Costly Technology Adoption and Capital Accumulation

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September 1997

JEL Classification: O1, O33

[1]We would like to thank conference particpants at the 1995 Midwest Macro Meetings, the 1996 Summer Meetings of the Econometric Society, seminar participants at the University of Virginia and Terry Fitzgerald for helpful comments.

Abstract

We develop a model of costly technology adoption where the cost is irrecoverable and fixed. Households must decide when to switch from an existing technology to a new, more productive technology. Using a recursive approach, we show that there is a unique threshold level of wealth above which a household will adopt the new technology and below which it will not. This threshold is independent of preference parameters, and depends only on technological parameters. Prior to adoption, households invest at increasing rates but consumption growth is constant. We also show that richer households adopt sooner which is consistent with the evidence from the Green Revolution. Our results are robust to households having access to loans.

1 Introduction

We study capital accumulation in an environment where technology adoption is costly. We develop a simple model in which a household faces a …xed cost to switch from a less productive technology to a more productive technology. Given the household's initial level of wealth, we examine how the presence of the …xed adoption cost in‡uences the evolution of the household's wealth and the length of time before the household adopts the higher productivity technology.

Several authors have recognized the important role of technology adoption in the process of economic development. Prominent examples of technology adoption and productivity growth episodes include the Industrial Revolution (Mokyr, 1993), the Green Revolution (van Zanden, 1991 and Alauddin and Tisdell, 1991) and the Information Revolution (David, 1990, and Greenwood, 1996). In studying the role of technology adoption, the literature has followed, essentially, two themes: (i) costs and bene…ts of adoption in a variety of environments and (ii) impact of adoption on macroeconomic variables such as growth, relative wages etc. In the former theme, the decision to adopt a new technology is cast in an environment with rich details on costs and bene…ts. For instance, Jovanovic and Nyarko (1996) and Pérez-Sebastián (1996) posit a trade-o¤ between the accumulated knowledge in the old technology through learning-by-doing and the risky, unknown, but higher average productivity of the new technology. In the latter theme, the focus is on the behavior of related variables in the economy using relatively simple trade-o¤s in the technology adoption decision. For instance, Parente and Prescott (1994), Parente (1995), and Easterly, Levine, King, and Rebelo (1994) try to explain persistent di¤erences in incomes across countries.

Our model belongs more to the latter theme. We subsume the cost of adopting a new technology entirely into an exogenous …xed cost. The bene…t of adoption is higher productivity. Capital, interpreted broadly to include both physical and human, is the sole factor of production and both the old and the new technologies are linear.[1]

[1] In our model, the household's wealth is, essentially, its capital stock, so we will treat the terms

Our aim is to characterize the path of capital accumulation. For an in…nitely lived household with time separable homothetic preferences, we show that the household accumulates capital at an increasing rate while it uses the low productivity technology i.e., the growth rate of capital is monotonically increasing in the level of capital. Once the household reaches a threshold level of wealth it pays the …xed cost and adopts the new technology. Consumption growth is constant during this period of increasing investment..

The threshold level of wealth at which the household switches to the new technology is independent of the initial level of wealth and, consequently, the duration with the low productivity technology is decreasing in the initial level of wealth. Furthermore, the threshold is independent of preference parameters, and depends only on the productivity of the two technologies and the …xed cost of adoption. The technological parameters in‡uence the date of adoption in our model directly through their e¤ect on the threshold level of wealth and indirectly through their e¤ect on the evolution of wealth. Higher productivity of the old technology tends to postpone the adoption date; higher productivity of the new technology makes the household adopt the new technology sooner; and, a higher …xed cost postpones the adoption date.

Our results are robust to the presence of loan markets. Financing either the …xed cost or consumption does not alter the threshold level of wealth or the savings behavior in our model. Even though wealthy households will adopt the technology sooner than poor households in our model, this result is not a¤ected by the presence or absence of loan markets; the cost of adoption is technologically …xed and the wealthy households will incur this cost sooner. This is in contrast to the two-period model of Eswaran and Kotwal (1989) where lack of access to loans markets is the key reason why poor farmers adopt later.

One may think of the …xed cost in our model as the cost of learning the new method of production. For instance, in the context of the Green Revolution, farmers had to

‘capital’ and ‘wealth’ as synonymous throughout the paper.

learn how to use the new variety seeds with other inputs. Foster and Rosenzweig (1995) document that the learning cost was non-trivial: knowledge about the new seeds was a signi…cant barrier to adopting the high-yield technology during the Green Revolution.

