure sold.

We shall focus on specialized dairy farms in the Netherlands, which are quite intensive and lead to a substantial nitrogen surplus (in our dataset the average was 416 kg nitrogen per hectare per year). Including beef and veal production, the dairy sector is reported to be responsible for 72% of manure production in the Netherlands, and 62% of the agricultural ammonia emmision (CBS 1995). Various restrictions have recently been put in place to curb the nitrogen problem. For example, as of 1998 the more intensive farms need to keep accounts of their mineral balances, and a levy has to be paid for every kg of nitrogen surplus above a certain threshold. Pricing this surplus and setting the threshold are key policy issues. Before policy decisions in this context can be discussed, however, it is crucial to be able to first define and then measure the success (efficiency) of farms in minimizing the release of nitrogen into the environment. Recently, the Commission of the European Communities (2000) has stressed the need for developing environmental efficiency indicators in this context to formally introduce environmental concerns into the Common Agricultural Policy.

There are many applications in economics where an agent (*e.g.* a farm, firm, country or individual) produces undesirable outputs such as pollution, in addition to desirable ones. In the application considered in this paper, Dutch dairy farms produce not only good outputs (which we will informally call "goods"), such as milk, but also undesirable outputs (or "bads"), such as excessive nitrogen. It is thus important to understand the nature of the best-practice technology available to farmers for turning inputs into good and bad outputs. Furthermore, it is important to see how individual farms measure up to this technology. In other words, evaluation of farm efficiency, both in producing as many good outputs and as few undesirable outputs as possible, is crucial. Whereas there is a longer tradition of measuring technical efficiency, related to production of goods, the question of how to define and measure environmental efficiency has only been considered much more recently. In this paper, we propose a definition of environmental efficiency along with a statistical framework for conducting inference. Currently, there is no generally accepted methodology to address these important questions. The state of the art in this emerging field is provided by Reinhard *et al.* (1999), who use a stochastic frontier approach, and Reinhard and Thijssen (2000), who base their analysis on a system of estimated shadow costs.

Thus, we deal with defining and estimating environmental (and technical) efficiencies. A third important issue is how to explain differences of measured efficiencies across farms. We shall provide a theoretical framework for doing so, by incorporating explanatory variables (*e.g.* farm characteristics) into the efficiency distribution. Such effects are typically explored through a second stage regression of estimated efficiencies on certain variables (see Hallam and Machado 1996, and Reinhard 1999, chap. 6). In contrast, we follow Kumbhakar, Ghosh and McGuckin (1991) and Koop, Osiewalski and Steel (1997) and make the dependence of the efficiency distribution on explanatory variables an integral part of our model.

This paper uses Bayesian methods, which automatically and formally allow for the calculation of finite sample measures of uncertainty (*e.g.* 95% posterior credible intervals) for all parameters and the efficiencies themselves. This is very difficult to do reliably using non-Bayesian statistical methods (see, *e.g.*, Horrace and Schmidt 1996). We have found substantial spread in the posterior distribution of efficiencies. Taking this uncertainty into account is crucial if we want to base policy decisions on differences in efficiencies.

The next section will outline the basic theoretical model we will use, while Section 3 formally describes the sampling model used in this paper. In the fourth section we look at various implications of our sampling model in terms of the underlying economic theory. The prior and the algorithm used for posterior inference are described in Section 5, while Section 6 presents our empirical application. The final section contains some policy conclusions.

2 Stochastic Frontiers

Stochastic frontier models are commonly used in the empirical study of efficiency and productivity (see Bauer 1990 for a survey). Such models are used to analyse the efficiency of dairy farms in *e.g.* Kumbhakar *et al.* (1991) and Ahmad and Bravo-Ureta (1995) for the USA and in Hallam and Machado (1996) for Portuguese data. All of these papers assume that a single good output is produced and ignore the possible presence of undesirable by-products. In the single output case, a sensible definition of efficiency can easily be found: the ratio of actual output produced to the maximum that could have possibly been produced with the inputs used. The extension to the case of multiple good outputs is more complicated since multivariate distributions must be used and various ways of defining efficiency exist. Fernández, Koop and Steel (2000a) provide a solution to these complications, and the present paper builds on this approach to allow for some of the outputs to be bad. We will distinguish between technical and environmental efficiency. The former is the standard efficiency concept which compares actual to maximum possible output, extended to a multi-output setting. Our definition of environmental efficiency aims to answer the question "How much could pollution be reduced, without sacrificing good outputs, by adopting best-practice technology?".

To fix ideas, y, b and x will, throughout, denote vectors of good outputs, bad outputs and inputs, respectively. The best-practice technology for turning inputs into outputs is given by a relationship between the inputs and best-practice vectors of good and bad outputs:

$$f(y_{bp}, b_{bp}, x) = 0. \tag{1}$$

In the simplest case of a single good output and no bads, the relationship above is typically assumed to allow for expressing $y_{bp} = h(x)$. The function $h(x)$ is known as the production frontier and corresponds to the maximum output level that can be obtained with input vector x. The technical efficiency of a firm producing y with inputs x is then defined as

$\tau \equiv y/h(x) \in [0,1]$. Statistical estimation of efficiency can be done by adding measurement error to the model and making appropriate distributional and functional form assumptions.

In the multiple good outputs case, still without bads, Fernández *et al.* (2000a) also assume that the relationship in (1) is separable, in the sense that there exist non-negative functions $\theta(\cdot)$ and $h(\cdot)$ such that $\theta(y_{bp}) = h(x)$, where y_{bp} is now a vector. $\theta(y) = constant$ maps out the output combinations that are technologically equivalent, and is thus defined as the production equivalence surface. By analogy with the single output case, $h(x)$ defines the maximum output [as measured by $\theta(y)$] that can be produced with inputs x and is referred to as the production frontier. For a firm producing y, technical efficiency is defined as $\tau \equiv \theta(y)/h(x)$. Fernández *et al.* (2000a) provide a detailed justification for this efficiency definition. Alternative definitions are examined in Fernández, Koop and Steel (2000b).

In the general multiple output case with both goods and bads, the present paper argues for a relationship in (1) that can be written as:

$$\theta(y_{bp}) = h_1(x) \text{ and } \kappa(b_{bp}) = h_2(y_{bp}), \tag{2}$$

for non-negative functions $\theta(\cdot)$, $h_1(\cdot)$, $\kappa(\cdot)$ and $h_2(\cdot)$. In other words, the general relationship can be broken down into two equations involving the "aggregate goods" $\theta(y)$, the "goods' production frontier" $h_1(x)$, the "aggregate bads" $\kappa(b)$, and the "bads' production frontier" $h_2(y)$. The assumption that the frontier for the goods depends only on the inputs, whereas the frontier for the bads is determined by the amount of good outputs produced is likely to be reasonable in many cases. For a firm producing (y, b) with inputs x, we can now define

$$\tau_1 \equiv \frac{\theta(y)}{h_1(x)} \text{ and } \tau_2 \equiv \frac{h_2(y)}{\kappa(b)}, \tag{3}$$

where τ_1 is technical efficiency and τ_2, environmental efficiency, is defined as the minimum possible bad aggregate output divided by the actual one. Both τ_1 and τ_2 are in [0,1].

