An Inquiry into the Nature and Causes of Walras's Theory of *tâtonnement* ⋆

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Abstract

In this short note, a broad analysis of the *tâtonnement* in Léon Walras theory of General Equilibrium is conducted. The possible origin of the *tâtonnement* in Walras's theory and the basic mechanics of this process are discussed. Using modern and conflicting interpretations of the Edgeworth-Walras debate, it is shown that the *tâtonnement* process is not part of a dynamic model.

The work of Léon Walras, together with other mathematical economists of his time, gave birth to Neo-Classical Economics, a school of thoughts which remains the dominant view in Economics. However, despite being considered as corner stones of modern thinking in Economics, there still exist an ongoing debate about the nature of the great masters's original thoughts. This short note conducts a broad analysis of the *tâtonnement* in Léon Walras theory of General Equilibrium. A synthesis of the literature is presented in section two in order to unveil the causes of the emergence of *tâtonnement* in Walras's theory. In section three, the mechanics of the process are explained and it is shown that regardless the stage of sophistication of Walras's model, it never belonged to the class of dynamic models. Section four concentrates on the *tâtonnement* process, and uses conflicting modern opinions on the Egeworth-Walras debate to demonstrate that Walras's original aim was not to build a dynamic model; section five concludes.

⋆ This working paper is at a very early stage of development. Opinions and feedback are greatly appreciated. The author is grateful to Michael McLure for helpful discussions and to James Fogarty for his generous editorial assistance. Do not quote without permission of the author.

1 Walras's link to reality

Let us begin by examining what is thought to be Walras's original aim. Paraphrasing Dockès (1999), let ask us what was Walras thinking when he wrote the Élements, not what he actually did. In his masterful presentation, Dockès (1999, p.14) makes the clear statement that Walras built a model of a perfectly competitive economy able to reach and work at general equilibrium, while he was actually thinking doing something else! The debate over what Walras actually thought at the the time he conceived his model still rages (Walker, 1987; Bridel and Huck, 2002). It is important to note as Reyberol (2002, p.541) explains "Historians of economic thought try to reconstruct the thought of past author and its genesis in an objective fashion." In the following sections we try synthesises the different opinions.

Looking at the early and the modern literature on Walras, there seems to exist a common agreement among experts that he was both a realist and an idealist. By realist, historians of economics thought like Dockès (1999, p.14), mean he believed in the existence of an outside world ruled by laws that exist independently of their discovery and the *savants* reveal step by step. On the other hand, he believed in ideals, and aimed to abstract them from the *imperfect* reality. Walras's science thus belongs to two worlds: The real world and the world of ideals. [1] In the words of Dockès (1999, p.18): [2]

> Il y a chez Walras une théorie des deux mondes. Le monde réel qui existe objectivement et le monde des Idées qui a également une existence objective. Le second est extrait du premier, mais le premier va être reconstruit à l'image du second.

In his analysis, Walras started by looking at the *objective reality* and proceeded to a *synthèse a posteriori* based on *induction* and experiences (Dockès, 1999, p.15) – i.e. he sought to generalise from empirical observations. Then came a *synthèse a priori* adding something without using experience or empirical observations. It is thus by a *deduction* on abstracted ideals that Walras's analysis of competitive markets was conducted. In other word, as Nakakubo (2002, p.5) explains, "Walras used the empirical method to abstract the essential elements from the real types and to get the ideal types in physical and social domain." The main point being the *real* aspect of the world has always played a central part in Walras theoretical works. In other words, Walras's main con-

[1] This point is particularly well addressed in Nakakubo (2002) and Dockès (1999). Nakakubo (2002) focuses on the reasons that pushed Walras to adopt this *mixed* approach and explains his *Scientific Socialism*, while Dockès (1999) explains carefully which aspect of Walras's analysis belong to each domain and shows the connection between the two.

[2] See appendix A for an approximate translation.

cern was to relate the deduction of his theories to the real world.[3] Moreover, the importance Walras attached to the connection of his theoretical models to the real world is crucial when attempting to understand the importance of his *tâtonnement* process.