We also make a methodological contribution in this paper. We set up a sequence of dynamic programs, one for each possible adoption date. Each dynamic program yields a value function that helps us evaluate whether the associated adoption date is optimal. One may use our dynamic programming technique to study a variety of situations where there is a one-time cost. For instance, consider a household deciding whether to migrate from a developing country to a developed country. If there is a one-time …xed cost at the time of migration, then our technique will be useful in determining the wealth accumulation prior to migration and the period in which the household chooses to migrate. Other examples include opening an economy to international trade or implementing a radical change in policy.

In related work, Greenwood and Jovanovic (1990) examine a growth model where agents must pay a one time start-up cost to avail themselves of the services of …nancial intermediaries who provide information on the pro…tability of a risky asset relative to a safe asset. In contrast to their analysis, we derive a unique threshold level of capital at which the household will adopt the higher productivity technology and show that this threshold is independent of preference parameters. We also explicitly solve for the path of capital accumulation. However, we abstract from issues involving uncertainty in the production technologies or intermediation. Bental and Peled (1996) develop a search-theoretic model where technological progress is endogenous. Improvements in technology depend on costly search and the search process is one of sequential sampling with a …xed cost associated with each sample. Their problem is to determine the optimal stopping time i.e., the number of samples above which one stops looking for improvements in technology. Our problem is to determine the optimal starting time, the period when one starts using the better technology. While growth is endogenous in our framework, technological progress is not.

2 The Model

Time is discrete and is indexed by t. Consider a household initially producing output, Y_t, using a technology which is characterized by a constant marginal product of capital whose value is $a > 0$, i.e., $Y_t = aK_t$, where K_t is the capital stock at the beginning of period t. As noted in the Introduction, K_t is also the sum of physical and human wealth. At any time it may pay a fixed cost $\Delta > 0$ and immediately gain permanent access to a more productive technology characterized by a constant marginal product $b > a$.

The household could be in one of three 'states' at any point in time: it could be operating the old technology or could have paid the fixed cost and just adopted the new technology or could be operating the new technology having adopted it earlier. Let S denote the time at which the household pays the cost of adopting the more productive technology. The household then faces the following constraints:

$$
\begin{aligned}
Y_t &= \begin{cases} aK_t & \text{if } t < S, \\ bK_t & \text{if } t \geq S, \end{cases} \\
C_t + I_t &\leq Y_t \text{ for } t = 0, 1, \ldots, S-1, S+1, \ldots, \\
C_S + I_S + \Delta &\leq Y_S.
\end{aligned}
$$

Capital depreciates at the rate $\delta \in [0, 1]$. The capital stock evolves in a manner similar to the one-sector growth model:

$$K_{t+1} = (1 - \delta) K_t + I_t.$$

Let us define $A = a + 1 - \delta$ and $B = b + 1 - \delta$, so we can combine the resource constraints and the capital accumulation equation as

$$\begin{aligned} C_t + K_{t+1} &\leq AK_t \text{ for } t = 0, 1, \ldots, S - 1, \\ C_S + K_{S+1} + \phi &\leq BK_S; \\ C_t + K_{t+1} &\leq BK_t \text{ for } t = S + 1, S + 2, \ldots \end{aligned} \tag{1}$$

The household's preferences are assumed to be time–separable. Let the subjective discount factor be $\beta \in (0, 1)$ and assume $\beta A \geq 1$.[2] The household's utility over consumption in period t, C_t, is given by $(1-\beta)\frac{C_t^{1-\sigma}}{1-\sigma}, \sigma > 0$. (We will interpret the $\sigma = 1$ case as logarithmic.) The costly technology adoption problem can be formulated as the following sequence problem:

$$\max_{S, \{C_t, K_{t+1}\}_{t=0}^{\infty}} (1-\beta) \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma}$$

subject to (1),

$$C_t \geq 0, \; t = 0, 1, \ldots,$$

K_0 given.