Alternatively, one could start with the transformation function in (1) and define, *e.g.*, the frontier for the goods as the maximal combinations of y given x and b, and the frontier for the bads as the minimal combinations of b given y and x. Under a separability assumption on the transformation function, this approach essentially reduces to treating the two types of output differently in the same aggregator, thus effectively reducing the problem to having a single frontier. This is pursued in Fernández *et al.* (2000b) and this basic idea also underlies some of the deterministic Data Envelopment Analysis literature (such as in Pittman 1983, and Färe, Grosskopf, Lovell and Pasurka 1989) and the papers by Koop (1998) and Reinhard *et al.* (1999), who include bads as inputs in a stochastic frontier for a single good. Technical and environmental efficiencies can then be defined as output- and input-oriented efficiency measures, respectively. Whereas we recognize the merits of such an approach, it does not allow for a natural separation of the efficiency of technical and environmental aspects of production. For example, a firm that is fully technically efficient will be on the (single) frontier and, therefore, will necessarily also be fully environmentally efficient. Furthermore, because the frontier for the bads depends not only on the goods but also on the inputs, firms that divert part of their inputs to bads' abatement can actually end up with a lower environmental efficiency than firms that produce more bads per unit goods but devote less resources to bads' abatement. By contrast, we argue for the frontier for the bads to depend only on the amount of goods produced, using bads' abatement inputs (if there are any) to explain environmental efficiency rather than to change the frontier for the bads. In our view, the frontier for the bads should correspond to use of the cleanest possible technology, and it is the firms' environmental efficiencies (rather than the frontier) that should depend on whether or not they are using this.

3 The Sampling Model

The cross-sectional unit of analysis will generally be referred to as a firm, which will be a farm in our application. We have data from a panel of $i = 1, \ldots, N$ firms, where the i^{th} firm has been observed for $t = 1, \ldots, T_i$ time periods. The average period of observation is denoted by $T = \sum_{i=1}^{N} T_i/N$, so that the total number of observations is NT. The i^{th} firm in the t^{th} period produces p good outputs $y_{(i,t)} = (y_{(i,t,1)}, \ldots, y_{(i,t,p)})' \in \Re^p_+$, and m bad outputs $b_{(i,t)} = (b_{(i,t,1)}, \ldots, b_{(i,t,m)})' \in \Re^m_+$, using k inputs $x_{(i,t)} = (x_{(i,t,1)}, \ldots, x_{(i,t,k)})'$.

First we model the production technology of the good outputs $y_{(i,t)}$ in the way proposed by Fernández *et al.* (2000a). Thus, we define the goods aggregator:

$$\theta_{(i,t)} = \left(\sum_{j=1}^{p} \alpha_j^q \, y_{(i,t,j)}^q \right)^{1/q}, \tag{4}$$

with $\alpha_j \in (0,1)$ for all $j = 1, \ldots, p$, $\sum_{j=1}^{p} \alpha_j = 1$ and with $q > 1$ to ensure a negative elasticity of transformation between any two outputs. The function in (4) is closely related to the "constant elasticity of transformation" specification of Powell and Gruen (1968), and implies an elasticity of transformation equal to $1/(1-q)$. The role of the p unknown parameters in α and q is further discussed in Fernández *et al.* (2000a). The particular choice of (4) for the aggregator function (albeit quite flexible) is a key element of our model, and different aggregators may well lead to different results.

Interpreting $\theta_{(i,t)}$ as aggregate good output, we define a production equivalence surface as the $(p-1)$-dimensional surface with a constant value for $\theta_{(i,t)}$. Now that we have reduced the problem to a unidimensional aggregate, we define the NT-dimensional vector

$$\log\theta = (\log\theta_{(1,1)}, \log\theta_{(1,2)}, \ldots, \log\theta_{(1,T_1)}, \ldots, \log\theta_{(N,T_N)})', \tag{5}$$

and we model $\log\theta$ through the usual stochastic frontier

$$\log\theta = V\beta - Dz + \varepsilon_g. \tag{6}$$

In (6), the matrix $V = (v(x_{(1,1)}), \ldots, v(x_{(N,T_N)}))'$ consists of exogenous regressors, where each $v(x_{(i,t)})$ is a function of the inputs $x_{(i,t)}$. The particular choice of $v(\cdot)$ defines the specification of the production frontier (*e.g.* Cobb-Douglas, translog, etc.). Typically, we impose regularity conditions on the coefficients β, which assign an economic meaning to the surface $V\beta$ in (6) as a frontier (*e.g.* monotonicity conditions ensuring that production does not decrease as inputs increase). In our farms application we use a Cobb-Douglas frontier (*i.e.* it contains an intercept and is linear in the logs of the inputs) and, accordingly, all the elements of β except for the intercept are restricted to be non-negative.

Another key element of (6) is the vector of *technical* inefficiencies $Dz \in \Re_+^{NT}$, where the matrix D gives some structure to the inefficiencies. In this paper we follow the usual practice in assuming that inefficiencies for each firm are constant over time. In our application $T_i \leq 4$ years, which makes this a reasonable assumption in our view. For balanced panels (where $T_i = T$ for all i) this corresponds to $D = I_N \otimes \iota_T$, where $\otimes$ denotes the Kronecker product and ι_T a vector of T ones, with the obvious generalization for unbalanced panels (where T_i varies with i). In both cases, $z \in \Re_+^N$ becomes a vector of firm-specific inefficiencies. Since the dependent variable in (6) has been transformed to logarithms, technical efficiency for the i^{th} firm is defined as $\tau_{1i} = \exp(-z_i)$. Extensions to more general D are straightforward.

Now we turn to the analysis of the bads, $b_{(i,t)}$, for which we specify a very similar model.

First we aggregate the m components of $b_{(i,t)}$ into

$$\kappa_{(i,t)} = \left(\sum_{j=1}^{m} \gamma_j^r \, b_{(i,t,j)}^r \right)^{1/r}, \tag{7}$$

with $\gamma_j \in (0,1)$ for all $j = 1, \ldots, m$ and such that $\sum_{j=1}^{m} \gamma_j = 1$ and now with $0 < r < 1$. In the case of bads a positive elasticity of transformation is more appropriate on the basis of economic theory considerations. We define $\log \kappa$ analogously to $\log \theta$ in (5) and model this through the following stochastic frontier:

$$\log \kappa = U\delta + Mv + \varepsilon_b, \tag{8}$$

where $U = (u(y_{(1,1)}), \ldots, u(y_{(N,T_N)}))'$, *i.e.* the matrix U is a function of the good outputs. This reflects our idea that the frontier for the bads should be measured relative to the amount of goods produced. Note that U is a function of the p-variate $y_{(i,t)}$ rather than of the aggregated scalar $\theta_{(i,t)}$ alone, since the aggregation in (4) relates only to the production of the good outputs, and it may well be that the influence of the various components of $y_{(i,t)}$ on the production of bads is very different from how they appear in (4). It is quite likely that different technologically equivalent $y_{(i,t)}$ vectors, *i.e.* situated on the same production equivalence surface corresponding to a particular value of $\theta_{(i,t)}$, can have very different consequences for the minimal amount of aggregated bads we can achieve. We impose regularity conditions on δ, so that a larger amount of goods cannot be commensurate with a smaller amount of bads. In the application we use a Cobb-Douglas specification and, accordingly, all the elements of δ except for the intercept are restricted to be non-negative.