Bridel (1996) explains that every time Walras exposed his study of the *tâtonnement* in the theories of exchange, production, capital and money, he explicitly tried to show the equivalence between his theoretical result and the market outcome. It thus becomes apparent Walras's *tâtonnement* is the key to understanding his actual aim. This is perfectly in line with Bridel and Huck (2002, p.514) statement that "the theory of *tâtonnement* [...] appears to be a well chosen theme to test the exact ambitions of Walras's *Élements*, and in particular, its alleged 'realism'." In this analysis, we argue it is *via* his elaborate *tâtonnement* that Walras tried to accomplish his aim of relating theoretical results to reality. It can thus be argued that the *tâtonnement* spawned from Walras's original will and followed him through the different extensions of his complex model, regardless of whether it would have been simpler to drop it on some occasions.[4]

2 Mechanics and Interpretations

Now that we have some idea of what Walras's original motivation was, and how *tâtonnement* relates to it, let us investigate what is indeed Walras's *tâtonnement* process in his theory of general equilibrium. According to Maks and van Daal (2002, p.10), "there is a great discrepancy between Walras's *tâtonnement* and what is called Walrasian *tâtonnement* in literature." Walras theory of general equilibrium is built on four distinct components: the Theory of Exchange, the Theory of Production, the Theory of Capital and the Theory of Money. All four parts are interrelated and interact to determine the general equilibrium in one period (we will come back later to this concept of period). Furthermore in all four theories, the process of adjustment toward the general equilibrium is achieved through a *tâtonnement* process. The *tâtonnement* processes varies in complexity across the four theories. We shall however, as did Maks and van Daal (2002), restrict ourselves to the simplest case, that is, the Theory of Exchange and the Theory of Production. As explained in Maks and van Daal (2002), Walker (1996) and Bridel and Huck (2002), Walras *tâtonnement* evolved greatly throughout the four editions of the Élements. However, all agree his *tâtonnement* in the Theory of Exchange

[3] This is also consistent with Walker (1996, pp.261-266), who explains extensively the importance of reality in Walras's thought.

[4] This was certainly one of the reason for Edgeworth's criticisms, which is addressed in the last section section.

stayed unaltered by the successive new editions, and that the main changes occurred in the fourth edition with the introduction of the *tâtonnement sur bons* that discarded disequilibrium production from his general equilibrium model.

In the Theory of Exchange, n traders are *thought* to be in a common place with given quantities of m different commodities, the *numéraires*. A price for a particular commodity is cried at random by both the buyer and the seller. Since the buyer will underbid and the seller will overbid, it is unlikely this first round yields an equilibrium price for that commodity and thus for the market as a whole. Note no transaction will take place without all prices being at equilibrium in all markets. Since this first set of prices for one market does not clear the market, another set of price is cried which is believed by Walras to be closer to equilibrium.[5] It is clear equilibrium in the market may not be accompanied by equilibrium in production; that is, prices of productive factors are not at equilibrium. In the first three editions of the Élements, Walras allowed for disequilibrium production and goods could be traded in the market when price were at equilibrium *without* prices for productive factors being at equilibrium (Maks and van Daal, 2002, pp.10-11). This disequilibrium possibility is then discarded in the fourth edition by the introduction of a *tâtonnement sur bons*.[6]

The transition to a *tâtonnement sur bons* added some ambiguity in determining what Walras was trying to achieve. Since production needs not be in equilibrium in one period and sets the parameters for the next period, one may think Walras tried building a dynamic model.[7] But was he trying to do so? Without entering into detailed explanation, let us explain briefly the implication of the existence of a disequilibrium of production up to the third edition of the Élements. First, was Walras dealing with time? According to Maks and van Daal (2002, p.4), "Walras stylised the economic process as a sequence of periods of time where production and trade per period take place determined by the working of a carefully devised mathematical model." At the beginning of a period, a set of parameters are given and are assumed to

[5] The mechanism generating this second set of prices is well described in Maks and van Daal (2002, p.10) where it is explicitly shown Walras was only guessing that the second set of prices – or vector of prices – will be closer the the equilibrium level. Since Walras, did not provide the reader with all the implied assumptions, modern investigations on the convergence of his *tâtonnement* yield different results with different assumptions. See for instance, Hahn and Negishi (1962) and more recently Morishima (1997).

[6] Some authors like Walker (1987) refer to the *tâtonnement sur bons* as the *pledges model*.