A few remarks are in order at this stage. First, the household may choose to adopt the new technology in period 0; the above formulation does not prevent the household from doing so. In fact, we will show later that a household with sufficiently large initial wealth will pay the fixed cost in period 0 and start operating the new technology. Second, the fixed cost is tightly linked to the time interval between model-periods in our framework. Suppose it takes five years of time and material expenditures to bring the new technology on line. Then the fixed cost is the present value of such expenditures and the time interval between model-periods is five years. Third, since the production technology is linear, ϕ units of output is the same as λ units of capital, where $\lambda = \phi / B$. That is, the fixed cost may be specified in terms of capital in which case the resource constraint in period S becomes $C_S + K_{S+1} \leq B(K_S - \lambda)$.

[2] If $\beta A < 1$, then it is possible that an intially poor household may never be able to accumulate enough capital to pay the fixed cost of adoption.

To solve the above sequence problem, a brute force approach is to proceed as follows: determine capital accumulation in the post-adoption state, determine capital accumulation in the pre-adoption state assuming a particular S, and then determine the optimal S to adopt the new technology. While the first two steps are relatively straightforward, the last step involves solving the problem for various values of S and then picking the one that yields the highest lifetime utility for the household. This is computationally intensive. Instead, we follow a recursive approach. We formulate a set of functions such that each element of this set is associated with a particular adoption period. We then use this set of functions to map the problem of finding the optimal S into one of finding the threshold level of wealth at which the household will switch to the new technology.

3 A Recursive Approach

To characterize the technology adoption problem, it is necessary to know the value of the household's wealth in three distinct scenarios: the value after adoption, the value of never adopting, and the value of adopting in an arbitrary period T. All our results below hold for $\sigma > 0$, but for expositional convenience we concentrate on the case $U(C_t) = (1 - \beta)\log C_t$ (i.e., $\sigma = 1$).

To begin, suppose that the household has adopted the higher productivity technology. Let $H(K)$ denote lifetime utility for a household with capital K, using the more productive technology. For such a household, the constraint correspondence $\Gamma_H(K) = \left\{K' \in R_+ \mid K' \leq BK\right\}$ is the set of feasible values for next period's capital, K'. Let the constraint set for K', when the less productive technology is used, be defined by $\Gamma_L(K) = \left\{K' \in R_+ \mid K' \leq AK\right\}$. As $B > A$, it follows that for any $K \geq 0$, $\Gamma_L(K) \subseteq \Gamma_H(K)$. Hence, the household will never want to switch back to the less productive technology even if switching back was costless. In other words, H will satisfy the following Bellman equation:

$$H(K) = \max_{K' \in \Gamma_H(K)} U\left(BK - K'\right) + \beta H\left(K'\right). \tag{2}$$

Existence of a unique H that solves the functional equation (2) follows from Alvarez and Stokey (1995). Define $K' = g_H(K)$ to be the optimal policy for next period's capital associated with the above dynamic program.

Similarly, define the value of never adopting the more productive technology as $L(K)$.

$$L(K) = \max_{K' \in \Gamma_L(K)} U\left(AK - K'\right) + \beta L\left(K'\right) \tag{3}$$

The policy function for this benchmark is denoted $g_L(K)$. It is easy to determine the functions H and L using a guess-and-verify method (see Sargent, 1987):

$$H(K) = \log(1-\beta) + \frac{\beta \log \beta}{1-\beta} + \frac{\log B}{1-\beta} + \log K, \tag{4}$$

$$L(K) = \log(1-\beta) + \frac{\beta \log \beta}{1-\beta} + \frac{\log A}{1-\beta} + \log K. \tag{5}$$

The associated policies are $g_H(K) = \beta BK$ and $g_L(K) = \beta AK$.

Now consider the value of adopting in an arbitrary period T. Define a family of value functions, $Z(K;T)$ where $T = 0, 1, \ldots$ represents the time until adoption; $T = 0$ implies immediate adoption. The constraint set during the switching period is given by $\left\{K' \in R_+ \mid K' \leq B(K-\lambda)\right\} = \Gamma_H(K-\lambda)$. The value function associated with immediate adoption, $Z(K;0)$, must satisfy the following functional equation:

$$Z(K;0) = \max_{K' \in \Gamma_H(K-\lambda)} U\left(B(K-\lambda) - K'\right) + H\left(K'\right). \tag{6}$$

Note that $Z(K;0) \equiv H(K-\lambda)$.

