# Productive Consumption and Growth in Developing Countries*

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**Abstract**: Productive consumption enables the satisfaction of current needs and, at the same time, increases the productive potential of labour. Theoretical as well as empirical evidence suggests that productive consumption is primarily relevant to low-income countries. From the perspective of growth theory, the productive consumption hypothesis is of fundamental interest because it modifies the "harsh" intertemporal consumption trade-off traditionally assumed. The analysis of the productive consumption hypothesis within a simple endogenous growth model reveals the following implications: (a) the possibility of a poverty-trap, (b) the rule of optimal consumption turns into a modified Keynes-Ramsey rule, (c) the (effective) IES is not only based on preferences but in addition on the technological possibilities to enhance human capital due to productive consumption, (d) a rising saving rate, and (e) transitional dynamics to an asymptotic balanced growth equilibrium.

Keywords: Productive Consumption, Human Capital, Growth in DCs

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JEL-Classification: D9, E21, I0, O1, O4, O41

# Contents

# 1 Introduction

The common interest of nearly all development and growth theories is the fundamental concept of a "harsh" intertemporal consumption trade-off: Current consumption inevitably reduces future consumption possibilities in a with-or-without sense. This is true for the early "low-level-equilibrium-trap" theories [Nelson (1956) and Leibenstein (1957)], the neoclassical growth theory [Solow (1956), Swan (1956), Ramsey (1928), Cass (1965), and Koopmans (1965)] as well as the endogenous growth theories [e.g. Lucas (1988), Romer (1990), and Rebelo (1991)].

In contrast, already since the fifties the possibility of productive consumption was recognised within development literature [Winslow (1951), Nurkse (1953), and in addition Wheeler (1980), Gersovitz (1983)].[1] *Productive consumption* enables the satisfaction of current needs and, at the same time, increases the productive potential of labour.[2] As a consequence, the potential for the satisfaction of future needs rises. Two interpretations of the productive effect of consumption can be distinguished: First, a rising level of per capita consumption can be considered to increase the efficiency of labour; this interpretation underlies the traditional efficiency wage theory [Leibenstein (1957), Stiglitz (1976), and Bliss and Stern (1978)]. Second, a rise in the level of per capita consumption can, on the other hand, be interpreted as increasing the stock of human capital [Blaug (1987)].

Gersovitz (1988) distinguishes three forms of productive consumption: (a) nutrition, (b) health, and (c) education.[3] All three forms serve the satisfaction of current needs, and, consequently, can be labelled as consumption expenditures; though occasionally this might be assessed differently in the case of education. Simultaneously, the efficiency of labour or – depending on the interpretation – the stock of human capital increases. From this point of view, the underlying consumption expenditures can be

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[1] The basic idea of productive consumption has already been recognised and discussed by classical economists in the context of the subsistence theory of wages [Mill (1848)]; for this see Blaug (1987).

[2] The question whether specific investment expenditures are registered as investment or consumption expenditures within the framework of national accounting is irrelevant for the purpose of theoretical analysis.

[3] See Gersovitz (1988), pp. 394.

classified as productive. Gersovitz (1988) expresses this notion as follows: „*Health and nutrition expenditures share some attributes of educational ones; they affect welfare beyond the period when they are made. To a much greater extent than in the case of education, however, these expenditures also affect current well-being, and it would be impossible to devise a convincing allocation of these expenditures between current and future consumption. For instance, at low nutritional levels, food consumption has joint effects on current and future well-being and productivity.*“[4]

For developing countries (DCs) special characteristics of preferences and technology exist, which are relevant to growth probably.[5] For example, intertemporal preferences are usually assumed to exhibit a constant time preference rate though a negative relation between the time preference rate and per capita income seems reasonable, especially for the lower range of income.[6] With respect to technology, the effect of enhancing the stock of a productive input (human capital) as a consequence of consumption activities presents a further characteristic relevant to growth, which is almost completely ignored in the context of growth theory.[7] After reviewing empirical evidence for a positive nutrition-productivity relation, Fogel (1994) stated recently: "*Although largely neglected by theorists of both the "old" and the "new" growth economics, these factors can easily be incorporated into standard growth models."*[8] The growth model presented in section 3 is an attempt to incorporate the productive consumption hypothesis into a simple endogenous growth model.