[7] Note that we focus here on the question of dynamic due to the disequilibrium transactions *and not* the question of dynamics in the *tâtonnement* that brings equilibrium. The later is addressed in the next section.

remain fixed during the period – the *within period* is thus *timeless*. At the end of this period (and really, we should not say *end of period*), the new equilibrium prices for commodities will determine the production of goods for the next period – since we explained production need not be at equilibrium for trade to take place. That is, transactions of goods with disequilibrium production in one period will determine the parameters that affect the quantities of goods offered to the market in the next period. This is merely what Maks and van Daal (2002) called the *in-period* and *inter-period* period equilibrium. But note that both production and exchange take place in the *same* period. That is, production *does not* change between periods but only in periods due to a new set of fixed parameters. Therefore, it would be careless and inaccurate to classify Walras's model as a dynamic model.

In the first half of last century, Pirou (1946, pp.289-290) devoted a section of his publication to addressing the question of dynamics in Walras model. Pirou (1946) explains that Walras was fully aware time has an effect on economic variables. However, while Walras gives laws about the determination of prices and variation of prices which encompass the effect of time, Pirou (1946) warns one should not deduct that Walras's model is dynamic. For this argument, Pirou (1946) explains Classical Economists (e.g., Adam Smith and Malthus) also took into consideration the effect of time in their model but were in no way modelling dynamical systems. Pirou (1946, p.290) then explains:

> "The theories of dynamical economics [...] start from the observation of facts and their evolutions *in a society which is not at equilibrium.* [...] [On the other hand, Walras's] theories about the laws of price *variations* always follow the laws of price *determination* in every market, and have for objective to show us how *from one economic equilibrium corresponding, in a given period, to the given parameters of the problem, but outdated because those parameters changed, we move to another equilibrium.*" [author emphasise] [8]

Pirou (1946, p.291) continues saying that for the theorists of dynamical economics "*there only exists successions of disequilibriums*, and equilibrium is *a pure creation of the mind.*" No reference to this publication was found in any of the papers reviewed for this essay. Given the clarity of Pirou's exposition of Walras position as a theorist, one must wonder how this document could be overlooked. [9]

[8] This translation was kept literal in order not to deviate too greatly from the original text. Please see Appendix A for the French text.

[9] Please see the Appendix B for a brief discussion of the *auctioneer* in Walras theory. Though not directly relevant to this section, it provides one of the latest example of the legacy of an old misinterpretation.

3 Edgeworth's Criticisms

We established above that Walras's model, *with or without* disequilibrium transactions taking place, does not belong to the class of dynamic economic models. This conclusion may, however, leave some readers feeling uncomfortable as the very nature of the *tâtonnement* process is an adjustment process, or groping, toward equilibrium prices. Is not this groping some sort of dynamic process? We argue below it is not; and a careful interpretation of the celebrated Edgeworth-(Bortkiewicz)-Walras debate provides the necessary arguments to support this claim.

The work of Walras aroused criticisms from Edgeworth, who published his critics formally in the September 1889 edition of *Nature*. Edgeworth's criticisms focused on three different aspects of Walras's theory: his theorem of the *utilité maxima*, his theory of the entrepreneur and his theory of *tâtonnement*. Walras answered Edgeworth via Ladislaus von Bortkiewicz, a young Polish statistician based in Russia.[10] In the following, we concentrate on two major interpretations of this debate in order to elucidate the *actual* nature of the *tâtonnement* – i.e., whether it is static or dynamic. The two papers of interest are Walker (1987) and Bridel and Huck (2002). Walker (1987) defends the thesis that Walras's *tâtonnement* is characteristic of a *dynamic* model, while Bridel and Huck (2002) subscribes to the view of a *static tâtonnement*.[11]

Drawing on various passages from the *Élements* and the *Correspondence*, Walker (1987, p.762) concluded "[t]his model of tatonnement, which contains within it the activity of exchange [...] is an attempt to model real dynamic phenomena." To support his claim, Walker (1987, p.762) explains that "Jaffè did not interpret accurately the position taken by Bortkiewicz and Walras." According to Walker, the two colleagues made it explicit in their correspondence that the *tâtonnement* aimed at modelling a dynamical system.[12] Interestingly, as we shall see, Bridel and Huck (2002) have a completely different explanation and show that the confusion originated from Bortkiewicz (not Jaffè), who misunderstood Edgeworth – but not Walras.