Similar to (6), we have a relationship between $Z(K;T+1)$ and $Z(K;T)$:

$$Z(K;T+1) = \max_{K' \in \Gamma_L(K)} U\left(AK - K'\right) + Z\left(K';T\right) \tag{7}$$

The constraint set prior to adoption is given by $\Gamma_L(K)$. As before, we define $K' = g_Z(K;T)$ as the optimal policy towards capital for the next period when the current state is $(K;T)$, that is, when there are T periods until adoption and K is the beginning of period level of capital.

Since we know H (see (4)), we may use (6) to obtain $Z(K;0)$:

$$Z(K;0) = \log(1-\beta) + \frac{\beta\log\beta}{1-\beta} + \frac{\log B}{1-\beta} + \log(K-\lambda). \tag{8}$$

The policy function is $g_Z(K;0) = \beta B(K-\lambda)$. Backward induction through (7) yields solutions for $Z(K;T+1)$, $T = 0;1;\ldots$:

$$\begin{aligned} Z(K;T+1) &= \log(1-\beta) + \frac{\beta\log\beta}{1-\beta} + \frac{\beta^{T+1}\log B}{1-\beta} \\ &\quad + \frac{\beta\left(1-\beta^{T}\right)}{1-\beta}\log A + \log\left(AK-\lambda A^{-T}\right). \end{aligned} \tag{9}$$

The associated optimal policies are

$$g_Z(K;T+1) = \beta AK + \frac{1-\beta}{A^T}\lambda. \tag{10}$$

3.1 The threshold level of wealth

At this stage, if one wanted to determine the optimal adoption time for a household with capital K then one has to search over different values of T and find a T^* such that $Z(K;T^*) \geq Z(K;T)$ for $T = 0;1;2;\ldots$. Instead, we determine the threshold level of wealth at which the household will adopt the new technology, using the family of time-until-adoption value functions. The adoption period can then be computed using the time taken to reach the threshold level of capital starting from the initial capital, K_0. The time taken, of course, will depend on the path of savings prior to adoption.

There are obvious bounds on the threshold level of capital. For instance, if the household's capital is less than λ then the household cannot adopt the new technology. This does not imply, however, that the household will switch as soon as it reaches a level of capital above λ. To see this, consider a household with λ units of capital. If this household never adopts the new technology then its lifetime utility is $L(\lambda)$. If the household decides to adopt the new technology then, it is clear from (4) and (5) that $\lim_{K \to \lambda} Z(K; 0) < L(\lambda)$. By continuity, the household will not adopt even if it has capital close to but greater than λ.

An upper bound on the threshold level of wealth may be found as follows. Let $\overline{K}$ be such that the output, net of fixed cost, under the new technology is the same as the output in the old technology i.e., $B(\overline{K} - \lambda) = A\overline{K}$ or $\overline{K} = B\lambda/(B - A)$. (See Figure 1.) A household with capital $K \geq \overline{K}$ will immediately adopt the new technology. This follows from a simple observation: any consumption sequence that is feasible when the household operates the old technology for $T \geq 1$ periods and then switches to the new technology is also feasible if the household adopts the new technology immediately. However, the converse is not true. Hence, the household is better off switching to the new technology if its capital is $K \geq \overline{K}$.

So far we know that the threshold level of capital lies in $[\lambda; \overline{K}]$. It turns out that there is a unique threshold level of wealth above which the household will adopt the higher productivity technology and below which it will not.

Proposition 1 Let $K^* = \frac{B-1}{B-A}\lambda$. For $K \geq K^*$; the household will immediately adopt the new technology and for $K < K^*$; the household will not immediately adopt the new technology.

The details of the proof are in the Appendix.[3] Briefly, our argument proceeds in three steps. First, we show that the household will immediately adopt rather than wait a period when its capital exceeds K^*. Second, we show that at this threshold

[3] The threshold level of capital is K^* even when $\sigma \neq 1$. The proof for this case is available from the authors upon request.

the household will adopt immediately rather than wait a finite number of periods. Third, we show that at K^* the household will adopt the new technology immediately rather than never.

Note that $K^* \in (\lambda, \overline{K})$ since $A > 1$. Furthermore, it is easy to show that the threshold level of wealth is decreasing in B, increasing in A, and increasing in λ. Later, we will examine the implications of these features for the length of time until adoption.