$U\delta$ will define the smallest feasible (frontier) production of the aggregate undesirable outputs for a given amount of desirable outputs. If there is any systematic (positive) deviation, this is labelled *environmental* inefficiency, which is grouped in the vector $Mv \in \Re_+^{NT}$. Generally, we can impose different structures through choosing the fixed matrix M. In this paper we assume that firms have a constant environmental inefficiency over time (*i.e.* individual effects), so that $M = D$ is as described above, and the vector $v \in \Re_+^N$ groups the environmental inefficiencies. The environmental efficiency of firm i is, thus, $\tau_{2i} = \exp(-v_i)$. Again, extensions to more general M, not necessarily equal to D, are straightforward.

In the microeconomic theory of production (see, *e.g.* Varian 1984), the production technology of a firm that uses input x to produce output y is defined in terms of the production possibilities set, that is, the set of all feasible input-output combinations. For a fixed value of the output y, the input requirement set, defined as the collection of input vectors x for which (x, y) belongs to the production possibilities set, is assumed to be convex [*i.e.* if (x_1, y) and (x_2, y) are feasible input-output combinations, so is $((1-a)x_1 + ax_2, y)$ for any $a \in (0,1)$]. The regularity conditions imposed on the frontier parameters β and δ ensure that the input requirement sets corresponding to (6) and (8) separately have this property, where "input" for bads production are the goods produced. From (6) and (8) it is easy to prove that if (x_1, y_0, b_0) and (x_2, y_0, b_0) are feasible input-goods-bads combinations (where b_0 is on or above the bads' frontier corresponding to y_0), so is $((1-a)x_1 + ax_2, y_0, b_0)$ for any $a \in (0,1)$, and thus convexity also holds when we consider goods and bads jointly.

We still need to introduce stochastics into the sampling model. The terms ε_g in (6) and ε_b in (8) capture the usual measurement error and model imperfections, and as such will be assigned a symmetric distribution. We allow for the two error terms to be correlated for the same firm and time period, and assume a bivariate Normal distribution. That is, if we let $f_N^R(\varepsilon|a, A)$ denote the R-variate Normal p.d.f. with mean a and covariance matrix A, evaluated at ε, we can write:

$$p(\varepsilon_g, \varepsilon_b|\Sigma) = f_N^{2NT}\left(\begin{array}{c} \varepsilon_g \\ \varepsilon_b \end{array} |0, \Sigma \otimes I_{NT}\right) \tag{9}$$

where Σ is a 2×2 positive definite symmetric matrix.

Through (4)-(9) we have specified a joint distribution for the aggregated goods and bads $(\theta_{(i,t)}, \kappa_{(i,t)})$. Since our aggregators contain unknown parameters, $(\theta_{(i,t)}, \kappa_{(i,t)})$ is not available for use as a sufficient statistic for the frontier parameters and inefficiencies. Thus, we need to add further stochastic assumptions in order to have a sampling model for $(y_{(i,t)}, b_{(i,t)})$ leading to a full likelihood. From a non-Bayesian viewpoint, one could consider limited-information approaches which do not require the full likelihood and could dispense with the distributions that we next specify. For example, in a context without bads, Adams, Berger and Sickles (1999) consider a linear aggregator for the goods and normalize the coefficient of one of the outputs to be one, while putting the remaining outputs on the right hand side. Problems which arise due to the correlation of the remaining outputs with the error term are resolved through semiparametric efficient methods. See also Löthgren (1997) for an alternative approach based on a polar transformation of the outputs.

We now complete the specification of the sampling model along the lines suggested by Fernández *et al.* (2000a). For the goods, when $p > 1$ we define the weighted output shares:

$$\eta_{(i,t,j)} = \frac{\alpha_j^q y_{(i,t,j)}^q}{\sum_{l=1}^p \alpha_l^q y_{(i,t,l)}^q}, \; j = 1, \ldots, p, \tag{10}$$

group them into $\eta_{(i,t)} = (\eta_{(i,t,1)}, \ldots, \eta_{(i,t,p)})'$, and assume independent sampling from

$$p(\eta_{(i,t)}|s) = f_D^{p-1}(\eta_{(i,t)}|s), \tag{11}$$

where $s = (s_1, \ldots, s_p)' \in \Re_+^p$ and $f_D^{p-1}(\cdot|s)$ is the p.d.f. of a Dirichlet distribution with parameter s. Similarly, if $m > 1$ we define a weighted vector of shares for the bads:

$$\zeta_{(i,t,j)} = \frac{\gamma_j^r b_{(i,t,j)}^r}{\sum_{l=1}^m \gamma_l^r b_{(i,t,l)}^r}, \; j = 1, \ldots, m, \tag{12}$$

stack them to form $\zeta_{(i,t)} = (\zeta_{(i,t,1)}, \ldots, \zeta_{(i,t,m)})'$, and assume independent sampling from

$$p(\zeta_{(i,t)}|h) = f_D^{m-1}(\zeta_{(i,t)}|h), \tag{13}$$

where $h = (h_1, \ldots, h_m)' \in \Re_+^m$.

Now (4) – (13) lead to a sampling distribution for Y, B which are matrices of dimensions $NT \times p$ and $NT \times m$, respectively, with elements ordered in the same manner as $\log\theta$ in (5). Taking into account the Jacobian of the transformation, we obtain the sampling density:

$$p(Y, B|\beta, z, \delta, v, \Sigma, \alpha, \gamma, q, r, s, h) = f_N^{2NT}\left(\begin{array}{c} \log\theta \\ \log\kappa \end{array} \Big| \begin{array}{c} V\beta - Dz \\ U\delta + Mv \end{array}, \Sigma \otimes I_{NT} \right) \tag{14}$$

$$\prod_{i,t}\left[f_D^{p-1}(\eta_{(i,t)}|s) \left(\prod_{j=1}^p q^{1-\frac{1}{p}} \frac{\eta_{(i,t,j)}}{y_{(i,t,j)}} \right) \right] \prod_{i,t}\left[f_D^{m-1}(\zeta_{(i,t)}|h) \left(\prod_{j=1}^m r^{1-\frac{1}{m}} \frac{\zeta_{(i,t,j)}}{b_{(i,t,j)}} \right) \right].$$

4 Implications of the Sampling Model

It is important to relate the statistical model specified in the previous section to the underlying economic theory. Given the independent sampling assumption, we will consider a single observation and suppress the (i, t) subscripts, and we will not explicitly indicate conditioning on model parameters in this section.