A major flaw in Walker (1987) analysis is the use of Walras's *own* definition

[10] Marchionatti (2003, p.4) writes "[...] Walras wrote a letter to the young Russian-Polish student Ladislaus Bortkievicz.", which is incorrect. For instance, in letter 955 of the *Correspondence*, relevant to Marchionatti (2003) analysis, Walras wrote to the mistaken Charles Gide "... qui (Bortkievicz), par parenthèses, est Polonais et non Russe" (i.e., Bortkievicz was Polish and not Russian).

[11] For a comprehensive overview of the debate using the *Correspondence*, see Marchionatti (2003).

[12] Following a careful review of the relevant letters, some points made by Walker appear open to interpretation or translation error.

of static and dynamic analysis to support his claims:

> [H]e (Walras) explained their difference clearly with reference to exchange. The "static theory of exchange," he wrote, may be defined as "the exposition of the equilibrium formula." The "dynamic theory," in contrast, is the attainment of that equilibrium through the play of the raising and lowering of prices until the supply and demand quantities are made equal." (Walker, 1987, p763)

Recalling Pirou (1946), The "dynamic theory," is *not* the attainment of an equilibrium since equilibrium *does not* exist in dynamical systems – only in thoughts. This is particularly important because Edgeworth never argued over the *existence* of Walras's equilibrium (Bridel and Huck, 2002). He was only concerned that the *path* described by Walras's *tâtonnement* was only "*a* way, but not *the* way of descent to equilibrium" (Edgeworth, 1889). Furthermore, the word *path* does not suggest Walras was dealing with a dynamic model, and that is precisely Edgeworth criticism: The static system of equations of Walras's model does not contain any information whatsoever about the trajectory that the variables will follow to reach equilibrium. [13]

According to Bridel and Huck (2002), the key is Jevons. In their brilliant paper, Bridel and Huck (2002) demonstrate in three pages what Walker (1987) failed to understand. [14] Walras, by the intermediary of Bortkiewicz, blamed Edgeworth for confusing static and dynamics in his theory (Bridel and Huck, 2002, p.526). Bridel and Huck (2002, p.526) explain that "[t]he invocation by both Walras and Edgeworth of two *different* notions of dynamics allegedly attributed to Jevons will add confusion to perplexity." According to the authors, the confusion is due to two successive concepts of dynamics presented on page 101 and 102 of the second edition of Jevons's *Theory of Political Economy*. Jevons first exposed a concept of the dynamics which can be regarded as a genuine dynamic process fitting the definition of Pirou (1946) – the 'p. 101 dynamics' in Bridel and Huck (2002). Bridel and Huck (2002) argues that "it is to this [type of] 'dynamics' that Bortkiewicz erroneously referred to as Edgeworth's interpretation of Walras's idea of 'dynamics' in connection with *tâtonnement*" while *both* Edgeworth and Bortkiewicz rightly attributed to Walras, Jevons's second concept of dynamics – the 'p. 102 dynamics': [15]

[13] Note this is the essence of Debreu's proof in the 1950s. For instance, he could prove by a trivial (!?) application the Kakutani fixed-point theorem the *existence* of a General Equilibrium but not *the way* to get to it. Note further, the Nash equilibrium is another application of the Kakutani fixed-point theorem and was used to prove the existence of an equilibrium in Cournot's model of duopoly which was, Jaffè (1965) explains, also criticised by Edgeworth for its *tâtonnement* process.
[14] Note chapter 12 in Walker (1996) reproduces Walker (1987).
[15] Bridel and Huck (2002, p.527) quoting Jevons

> It is as a purely statical problem that I can venture to treat the action exchanges. Holders of commodities will be regarded not as continuously passing on these commodities in streams of trade, but as possessing certain fixed amounts which they exchange until they come to an equilibrium.

This type of 'dynamics' *do not* imply a dynamic model and are part of the static model taking place in a *notional time* (i.e., the *in-period* of Maks and van Daal (2002)) rather than in *real time*.[16] This implies Edgeworth understood Walras, while the later, confused, failed to recognise this. Bridel and Huck (2002, p.527) concluded:

> In the last analysis, and despite the confusion introduced by Bortkiewicz (though in an article written under Walras's close supervision), one can safely conclude that both Walras and Edgeworth adopt Jevon's 'p. 102 dynamics', and, hence, the same 'static' reading of *tâtonnement*.