4 Capital Accumulation and Technology Adoption

The evolution of wealth after adoption is completely described by the policy function $g_H(K) = \beta BK$. The path of wealth prior to adoption depends on when the new technology is adopted (see equation (10)). However, since we know the threshold level of wealth at which the household will adopt the new technology, we can compute the time taken to reach K^* from K_0 using the policy function (10).

4.1 Capital Accumulation

Suppose that the household plans to adopt the new technology in period S and has K_0 units of initial capital. Then, setting $T = S - t$ in (10), we have

$$K_t = \frac{(1-\beta)\lambda}{A^{S-t}} + \beta A K_{t-1}; \quad t = 1; \ldots; S. \tag{11}$$

Iterating backwards to period 0, we can derive the path of capital from period 0 to period S:

$$K_t = \frac{(1-\beta^t)\lambda}{A^{S-t}} + (\beta A)^t K_0; \quad t = 1; \ldots; S. \tag{12}$$

Define the growth rate in period t in the pre-adoption state to be $\gamma_t \equiv \frac{K_{t+1}}{K_t}; t = 0; 1; \ldots; S-1$. Using (11) and (12), we have

$$\gamma_t = \frac{(1-\beta)\lambda A}{\lambda + \beta^t(A^S K_0 - \lambda)} + \beta A.$$

Two observations help us realize the increasing growth rate result. (i) Starting from K_0, the highest level of capital that the household can possibly have in period S is $A^S K_0$ (by allocating all its wealth in each period to the next period's capital stock). For S to be the switching period it must be the case that $K_S - \lambda > 0$ i.e., $A^S K_0 - \lambda > 0$ is a necessary condition. (ii) Since $\beta < 1$, β^t is a decreasing function of t. Hence, $\gamma_t > \gamma_{t-1} > \gamma_{t-2} > \ldots > \gamma_0$.

The increasing growth rate of capital implies that the investment rate, $\frac{K_{t+1} - K_t}{aK_t}$, is increasing over time. The reason for this is intuitive. While the household would like to access the high productivity soon, it would also rather have more current consumption than more future consumption. Consequently, the investment rate is low in the earlier periods and high in the later periods. The household begins K_0 units of capital, invests at an increasing rate till reaches the threshold K^*, pays the fixed cost of λ units of wealth and switches to the new technology. We thus have the following proposition:

Proposition 2 Prior to technology adoption, the growth rate of capital and the investment rate are both monotonically increasing over time.

4.2 The Switching Period

We can use (12) to determine the switching period. The wealth at the beginning of period S is given by

$$K_S = (1 - \beta^S)\lambda + (\beta A)^S K_0. \tag{13}$$

For S to be switching period we must have $K_S \geq K^*$. By setting $K_S = K^*$ in (13), we can solve for S (subject to an integer constraint that S is the smallest integer that exceeds the solution to (13)). An implication of (13) is that the switching period depends on the initial wealth. Holding the left hand side fixed at the threshold level of wealth, a lower K_0 implies a higher S for equation (13) to be satisfied.

Proposition 3 Poorer households will adopt later.

As noted in the Introduction, the reason for this is technological. The threshold level of wealth is pinned down by the technological parameters, $A; B;$ and $\ast$. Poor households will take longer to reach K^* even if they accumulate wealth at the same rate as rich households. In fact, in our setup, the poor households save a smaller fraction of their income relative to the rich households (see Proposition 2). Hence, the new technology is adopted at a later date by households that are relatively poor.

Proposition 3 is also consistent with the findings in Wozniak (1987), given our interpretation of K as a mix of physical and human capital. He finds that the decision to adopt is a human capital intensive activity and that early adopters tend to be those with high levels of human capital.

To see how the switching period depends on the technological parameters $B; A;$ and λ, combine equation (13) with Proposition 1 to yield:

$$\frac{B-1}{B-A}\lambda = (1-\beta^S)\lambda + (\beta A)^S K_0. \tag{14}$$

The left hand side is, of course, the threshold level of wealth. The right hand side is the level of wealth in period S, assuming that the switching period is S. Loosely speaking, if one graphs the two sides of equation (14) as functions of S, then the intersection point gives us the switching period (subject to integer constraints).

Consider, for instance, the effect of higher productivity, $B' > B$. Intuition suggests that the household will want to adopt the new technology sooner since the return to adoption is higher under B'. As noted in the previous section, B' implies a lower threshold than B. The path of wealth accumulation in the pre-adoption state, however, does not depend on the productivity of the new technology. That is, the left hand side of (14) is lower but the right hand side remains the same. Hence, the switching period falls.