Economic intuition relates largely to the stochastic frontiers of equations (6) and (8), which only involve $\log\theta$ and $\log\kappa$ instead of the entire vectors y and b. This suggests focussing on the marginal and conditional distributional properties of $\log\theta$ and $\log\kappa$. Using (14) with a Cobb-Douglas structure for the bads' frontier [*i.e.* $U = (1, \log y_1, \ldots, \log y_p)$] and changing variables from y to $(\log\theta, \eta)$ and from b to $(\log\kappa, \zeta)$, we obtain:

$$p(\log\theta, \eta, \log\kappa, \zeta) = f_N^2\left(\begin{array}{c}\log\theta\\ \log\kappa\end{array} \mid \begin{array}{c} V\beta - z\\ \overline{\delta}(V\beta - z) + l(\eta) + v\end{array}, W\right) f_D^{p-1}(\eta|s) f_D^{m-1}(\zeta|h), \quad (15)$$

where $\overline{\delta} \equiv \sum_{j=2}^{p+1}\delta_j$, $l(\eta) \equiv \delta_1 + \sum_{j=1}^{p}\delta_{j+1}\log(\alpha_j^{-1}\eta_j^{1/q})$ and, denoting by σ_{ij} the $(i,j)^{th}$ element of Σ, the elements of W are $w_{11} = \sigma_{11},\ w_{12} = \overline{\delta}\sigma_{11} + \sigma_{12},\ w_{22} = [det(\Sigma) + w_{12}^2]/\sigma_{11}$.

The marginal distributions of y and b correspond to modelling the goods ignoring the bads and modelling the bads ignoring the goods. From (15) it is immediate that, in the marginal distribution, $\log\theta$ and η are independent, with p.d.f.'s respectively given by $f_N^1(\log\theta|V\beta - Dz, \sigma_{11})$ and $f_D^{p-1}(\eta|s)$. This is the specification in Fernández *et al.* (2000a), who consider the problem with multiple goods and no bads. Marginally, $\log\kappa$ and ζ are also independent, with $\log\kappa$ distributed as a location mixture of Normals with expectation $E[\log\kappa] = \delta_1 + \sum_{j=1}^{p}\delta_{j+1}E[\log y_j] + v$. Clearly, the economic regularity conditions imposed previously on β and δ exactly correspond to regularity on the marginal distributions.

We now consider the conditional distributions of $\log\theta$ given (η, b) and of $\log\kappa$ given (ζ, y). From (15), it is immediate that both these distributions are Normal with means given by:

$$\begin{aligned} E[\log\theta|\eta, b] &= V\beta\left(1 - \overline{\delta}\frac{w_{12}}{w_{22}}\right) + \frac{w_{12}}{w_{22}}[\log\kappa - l(\eta)] - \left[\left(1 - \overline{\delta}\frac{w_{12}}{w_{22}}\right)z + \frac{w_{12}}{w_{22}}v\right], \quad (16)\\ E[\log\kappa|\zeta, y] &= \left(\overline{\delta} - \frac{w_{12}}{w_{11}}\right)E[\log\theta] + \frac{w_{12}}{w_{11}}\log\theta + l(\eta) + v. \end{aligned}$$

Note that the conditional mean of $\log\theta$ given (η, b) depends on $\log\kappa$, similarly to analyses where bads are treated as inputs (*e.g.* Koop 1998, or Reinhard *et al.* 1999). Thus, the treatment of pollutants as inputs arises naturally in our framework if we focus on the conditional distribution of y given b. In order to interpret the means in (16) as frontiers with inefficiencies, we need to impose regularity conditions, so that aggregate goods increase with inputs and $\log\kappa - l(\eta)$, and aggregate bads increase with $E[\log\theta]$ and goods. When V corresponds to a Cobb-Douglas specification, these conditions amount to

$$-\min\left(\overline{\delta}\sigma_{11}, \frac{\sigma_{22}}{\overline{\delta}}\right) \leq \sigma_{12} \leq 0. \quad (17)$$

We will impose this regularity condition in addition to those discussed in Section 3.

5 The Prior and the MCMC Algorithm

We will use the following proper prior structure:

$$p(\beta, \delta, \Sigma, z, v, \alpha, \gamma, q, r, s, h) = p(\beta, \delta, \Sigma)p(z, v)p(\alpha)p(\gamma)p(q)p(r)p(s)p(h), \tag{18}$$

where independence is assumed between most parameters, but not between the frontier parameters (β and δ) and between the inefficiency terms (z and v). In addition, restriction (17) links Σ and δ. Building upon earlier work with stochastic frontiers (see *e.g.* Koop *et al.* 1997) and using the intuition that only T_i observations (where $T_i \leq 4$ in our present application) are available for the inefficiency terms of firm i, it is the prior on (z, v) that is most critical to the analysis. We shall thus spend considerable effort on its elicitation.

Prior for (z, v):

For each $i = 1, \ldots, N$, we take a truncated Normal inefficiency distribution:

$$p(z_i, v_i|\mu_i, \Omega) = f_N^2((z_i, v_i)'|\mu_i, \Omega)f^{-1}(\mu_i, \Omega)I_{\Re_+^2}(z_i, v_i), \tag{19}$$

where $f(\mu_i, \Omega)$ is the integrating constant of the truncated Normal, $I_{\Re_+^2}(\cdot)$ is the indicator function for $\Re_+^2$ and we assume independence between firms. The reason for taking a truncated Normal is mainly the ease with which correlation can be handled, which is generally not the case for other distributions.

It is often desirable to allow for firms' efficiencies to depend on their characteristics (*e.g.* size, ownership, etc.). In order to incorporate this, we allow for the inefficiencies to depend on d variables in $g_i = (1, g_{i2}, \ldots, g_{id})'$ through the mean of the underlying Normal in (19):

$$\mu_i = (\phi, \psi)'g_i, \tag{20}$$

and assume the following prior on the parameters ϕ and ψ:

$$p(\phi, \psi|\Omega) = f_N^{2d}\left(\begin{array}{c} \phi \\ \psi \end{array} |0, \Omega \otimes c \left(\begin{array}{cc} a - \frac{d-1}{3} & 0' \\ 0 & I_{d-1} \end{array} \right) \right) \tag{21}$$

where we need to choose $a > (d-1)/3$ and $c > 0$. We could easily generalize this to let both elements of μ_i also depend on different explanatory variables. The half-Normal inefficiency distribution (*i.e.* $\phi = \psi = 0$) is an extremely common choice in the literature, and is less prone to problems in distinguishing between the symmetric error terms and the inefficiencies than a general truncated Normal (see Ritter and Simar 1997, for a related discussion of Gamma inefficiency distributions). By keeping the prior centered over $\phi = \psi = 0$, we allow some flexibility relative to this well-known benchmark. The prior inefficiency distribution is now completed by an Inverted Wishart prior on Ω:

$$p(\Omega) = f_{IW}^{2}\left(\Omega|\Omega_0, \nu_0\right), \tag{22}$$

which has expectation equal to the matrix $\Omega_0/(\nu_0 - 3)$ if $\nu_0 > 3$.