From the perspective of growth theory, the productive consumption hypothesis seems to be of fundamental interest because of two reasons: First, productive consumption essentially modifies, that is partially eliminates, the intertemporal

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[4] Gersovitz (1988), p. 396.

[5] Azariadis (1996) gives an overview of different growth models which can explain poverty-traps.

[6] This relation is known as the Fisher-hypothesis [Fisher (1907)]. Usually, if the time preference rate is allowed to be endogenous, a positive relation between the time preference rate and per capita income is postulated, mainly because of technical reasons in order to guarantee stability. For a discussion on this subject see Obstfeld (1990) as well as Zee (1994).

[7] To the best of my knowledge the only exception is Wichmann (1995), who formulates a two-sector growth model with the labour efficiency being dependent on nutrition. However, Wichman interprets the nutrition-productivity relation as an external effect which is not relevant for the individual choice of an optimal consumption path. Therefore, this model does not capture the crucial point considered in this paper, which is the modification of the harsh intertemporal consumption trade-off.

[8] Fogel (1994), p. 385/386.

consumption trade-off.[9] Second, theoretical as well as empirical evidence suggests a systematic, that is negative relationship between the level of per capita consumption and the marginal productive effect of consumption.[10] Concentrating on the importance of productive consumption for economic growth does surely not intend to neglect the importance of other factors that undoubtedly influence growth and development, e.g. the stability of the political system, the openness of the economy, or the development of the financial sector.

The rest of this paper is organised as follows: In section 2, a brief outline of the current theoretical and empirical work on the subject "productive consumption" is given first of all. In section 3, the hypothesis of productive consumption is specified in the form of a "human-capital-enhancement-function" and then integrated into a simple endogenous growth model. The transition process to an asymptotic balanced growth equilibrium is illustrated by means of a simulation. Section 4 summarises the main results and concludes with some final considerations.

# 2 Productive consumption: an Overview

## 2.1 Empirical evidence

The relation between labour productivity as well as output growth on the one hand and nutrition, health, and education on the other hand has been analysed empirically mainly against the background of two different questions: (a) In the wake of the traditional efficiency wage theory, it was attempted to uncover empirical evidence supporting or refuting the impact of nutrition and health expenditures on labour productivity within the framework of microeconomic empirical analyses.[11] (b) On the other hand, the contribution of nutrition, health, and education to output growth was examined on a macroeconomic level using the methodology of growth accounting. These empirical investigations were partly motivated by the question whether a development strategy primarily focusing on the satisfaction of basic needs prevents an

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[9] That is, as far as the possibility of productive consumption exists; the hypothesis of productive consumption does not assert that every consumption activity is productive.

[10] For empirical evidence see section 2.1. Within the framework of the efficiency wage theory, this assumption is illustrated in the form of the so-called effort-function; see section 2.2.

[11] Bliss and Stern (1978), Part II, outline the empirical investigations concerning the efficiency wage theory.

economy from growing or even stimulates growth [Hicks (1979), Wheeler (1980), and Barro and Sala-i-Martin (1995), chapter 12]. Some selected empirical investigations and their most important results are outlined in the following.

## Microeconomic analyses

On the basis of microeconomic cross-sectional data for small-scale farming enterprises in Sierra Leone (1974/75), Strauss (1986) estimates the coefficients of an agricultural Cobb-Douglas production function. The production function is specified to account for a dependence of the agricultural workers' efficiency upon daily nutrient intake per worker. The approach takes into account the simultaneity of household choices, the levels of variable farm inputs and it considers the possible influence of other variables on agricultural output, e.g. land quality. The coefficients of nutrient intake show the expected positive sign and are highly significant. The positive impact of nutrient intake on labour productivity is especially marked at low levels and decreases with an increasing level of calorie intake. The estimation results of Strauss (1986) imply remarkably high values for the output elasticity of nutrition at low levels of calorie intake. The corresponding values vary from 0.49 at a daily intake of 1500 calories via 0.34 at the sample mean value for a daily calorie intake up to 0.12 at a daily intake of 4500 calories.[12] Accordingly, at the mean value of daily calorie intake, an increase by 1 percent results in a rise in output by 0.34 percent.