Similarly, consider the effect of a higher fixed cost of adoption, $\lambda' > \lambda$. For each unit increase in λ it is easy to see that the left hand side of (14) increases by $\frac{B-1}{B-A} > 1$ while the right hand side increases by $1 - \beta^S < 1$. Thus, the switching period has to increase to satisfy equation (14). The intuition for this is fairly straightforward. A higher fixed cost raises the barrier to adoption, so starting from the same initial condition the household will adopt later. Clearly, earlier adoption in this case would suggest that the household's behavior was not rational when the fixed cost was lower.

We summarize our results in the following proposition.

Proposition 4 *(i) A more productive new technology implies that the household will adopt earlier. (ii) Higher adoption cost delays the adoption date.*

Now consider the effect of higher productivity in the initial technology i.e., $B > A' > A$. The return to adoption falls, so one would guess that the household will adopt the new technology later. When the productivity is A', the threshold wealth is higher but the investment rate in the pre-adoption state is also higher. Consequently, the path of wealth accumulation shifts up. It is not obvious whether switching period falls or rises. We present a simple numerical example to determine the effect on the switching period. Set $B = 1.065$ to reflect the average rate of return on capital in developed economies and set $\beta = 0.9577$ so that the growth rate in the post-adoption state, βB, is 2 percent. (This is also the long-run growth of average income in developed economies.) Let $\lambda = 1$, $K_0 = 2.5$, $A = 1.001/\beta$ and $A' = 1.005/\beta$. Figure 2 illustrates the two sides of equation (14). It is clear from the figure that the household adopts the new technology later under A' than under A. This result is robust to different values for A'.

4.3 Consumption growth

A notable feature of our model is that the household's consumption and wealth do not grow at the same rate prior to adoption. To see this, consider the household that plans to adopt the new technology in period S and has K_0 units of initial capital. From the resource constraint (1) we know that the gain to giving up a unit of consumption in period 0 is A units of consumption in period 1. In utility terms, we have the following Euler equation:

$$\frac{1}{C_t} = \beta \frac{1}{C_{t+1}} A; \quad t = 0; \ldots; S - 2.$$

During these periods, the consumption growth is constant, equal to βA.[4] We have already shown in Proposition 2 that wealth exhibits increasing growth during this period. The contrast is especially stark when one considers the case $\beta A = 1$: the level of consumption stays constant, but wealth exhibits growth. The reason for saving despite the low rate of return is that even though the 'current' return to savings is low, the 'anticipated rate of return' to savings is high. The household takes this into account when making its intertemporal decisions.[5]

4.4 Markets for Loans

Our analysis assumes throughout that the household does not have access to loans either to finance consumption or the adoption cost. Suppose there were such markets.

[4] When the household reaches period $S - 1$, the tradeoff is different: the gain to giving up a unit of current consumption is B units of future consumption. Thus, the Euler equation is

$$\frac{1}{C_t} = \beta \frac{1}{C_{t+1}} B; \quad t = S - 1; S; \ldots.$$

Once the technology is adopted, the rate of growth of capital is the same as that of consumption.

[5] In contrast, models with minimum consumption requirements (such as Chatterjee and Ravikumar, 1997) would imply an increasing growth rate of consumption. For instance, suppose $U(C_t) = \log(C_t - \underline{C})$ and the rate of return to saving is constant. Then, as households accumulate capital they move away from $\underline{C}$ and the growth rate of consumption increases.

What might the interest rate be in such markets? Since the gross rate of return on the new technology is B, an interest rate $r > B - 1$ would imply that there is no point in adopting the new technology and all households will immediately choose consumption paths consistent with the interest rate r. If $r < B - 1$, then there is an arbitrage opportunity for those who have adopted the new technology: they can make infinite profits by borrowing at a rate r and earning a rate of return greater than r. Hence, the interest rate in such a loan market must equal $B - 1$.