Our aim is to obtain a prior for (μ_i, Ω) (and, thus, for the inefficiencies), which is approximately the same for a range of choices of d and g_i. This is achieved by standardization of the variable g_i so that $0 \leq g_{ij} \leq 1$ for $j = 2, \ldots, d$. Then we have

$$p(\mu_i|\Omega) = f_N^2\left(\mu_i|0, c\left[a - \frac{d-1}{3} + \sum_{j=2}^{d} g_{ij}^2\right]\Omega\right) \approx f_N^2(\mu_i|0, ac\Omega), \tag{23}$$

where the approximation is based on the fact that if the g_{ij}'s were independent and uniformly distributed on $[0, 1]$ then $E[\sum_{j=2}^{d} g_{ij}^2] = (d-1)/3$. This approximation should be reasonably accurate if a is large with respect to $|\sum_{j=2}^{d} g_{ij}^2 \; -(d-1)/3|$ and is exact when $d = 1$. For (empirically relevant) situations with $d \leq 4$ we find that choosing $a = 3$ gives reasonable results. From (21), the variance for the first elements of ϕ and ψ is then two to three times larger than for the others, which reflects a moderate prior belief that firm characteristics influence the efficiency distributions less than the common intercept. The ratio of the prior variance of the last $d-1$ components of ϕ and ψ to that of the underlying Normal on inefficiencies in (19) is given by c. We choose $c = 1$ as a reasonable value which allows for sufficient uncertainty on ϕ and ψ, given a realistic prior efficiency distribution.

In (22), we choose a diagonal Ω_0 and set $\nu_0 = 6$ which is the smallest (and hence least informative) integer value for which the prior covariance of Ω exists. For the diagonal elements of Ω_0 we take 0.65, on the basis of extensive simulations of the resulting distributions of technical efficiency $\tau_{1i} = \exp(-z_i)$ and environmental efficiency $\tau_{2i} = \exp(-v_i)$. Figures 1-4 graph the resulting marginal prior density function (which is the same for both efficiencies) computed using the approximation in (23). Prior median efficiency is 0.72 with a 95% credible interval of (0.11, 0.99) and the correlation between both efficiencies is 0.09. The exact prior was found to be very similar for a range of values of d and g_i. Experimentation with simulated data revealed the resulting prior to be easily dominated by data.

Prior for other parameters:

All other parameters of the model are assigned relatively flat priors. In selecting these, we base ourselves on the experience gained with the multiple good output analysis with no bads of Fernández *et al.* (2000a). In particular, we adopt the following structure:

$$p(\beta, \delta, \Sigma) \propto f_N \begin{pmatrix} \beta \\ \delta \end{pmatrix} |b_0, H_0^{-1} \Big) f_{IW}^2(\Sigma|\Sigma_0, \lambda_0) I_{\mathcal{RR}}(\beta, \delta, \Sigma), \tag{24}$$

where $I_{\mathcal{RR}}(\cdot)$ is the indicator function for the region where regularity conditions are met,

$$p(\alpha) = f_D^{p-1}(\alpha|a_0), \quad p(\gamma) = f_D^{m-1}(\gamma|g_0) \tag{25}$$

$$p(q) \propto f_G(q|1, q_0) I_{(1,\infty)}(q), \quad p(r) \propto f_G(r|1, r_0) I_{(0,1)}(r), \tag{26}$$

where $f_G(.|a, b)$ denotes a Gamma density function with shape parameter a and mean a/b,

$$p(s) = \prod_{j=1}^{p} f_G(s_j|d_j, k_j), \quad p(h) = \prod_{j=1}^{m} f_G(h_j|l_j, n_j). \tag{27}$$

We make the relatively noninformative choices of $b_0 = 0$, H_0 equal to 0.0001 times the identity matrix of the appropriate dimension, $\Sigma_0 = I_2$ and $\lambda_0 = 2$. In the previous sections we have shown the form of $I_{\mathcal{RR}}(\beta, \delta, \Sigma)$ when V and U are chosen to imply Cobb-Douglas frontiers. For the Dirichlet priors in (25) we set a_0 and g_0 to appropriately dimensioned vectors of ones. For the truncated exponential priors in (26) we choose $r_0 = q_0 = 0.1$. In (27) we set $d_j = k_j = l_j = n_j = 0.1$, thus centering $\eta_{(i,t)}$ and $\zeta_{(i,t)}$ at the equal output share values. In Section 6 we will comment on the effect of departures from this base prior.

The Posterior MCMC Algorithm:

We use a Markov chain Monte Carlo (MCMC) algorithm on the space of the parameters in (14) and the inefficiencies (z, v), augmented with ϕ, ψ, Ω [from the hierarchical prior of (z, v) in (19)-(22)]. The Markov chain will be constructed from Gibbs steps for $(z, v), (\beta, \delta), \Sigma$, where we can draw immediately from the conditionals, and Normal random walk Metropolis samplers (see *e.g.* Chib and Greenberg 1995) for $\Omega, \phi, \psi, \alpha, \gamma, q, r, s, h$.

6 Application to Nitrogen Surplus in Dairy Farms

6.1 The data

The data set used in this paper was compiled by the Agricultural Economics Research Institute in the Netherlands using data on highly specialized dairy farms that were in the

Dutch Farm Accountancy Data Network, a stratified random sample. Reinhard *et al.* (1999) and Reinhard (1999) describe the data in detail. The latter papers use a deterministic linear aggregator (based on prices) to aggregate the two types of good outputs mentioned below into one. The panel is unbalanced and we have $NT = 1,545$ observations on $N = 613$ dairy farms in the Netherlands for some or all of 1991-94. We assume Cobb-Douglas forms for both frontiers. The dairy farms produce three outputs, $p = 2$ of these are good and $m = 1$ is bad, using $k = 3$ inputs:

- *Good outputs*: Milk (millions of kg) and Non-milk (millions of 1991 Guilders).
- *Bad output*: Nitrogen surplus (thousands of kg).
- *Inputs*: Family labor (thousands of hours), Capital (millions of 1991 Guilders) and Variable input (thousands of 1991 Guilders).

Non-milk output contains meat, livestock and roughage sold. Nitrogen surplus is the emission of nitrogen into the environment, *i.e.* the difference between nitrogen inputs (such as fertilizer) and its incorporation in marketable outputs (milk, meat etc.). In particular, nitrogen surplus takes the form of nitrogen exchange with the soil, ammonia emmission from land and stables and manure sold. Variable input includes hired labor, concentrates, roughage and fertilizer. Capital includes land, buildings, equipment and livestock.

As explanatory variables for the inefficiency distributions we use:

- *Labor quality:* agricultural education of the farmer (a dummy variable).
- *Nitrogen content of inputs:* kg of nitrogen fertilizer per hectare.
- *Capital composition:* number of cows per unit capital.

We tried most of the explanatory variables mentioned in Reinhard (1999, Chapter 6), with the exception of time dummies (since we have time-invariant efficiencies) and variables that can not be assumed exogenous with respect to our sampling model. In the end, we only retained the three explanatory variables above, since the others were found to be either unimportant in explaining efficiencies or strongly correlated with the inputs in the frontiers.

6.2 Posterior results

We present results based on retaining every 3^{rd} value from a Markov chain of 105,000 drawings after discarding the first 20,000. Convergence of the sampler as well the length of the chain were found to be adequate following the method of Raftery and Lewis (1992).