4 Epilogue

This has conducted an overview of Walras's theory of *tâtonnement*. First, it demonstrated Walras's desire to link theoretical results to reality lead to the creation of this complex *tâtonnement*. The two subsequent sections were devoted to explaining thoroughly the importance Walras place on his *tâtonnement* did not imply he attempted to build a dynamic model. Some additional conclusions can be drawn from this analysis. First, it is clear if all the assumptions of Walras's *early* mathematical models were clearly specified, we would not be witnessing such controversy a hundred years later. Second, this particular controversy implies, as Maks and van Daal (2002) explained, there exists a great discrepancy between what is thought to be known about early economist like Walras, and what is really known. "Mais, plus on étudie l'histoire – de la science – plus on se convainc que l'humanité avance ainsi en s'emballant tantôt à droite, tantôt à gauche, avant de prendre la bonne voie."[17] (Letter from Walras to Gide, II:1000)

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[16] I am grateful to Timothy Kam for this clarification.
[17] See Appendix A.

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A Translation notes

Disclaimer While the best was done to keep the general idea, please be aware the following translations might actually disfigure the authors's original thoughts.

A.1 (Dockès, 1999, p.18)

Il y a chez Walras une théorie des deux mondes. Le monde réel qui existe objectivement et le monde des Idées qui a également une existence objective. Le second est extrait du premier, mais le premier va être reconstruit à l'image du second.

Walras' theory belong to two worlds. The real world which exists objectively and the world of Ideas which also has an objective existence. The later is extracted from the former, but the former will be reconstructed at the image of the later.

A.2 (Pirou, 1946, p.289)

Les Théories de la dynamique économique [...] partent de l'observation des faits et de leur évolution *dans une société qui n'est pas en équilibre.* [...] [Les théories de Walras] sur les lois *de variation* des prix suivent toujours les théories sur les lois de *détermination* des prix pour chacun des marchés, et qu'elles ont pour objectif de nous montrer comment *d'un équilibre économique correspondant, en un certain moment, aux donnés du probleme, mais périmée parceque ces données ont chagé, on passe à un autre équilibre.*

A.3 Walras, Correspondence, II:1000)

Mais, plus on étudie l'histoire – de la science – plus on se convainc que l'humanité avance ainsi en s'emballant tantôt à droite, tantôt à gauche, avant de prendre la bonne voie.

But, the more we study the history – of science – the more we are convinced that humanity progress in this fashion, sometime carrying away to the right, sometime to the left, before taking the good direction.

B The *auctioneer*

There is one last question that could be addressed in section two: Was there an *auctioneer*? Maks and van Daal (2002) clearly state that "Léon Walras himself never made use of the fiction auctioneer." Walker (1996, pp.266-267) goes a step further explaining why there is no need for and auctioneer in Walras theory giving the claim more strength. Another important and clarifying source is Dockès (1999, p.24). This work, perhaps less well-known, provides an

interesting explanation about the *origin* of the confusion. Dockès (1999, p.18) explains in a footnote, referring to a translation note by Jaffè, it was clear to Jaffè that Walras had never thought about a fictitious auctioneer lowering or raising price centrally. However, Dockès continues, "it is unfortunate that he [Jaffè] was the one to translate *crieur* by *auctioneer*" Dockès (1999, p.18). In another footnote, Dockès (1999, p.24) explains that the confusion might have occurred when Walras made reference to a *on*, the singular version of *we* in English: [18]

> Si c'est la demande qui est supérieure à l'offre, *on* fait la hausse du prix... *on* crie $m(m-1)$ prix des m marchandises les unes en les austres. [author emphasis]

Paraphrasing Dockès (1999), this potentially misleading *on* was no-one else but the *mathématicien-économiste* in Walras theory.

What motivated the inclusion of this extra clarification (while above the word limit!) was Marchionatti (2003). While acknowledging his paper is at an early stage of development, it was surprising to find the following:

> Walras conceives the general market as an auction market and *introduces an auctioneer who continues to change prices* until supply and demand imbalances with respect to all commodities disappear. Marchionatti (2003, p.12) [emphasise added]

What is even more striking is Marchionatti (2003) is heavily based on the correspondence of Walras, which is more often than not, written in French. This unambiguously implies that Marchionatti has proficiency in the French language and thus would use the original version of the *Élements* for his analysis. Since, as previously explained, there is no mention whatsoever of an auctioneer in the original text, one must wonder what sources Marchionatti used to develop his analysis.

[18] Dockès (1999, p.24) quoting Walras (1988, p.842).