If a household decides to borrow, the only reason for doing so is to switch to the new technology immediately. Borrowing resources and operating the old technology is easily dominated by a strategy of lending at the interest rate r since $r > A - 1$. Consider a household with K_0 units of capital borrowing Λ units of goods and repaying the loan in constant amounts starting the next period. The per period payment, x, must equal $r\Lambda$. Now the household's resource constraint is modified as follows:

$$
\begin{aligned}
C_0 + K_1 &\leq BK_0, \\
C_t + K_{t+1} &\leq BK_t - r\Lambda \text{ for } t = 1, 2, \ldots.
\end{aligned}
$$

In period 0, the household uses the borrowed resources Λ to pay the fixed cost (or finance consumption). From period 1 on the household repays the loan by the constant amount $r\Lambda$ each period. We can rewrite the above constraint as

$$
\begin{aligned}
C_0 + K_1 - y &\leq B(K_0 - \frac{y}{B}), \\
C_t + K_{t+1} - y &\leq B(K_t - y) \text{ for } t = 1, 2, \ldots.
\end{aligned}
$$

where $y = \frac{r\Lambda}{B-1} = \Lambda$. Define $\hat{K}_t \equiv K_t - y$ for $t = 1, 2, \ldots$. Since $\frac{y}{B}$ is nothing but λ, one can think of the household's problem as follows: it begins with $K_0 - \lambda$ units of capital and has to choose a sequence $\hat{K}_1, \hat{K}_2, \ldots$ subject to the above constraint. It is easy to see that the value to the household is $H(K_0 - \lambda)$, and from the functional equation (6), this value is the same as $Z(K_0; 0)$. That is, the access to loans helps the household immediately adopt the new technology and realize the value $Z(K_0; 0)$.

The surprising result is that the presence of such loan markets does not make the household better off. To see this, suppose that $K_0 < K^*$. In this case, the household would be better off by ignoring the loan markets and waiting one or more periods before switching (see Proposition 1 and Lemma 5). If $K_0 \geq K^*$, the household would indeed switch immediately, but it would have done so with or without loans. Thus, the path of wealth and the threshold level of wealth in our model is robust to the introduction of loan markets.

5 Concluding Remarks

We have developed a model of costly technology adoption where the cost is irrecoverable and fixed. We study the path of wealth in this model using a recursive approach. We formulate a sequence of value functions, one for each possible adoption date, and show that there is a unique threshold level of wealth above which the household will adopt the more productive technology and below which it will not. This threshold is independent of initial wealth and depends only on technological parameters. Prior to adoption, the household saves at an increasing rate until it reaches the threshold level of wealth, but consumption growth is constant. We also show that richer households adopt sooner. These results are robust to whether the household has access to loan markets or not.

A crucial assumption in our model is the lack of markets that channel the endowments of those operating the low productivity technology to those operating the high productivity technology. Given that some farmers had already adopted the high productivity technology during the Green Revolution, an interesting question is why the remaining farmers did not rent out their factors of production.

Appendix

Proof of Proposition 1

Our proof of Proposition 1 proceeds along the three steps outlined in the text.

Lemma 5 For a household with $K \geq K^*$, it is better to switch immediately rather than wait one more period. Formally, $Z(K;0) \geq Z(K;1) \iff K \geq K^*$.

Proof. The functional equations (8) and (??) imply that

$$Z(K;0) \geq Z(K;1) \iff \log B + \log(K - \lambda) \geq \log(AK - \lambda).$$

The result follows from rearranging the right hand side. ■

Lemma 6 If a household has $K \geq K^*$, then it is better to switch now than wait a finite number of periods i.e., for $K \geq K^*$, $Z(K;0) \geq Z(K;T)$, $T = 2;3;\ldots$.

Proof. Suppose that the household prefers to switch T periods hence instead of switching immediately. Note from the optimal policy (10) that the growth rate of wealth exceeds βA. Since $\beta A \geq 1$ by assumption, the wealth $T-1$ periods hence will be greater than K^*. By Proposition (5) it is optimal to switch in period $T-1$ rather than wait until period T. A similar argument applies to $T-2$ versus $T-1$. Working backwards, it is easy to see that it is optimal to switch immediately. ■

Lemma 7 If a household has $K \geq K^*$, then it is better to switch to the new technology now than never switch i.e., for $K \geq K^*$, $Z(K;0) \geq L(K)$.