It should be understood that our model requires many assumptions for the definition of the key concepts and its stochastic implementation. Thus, our results are inevitably dependent on *e.g.* the specification of the frontiers, the output aggregators and the inefficiency prior. However, we feel the particular assumptions made were reasonable from an economic point of view. Unreported results with various sets of artificial data show that the resulting model allows us to extract useful information on key concepts, such as efficiencies, from practically relevant sample sizes. In addition, by considering prior assumptions on efficiencies at odds with the data, it was revealed that the prior is easily dominated by data. The results presented here are obtained with the base prior described in Section 5. However, careful elicitation was only conducted for the prior on the inefficiency terms (z, v) and the only other real prior information comes from economic theory in the form of the regularity conditions and the indicator functions in (26). The other aspects of the prior are much more arbitrarily chosen, and aim at reflecting the absence of real prior information. In order to assess whether we have not inadvertently introduced prior information, we have tried various alternative prior assumptions, which attempt to be even less informative than the base prior. To this end, we have divided H_0, a_0, q_0, d_j and k_j by 100 and changed λ_0 to 1.01 (it needs to be larger than one for a proper prior). Recall that we only have one bad in our application, *i.e.* $m = 1$. An even further deviation from the base prior was achieved by also changing the prior on q and s from Gamma priors to inverted Beta (or Beta prime) priors with density function $f_{IB}(x|1,1) = (1+x)^{-2}$. This prior has zero mode and no mean. Neither of the alternative priors resulted in any appreciable change in results (changes in posterior medians were typically less than one tenth of the interquartile ranges).

Table 1 shows that most of the parameters are reasonably precisely estimated. It is interesting to note the relative magnitudes of the coefficients δ in the production frontier for nitrogen surplus. These coefficients can be interpreted as elasticities, and the elasticity of bad output production with respect to milk production is much greater than the elasticity with respect to non-milk production. Thus, it is the milk side of dairy farming that is most associated with the production of excess nitrogen. "RTS" denotes returns to scale, which reflects the relative effect of a proportionate change in all inputs on attainable good (aggregate) output or the effects of such a change in goods on attainable bads. The production frontier for goods shows increasing RTS, while the production frontier for bads shows decreasing RTS, indicating advantages of large farms, both technically and environmentally.

The shape of the production equivalence surface for the good outputs depends on α and q. The values $\alpha_1 = \alpha_2 = 1/2$ and $q = 1$, imply a linear production equivalence surface with a one-for-one tradeoff (in the units of measurement mentioned above) between the two outputs (see Fernández *et al.* 2000a). Table 1 indicates that we are close to this case.

With stochastic frontier models, interest typically centers on the efficiencies. Space precludes the presentation of results on environmental and technical efficiency for each individual farm, but Table 2 presents results for ten farms with a wide range of efficiencies. Based on an earlier run, we select these as the five farms with the minimum, 25^{th} percentile, median, 75^{th} percentile and maximum mean technical efficiency and similarly for environmental efficiency (these are not the same farms for both types of efficiencies). The information in Table 2 about the posterior technical and environmental efficiencies is supplemented by Figures 1 and 2 which plot the posterior efficiency distributions of the minimum, median and maximum efficiency farms. Table 2 also presents results on the effect that the explanatory variables have on the inefficiency distributions. Education is found to be quite unimportant for environmental efficiency, but the amount of nitrogen fertilizer per hectare has a large negative effect on environmental efficiency, while being conducive to technical efficiency. This is in line with our expectations and these results are not inconsistent with the findings of Reinhard (1999, chap. 6), who uses a second-stage regression of his measure of environmental efficiency on some of the same variables. Interestingly, as cows constitute a larger fraction of total capital, both types of efficiencies tend to decrease substantially. The latter may well be related to less investment in new technology. An alternative explanation for the effect on environmental efficiency is that this characteristic is a proxy for the degree of intensity of the dairy farm (usually measured as number of cows per hectare, which is unavailable to us). LEI (2001) comments on the fact that more intensive farms have a far higher nitrogen surplus (1999-2000 data). Our model will account for this by the fact that these intensive farms will tend to have a larger production share of milk, which has a much higher weight (δ_2) than non-milk output in defining the environmental frontier. Nevertheless, it is possible that aspects of the degree of intensity of dairy farming remain that are not totally captured in our frontier, and that these are now recovered in the efficiency distribution.

The posteriors for technical and environmental efficiency for the observed farms are fairly dispersed. A key advantage of Bayesian methods is that they lead to finite sample measures of uncertainty (*e.g.* 95% posterior credible intervals) for farm-specific efficiencies. In empirical work, it often turns out that such 95% posterior credible intervals are quite wide and the present application is no exception. Ranking farms in terms of their efficiencies could be of interest for policy purposes. A naive ranking based on point estimates could be highly misleading, in view of the posterior uncertainty in farm efficiencies. Perhaps a more realistic use of efficiency results is to see whether different groups of farms can be statistically distinguished on the basis of their entire posterior distributions (*e.g.* can the best and worst farms be reliably differentiated in terms of their 95% posterior credible intervals?). Figures 1 and 2 indicate that this is possible, since the efficiency distributions of the minimum, median and maximum farms do not overlap much. Tables 3 and 4 present additional evidence on this issue, in the form of the probability that, *e.g.,* the farm we label the minimum efficient one really is less efficient than the farm that we label as median efficient. The tables indicate that we are able to distinguish well between the various quartiles.

6.3 Predictive results

It might be illuminating to conduct efficiency inference for an unobserved farm, rather than for farms we actually observe. Technical and environmental efficiencies for an unobserved farm are defined as out-of-sample predictive efficiencies, obtained by integrating out (19) with the posterior distribution on (ϕ, ψ, Ω) and transforming to efficiencies. If we denote the inefficiency of an unobserved farm, say f, by (z_f, v_f), we have $\tau_{1f} = \exp(-z_f)$ as technical efficiency and $\tau_{2f} = \exp(-v_f)$ as environmental efficiency. Since the efficiency distributions depend on a set of farm characteristics, we have to specify which combination of regressors g_f we wish to consider. We choose one farm (labelled "type A") as having the following configuration: educated farmer, nitrogen fertilizer at the 90th percentile of the observed sample and median cows/capital. Another type of farm ("type B") is chosen to have a farmer without agricultural education, and uses both nitrogen fertilizer and cows/capital at the 10th percentile. In addition, we consider the analysis without explanatory variables for efficiencies (labelled "$d = 1$"), and from that we compute the predictive efficiency distribution. Figures 3 and 4 display the predictive (out-of-sample) efficiency distributions for these three basic types of farms. These distributions exhibit a high variance, indicating a great degree of spread in efficiencies across farms. Clearly, if we have not actually observed the farm, our inference on its efficiency is bound to be less precise than for the observed farms in Figures 1 and 2. Some farms are much more efficient than others, even if they have the same characteristics, and this is reflected in the spread of the predictive distribution. If, in addition, we do not use any information on farm characteristics (*i.e.* $d = 1$), we lose even more precision. In any case, these predictives are very different from the prior efficiency distribution, so we have learned something from the data. From Table 2, we also see that the predictive correlation between the two types of efficiencies is always moderately positive, indicating that there is a tendency for farms which are technically inefficient to also be environmentally inefficient. A similar finding was reported by Reinhard *et al.* (1999). Note that this correlation is stronger when we take into account explanatory variables.