Deolalikar (1988) investigates, based upon Strauss (1986), the relation between labour productivity in agriculture as well as the wage rate of rural workers on the one hand and individual calorie intake per day and weight-for-height (kg per cm) on the other hand by using panel data for 240 households in different rural areas of southern India (1976-77 and 1977-78). In this case, the weight-for-height variable is interpreted as a medium-term indicator of the nutritional status as well as an indicator of the health status. The results are not unambiguous: The coefficients of calorie intake per day are not significant, while the coefficients of weight-for-height prove to be significant. However, Deolalikar does not interpret these results as an evidence against a nutrition-productivity relationship: *"What the empirical results then suggest is that, even if the short-run effects of nutrition on labor productivity are insignificant, the medium-run* *effects are large and positive. ...Another interpretation may simply be that weight-for-height is a better indicator than average daily calorie intake.*“[13]

In the context of a mesoeconomic study, Ram and Schultz (1979) analyse the relation between the health status and labour productivity in agriculture on the basis of data for different Indian states. The rate of mortality is employed as an indicator of the health status in such a way that a decrease in the rate of mortality is interpreted as an improvement in the health status. Ram and Schultz regress the percentage change in rural labour productivity on the percentage change in the rate of mortality for the period from 1958 to 1967. This single regression explains 28 percent of the interstate variation in agricultural productivity; the corresponding coefficient has a value of 0.3 and is highly significant. Consequently, a reduction in the rate of mortality by 1 percentage point increases the labour productivity by 0.3 percentage points.[14]

## Macroeconomic analyses

On a macroeconomic level, Wheeler (1980) examines for 54 DCs the relation between the growth rate of output on the one hand and the growth rate of different indicators for the nutritional status (calorie availability per day), the health status (life expectancy at birth), and education (adult literacy rate), on the other hand, for the period from 1960 to 1970. For this purpose, Wheeler formulates a simultaneous four-equation model, consisting of a macroeconomic production function and one equation for nutrition, health, and education, respectively (which are called "welfare equations").[15] The production function includes capital in addition to labour in efficiency units as inputs, with the latter again depending on the level of nutrition, health, and education.[16]

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[12] See Strauss (1986), pp. 313; Wolgenmuth et. al. (1982) find similarly high values for output elasticity with respect to calorie intake for Kenyan road construction workers.

[13] Deolalikar (1988), p.412.

[14] It appears problematic to interpret the observed correlation in the sense of one-sided causality of an improvement in the health status resulting in an increase in labour productivity. Ram and Schultz refer in this context to enormous public health programmes and the abatement of malaria during the examined period and, therefore, interpret the fall in mortality rates as an exogenous variable.

[15] This model is purely econometric in order to derive estimation equations for the growth rate of output while explicitly taking into account simultaneity. The model is not able to achieve a theoretical analysis of the importance of the productive effects of nutrition, health, and education for the growth process.

[16] Consequently, the production function in Cobb-Douglas form reads: $Q_t = A_t K_t^{\gamma_1} (L_t H_t^{\theta_1} N_t^{\theta_2} E_t^{\theta_3})^{\gamma_2}$, where $Q_t$ denotes Output, $A_t$ the level of technology, $K_t$ the