Proof. The functional equation (9) implies that

$$\lim_{T \to \infty} Z(K;T) = \log(1-\beta) + \frac{\beta \log \beta}{1-\beta} + \frac{1}{1-\beta} \log A + \log K \tag{15}$$

since $\beta \in (0;1)$ and $A > 1$. Note that the right hand side is the same as $L(K)$. Thus, the sequence $Z(K;T)$ converges to $L(K)$. The result follows from Lemma 6. ■

It is clear from lemmas 5, 6, and 7 that K^* is the threshold level of wealth, as stated in Proposition 1.

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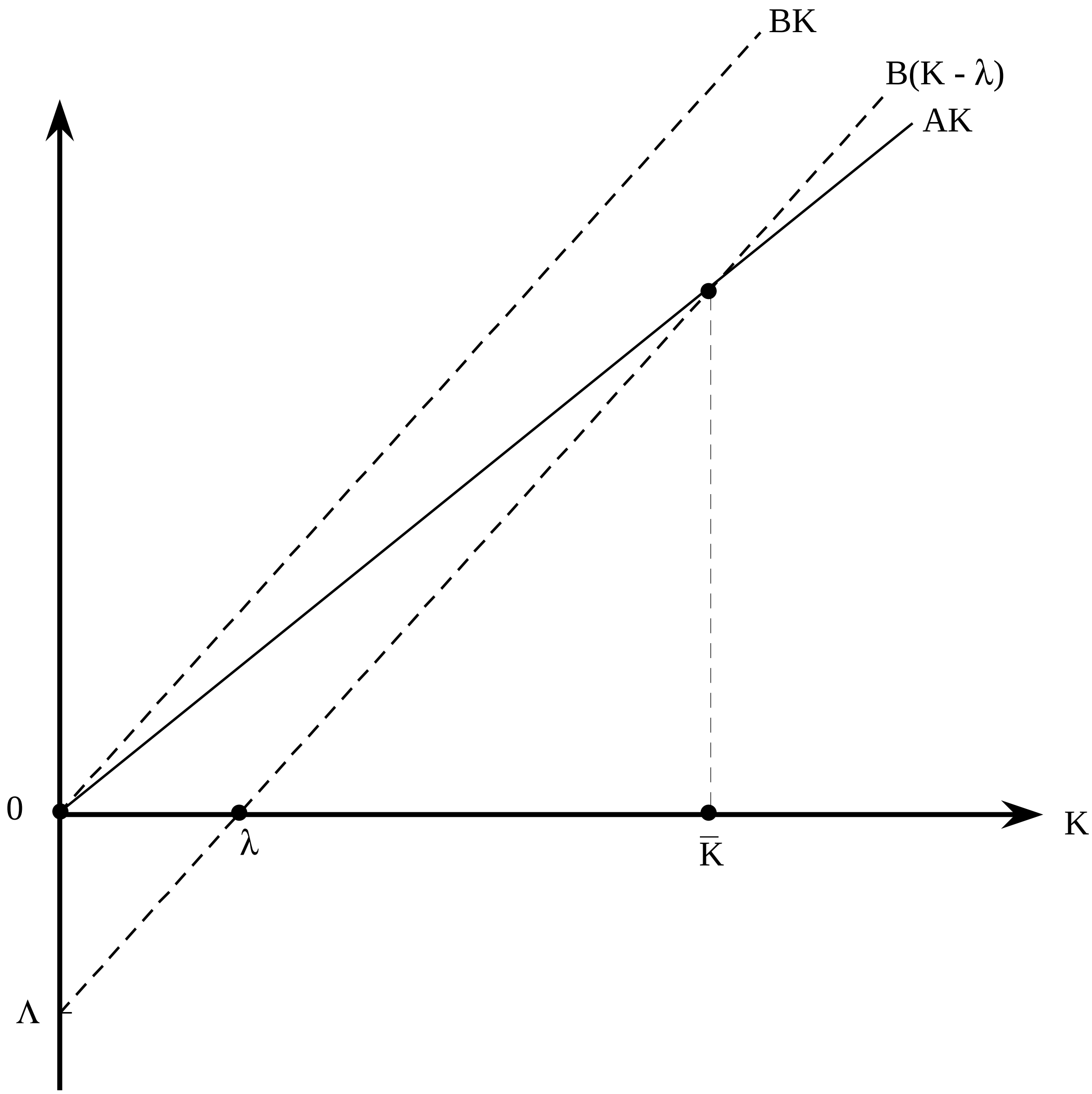

Figure 1. Bounds on the threshold capital

Figure 2. Low productivity technology and the period of adoption