In order to get a more direct interpretation of the effect of the explanatory variables, we also consider the predictive efficiencies of a number of firms with characteristics that differ only in one explanatory variable from the firm with median characteristics (denoted by "median g_i" in Table 2). In particular, we consider firms with nitrogen fertilizer at the 25^{th} and 75^{th} sample percentiles, and similarly for the cows per capital variable. A final firm has a farmer without agricultural education. Note that these are all reasonable configurations for the farms in this industry, and thus provide important indications as to the effects of different policy initiatives. The efficiency distributions display a shift in location with respect to the firm with median g_i. Here we briefly summarize the main findings. Even moderate changes can have appreciable effects on the median predictive efficiencies: reducing nitrogen fertilizer from the median (about 256 kg per hectare) to the 25^{th} percentile (about 197 kg) increases environmental efficiency by 0.038 or 3.8%, while only reducing technical efficiency by 1.1%. Reducing the number of cows per unit capital from the median of 32.9 cows per million guilders of capital to 29.3 cows leads to a median increase in technical and environmental efficiency of 2.1% and 1.3%. For both variables, changing to the 75^{th} percentile has roughly the same effect, but with opposite signs. Education improves technical efficiency by 2.6%, but leaves environmental efficiency virtually unchanged. The latter could reflect a focus of agricultural education on technical rather than environmental issues (maybe partly because many farmers in the sample were educated some time ago, their median age being 47 years).

It is apparent throughout that environmental efficiency tends to be lower than technical efficiency. If we take $d = 1$ (to abstract from the influence of explanatory variables), the predictive median of the environmental efficiency is only 0.39, corresponding to the "typical" farm being only 39% as efficient as the (hypothetical) best-practice farm in terms of pollution control. Thus, the typical farm is producing two to three times as much nitrogen surplus than is consistent with best practice! This result seems to be driven by a minority of farms who are leading the way in minimizing nitrogen surplus with a majority of farms falling far behind this lead. This finding is consistent with the incentives farmers face: there are economic incentives to eliminate technical inefficiencies (*i.e.* profit maximization). However, there were, at the time this data was collected, no financial incentives for farms to improve their environmental efficiency, since no levy for nitrogen surplus was imposed. Note that Reinhard *et al.* (1999) also find lower environmental efficiencies than technical ones.

7 Policy Conclusions

In this paper we have introduced a framework for measuring environmental and technical efficiency in the context of a model for multiple good and bad outputs. A Markov chain Monte Carlo algorithm is used to conduct Bayesian statistical inference. Results using artificial (not reported) and real data show that this algorithm is computationally practical and our model yields reasonable results.

An application to Dutch dairy farms indicates that:

- Farms tend to be more efficient technically than environmentally. Without taking into account farm characteristics, we expect an unobserved dairy farm to have a median technical efficiency of 67% and a median environmental efficiency of only 39%.
- However, there is a large spread of efficiencies. This manifests itself in large differences between the 2.5^{th} and 97.5^{th} percentiles of the predictive efficiencies of unobserved farms: technical efficiency ranges from 48% to 92%, and for environmental efficiency these percentiles are 23% and 63%.
- The (moderate) positive correlations between efficiencies indicate that farms which tend to be less efficient technically also tend to be less efficient environmentally.
- Agricultural education has a positive impact on technical efficiency, but virtually none on environmental efficiency. A large proportion of livestock in capital affects both efficiencies negatively, whereas nitrogen fertilizer has a positive effect on technical, but

a large negative effect on environmental efficiency.

- Milk output has a much larger effect on the nitrogen frontier than non-milk output. In particular, increasing milk output by 10% increases the minimum attainable nitrogen production by 8.5-9%, whereas a 10% increase of non-milk production only raises the efficient nitrogen production by 0.8-1.1%.
- Increasing returns to scale seem to exist for good output production, while slightly decreasing returns exists for bad output production.

We hesitate to draw firm policy conclusions based solely on this one set of empirical results for one model specification. However, to illustrate the types of issues that our model can be used to address, we offer the following comments. The relatively large degree of environmental inefficiency indicates that pollution can be reduced in many farms at no cost in terms of foregone good output. That is, if inefficient farms were to adopt best-practice technology and move towards their environmental production frontiers, production of pollutants could be reduced at no cost to milk or non-milk production. The positive correlation between the two types of efficiencies indicates that improving environmental efficiency could be associated with improvements in technical efficiency. Whereas farms can theoretically reach their environmental (and technical) frontiers, we have tried to model some of the pathways along which improvement could occur. Firstly, we observe that the amount of nitrogen fertilizer per hectare has a large negative impact on the environmental efficiency distribution. As could be expected, this variable affects technical efficiency in the opposite way (albeit less importantly), so that policy initiatives towards reducing the application of nitrogen fertilizer could (and probably should) take place, but may meet with some resistance. Altering the composition of capital away from livestock, though, could have a large beneficial effect on both types of efficiencies. For environmental efficiency this could partly reflect the fact that more land will be able to take up more nitrogen, but we think this is also linked with investment in new and more efficient technologies. Thus, subsidies for such investments might well be very beneficial, both on technical and environmental grounds. Finally, the pattern of returns to scale results indicate that larger farms have advantages, both technically and environmentally. Hence, policies which promote rationalization of farms (*e.g.* encouraging larger farms to purchase smaller farms) could result both in more production of milk and non-milk outputs (due to increasing returns in the good production frontier) and less pollution (due to decreasing returns in the bads' frontier).

As mentioned in Section 1, a levy on nitrogen surplus above a certain threshold was introduced in the Netherlands in 1998 for intensive farms. There is currently an active debate concerning this issue, both at Dutch national (see LEI 2001) and EU level (Brouwer *et al.* 1999, CLM 2001). Our present framework feeds directly into this issue. In setting the levy-free surplus, *i.e.* the threshold up to which nitrogen pollution is not priced, the regulatory agency can now directly take account of the production of good outputs. It is, in our view, much less desirable to take into account other characteristics of the farm, such as the presence of pollution-abatement technologies, since it is exactly the responsibility of the farmer to introduce the latter and the environmental target should not depend on the efforts made towards reducing pollution. Thus, it seems quite natural to make the threshold proportional to the bads' frontier (for the amount of goods produced). If the government would set it at $l\%$ above the bads' frontier, it would implicitly start taxing nitrogen surplus if environmental efficiency fell below $(1 + l/100)^{-1}$. In contrast, current government policy is formulated in terms of absolute values of nitrogen surplus per hectare, without taking into account the production of goods. Clearly, this latter policy does not take into account that the production of certain goods inevitably leads to polluting side-effects, and risks introducing unwanted shifts in economic activities (such as from the production of milk to non-milk outputs, *i.e.* towards less intensive farming). Basing the levy-free surplus on the environmental frontier would avoid penalizing (necessary) economic activities with high unavoidable bad output, but would rather penalize production activities that fail to stay close to the relevant environmental best-practice.

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Table 1: Posterior Median and Percentiles for Parameters

	2.5%	Median	97.5%
β_1(Intercept)	-3.31	**-3.17**	-2.99
β_2(Labor)	0.08	**0.10**	0.13
β_3(Capital)	0.53	**0.55**	0.58
β_4(Variable)	0.40	**0.42**	0.44
RTS Goods	1.05	**1.08**	1.10
δ_1(Intercept)	2.45	**2.67**	2.86
δ_2(Milk)	0.84	**0.87**	0.90
δ_3(Non-milk)	0.08	**0.09**	0.11
RTS Bads	0.93	**0.96**	0.99
q	1.00	**1.00**	1.02
α_1	0.56	**0.59**	0.61

Table 2: Posterior and Predictive Results for Efficiencies. The first five farms are observed farms selected on basis of their posterior mean efficiencies (for each efficiency type). The following four rows present predictive efficiency results for firms with various configurations of characteristics ("$d = 1$" corresponds to the analysis without any explanatory variables in the inefficiency distribution). The last column reports the predictive correlation between both types of efficiencies. The last four rows present posterior quantiles of the coefficients of explanatory variables (ϕ, ψ).

	Tech. eff.			Env. eff.			
	2.5%	Median	97.5%	2.5%	Median	97.5%	Corr
Min	0.30	**0.36**	0.41	0.14	**0.18**	0.23	
25th	0.49	**0.60**	0.69	0.25	**0.33**	0.43	
Med	0.56	**0.67**	0.74	0.29	**0.38**	0.48	
75th	0.61	**0.72**	0.80	0.33	**0.42**	0.54	
Max	0.79	**0.94**	1.00	0.61	**0.82**	0.99	
Type A	0.52	**0.69**	0.89	0.22	**0.32**	0.46	0.40
$d = 1$	0.48	**0.67**	0.92	0.23	**0.39**	0.63	0.24
Type B	0.49	**0.65**	0.85	0.33	**0.48**	0.69	0.39
Median g_i	0.51	**0.67**	0.87	0.26	**0.38**	0.55	0.40
Intercept	0.05	**0.15**	0.31	-0.06	**0.15**	0.35	
Education	-0.08	**-0.04**	-0.01	-0.06	**-0.01**	0.03	
N fertilizer	-0.23	**-0.16**	-0.09	0.82	**0.91**	1.01	
Cows/capital	0.72	**0.82**	0.93	0.74	**0.89**	1.04	

Table 3: Probability that Farm in Column is Less Technically Efficient than Farm in Row

Farm	Min	25th	Median	75th
25th	1.00			
Median	1.00	0.93		
75th	1.00	1.00	0.96	
Max	1.00	1.00	1.00	1.00

Table 4: Probability that Farm in Column is Less Environmentally Efficient than Farm in Row

Farm	Min	25th	Median	75th
25th	1.00			
Median	1.00	0.87		
75th	1.00	0.98	0.86	
Max	1.00	1.00	1.00	1.00

Figure labels

Figure 1. Technical efficiency for Dutch dairy farms observed in the sample. Prior density based on the approximation in (23) (dotted line) and posterior densities for three farms, chosen on the basis of their mean technical efficiency: minimum (short dashes), median (solid line) and maximum (long dashes)

Figure 2. Environmental efficiency for Dutch dairy farms observed in the sample. Prior density based on the approximation in (23) (dotted line) and posterior densities for three farms, chosen on the basis of their mean environmental efficiency: minimum (short dashes), median (solid line) and maximum (long dashes)

Figure 3. Technical efficiency for Dutch dairy farms not observed in the sample. Prior density based on the approximation in (23) (dotted line) and predictive densities for two types of farms, corresponding to type A: educated farmer, high nitrogen fertilizer and median cows/capital (short dashes), and type B: farmer without agricultural education, and low nitrogen fertilizer and cows/capital (long dashes). In addition, the solid line represents the predictive of a typical farm without taking into account any explanatory variables for the inefficiencies.

Figure 4. Environmental efficiency for Dutch dairy farms not observed in the sample. Prior density based on the approximation in (23) (dotted line) and predictive densities for two types of farms, corresponding to type A: educated farmer, high nitrogen fertilizer and median cows/capital (short dashes), and type B: farmer without agricultural education, and low nitrogen fertilizer and cows/capital (long dashes). In addition, the solid line represents the predictive of a typical farm without taking into account any explanatory variables for the inefficiencies.

Figure 1

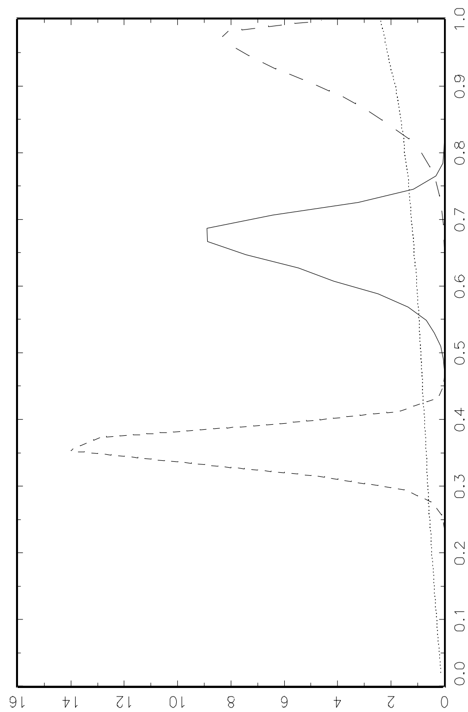

Figure 2

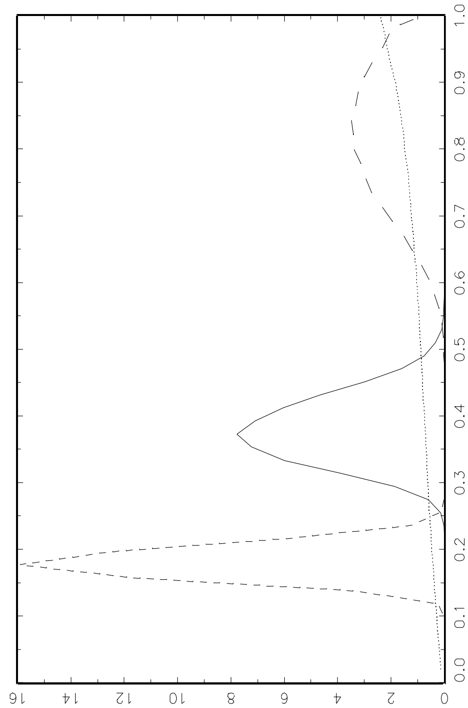

Figure 3

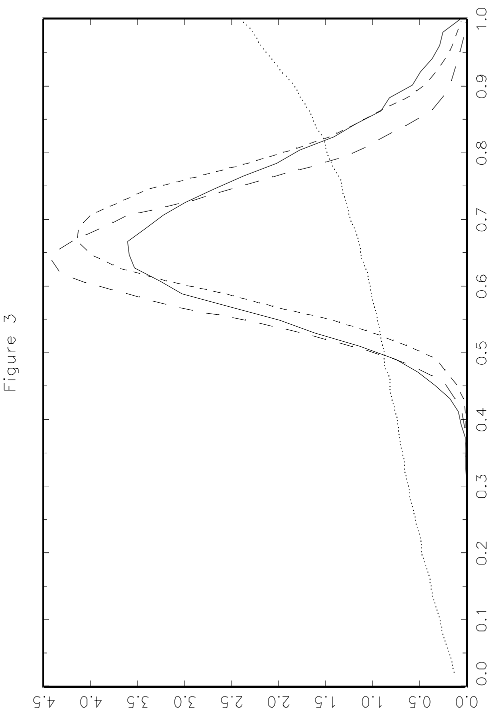

Figure 4