The three "welfare equations" represent the level of nutrition, health, and education as a function of per capita income as well as some exogenous variables. By this formulation, a mutual causality between the growth rate of output on the one hand and the change in nutrition, health, and education on the other hand can be taken into consideration. Wheeler finds a strong labour augmenting effect of the nutrition and health variables in the determination of the change in output for "poor countries". The parameter estimates imply elasticities for labour in efficiency units with respect to nutrition of 11.14 and for labour in efficiency units with respect to health of 7.13. Multiplying these elasticity values with the value of the output elasticity with respect to labour in efficiency units yields the output elasticities for nutrition ($0.26 \cdot 11.14 \cong 2.90$) and for health ($0.26 \cdot 7.13 \cong 1.85$). These values are several times higher than the microeconomic elasticities of Strauss (1986). A possible interpretation is based on the assumption of positive external effects of labour efficiency: Well nourished, healthy, appropriately educated, and economically active individuals at the same time increase the productivity of other economic active individuals.[17] Surprisingly, Wheeler finds no significant influence of the education variable on the growth rate of output; not even at the 10 percent level of significance. Furthermore, the analysis of the productive contributions of nutrition, health, and education reveals a strong influence of nutrition and health especially for low per capita incomes and a decreasing marginal contribution with a rise in per capita income, while the positive influence of education increases with a rise in per capita income.[18] Wheeler summarises his econometric analysis with the words: *"Thus, the available data are shown to be consistent with the notion that some basic needs expenditures can legitimately be regarded as investments in human capital.*“[19]

The above-mentioned results are confirmed by Hicks (1979) insofar as he finds within the framework of different multiple regressions on the basis of cross-sectional data for 69 non-oilexporting DCs (1960-73), without exception, a strong and significant

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capital stock, $L_t$ physical labour, $H_t$ a health indicator, $N_t$ a nutritional indicator, and $E_t$ an indicator of the educational level at time t.

[17] See for this Lucas (1988), who assumes positive external effects of the average human capital stock in the production of output. However, a direct comparison between the results of Strauss (1986) and the results of Wheeler (1980) is problematic on account of different references to space and time.

[18] It should be noted that the data set records only countries with low and middle per capita income and no countries with high per capita income.

[19] Wheeler (1980), p. 450.

influence of different "basic-needs" indicators (life expectancy at birth, adult literacy rate, primary school enrolment rates) on the growth rate of real per capita income. Finally, the results of a broadly designed cross-sectional analysis by Barro (1991) are pointed out, according to which life expectancy at birth, interpreted as a nutrition and health indicator, is positive and highly significant for the explanation of the growth rate of real per capita income; whereas the results for various indicators of the educational status are not unambiguous.[20]

Lastly, Fogel (1994) estimates the importance of a nutrition-productivity relationship for the development process of Britain. He concludes that improvements in nutrition explain 30 per cent of per capita income growth between 1790 and 1980. One third of this effect is assigned to increased labour force participation while the remaining two thirds are assigned to an increased labour productivity in production.

## 2.2 Theoretical approaches

### 2.2.1 Preliminary remarks

Traditional efficiency wage theory assumes a positive relation between the level of consumption and the efficiency of labour. This hypothesis bears far-reaching theoretical implications with respect to the labour market: Profit-maximising producers are willing to pay that wage rate which minimises labour cost per efficiency unit of labour; this wage is called the efficiency wage. If the market-clearing wage rate lies below the efficiency wage, unemployment arises. The efficiency wage theory is, above all, an approach to explain the widespread phenomenon of rural unemployment in DCs [Stiglitz (1976) and Bliss and Stern (1978)].[21]

The implications of a positive relationship between consumption and the efficiency of labour - or in other words the implications of the productive consumption hypothesis - for the consumption/saving behaviour were hardly analysed within the economic

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[20] See also Barro and Sala-i-Martin (1995), chapter 12.

[21] During the eighties, the efficiency wage theory was further developed within the framework of new-keynesian theory and used to explain real wage rigidities and unemployment in industrialised economies; for an overview see Yellen (1984) and Romer (1996) chapter 10.

literature.[22] Gersovitz (1983) represents an important exception. He discusses two complementary approaches, which explicitly analyse the importance of productive consumption for the consumption/saving behaviour within the framework of a two-period model.[23] Both approaches are based on the idea that consumption might have a second positive effect in addition to the direct satisfaction of current needs. This consists in an increase in the probability of survival and in an increase in the efficiency labour. Both approaches offer the possibility to derive a positive relation between the average saving rate and income on a sound microeconomic foundation.[24] On account of the exceptional importance of both approaches for the theoretical analysis both are outlined in their essential features.

### 2.2.2 Consumption and the probability of survival

The crucial hypothesis of this model is a positive relation between the standard of living and the probability of survival for the lower range of income. Consequently, consumption increases welfare in two different respects: The satisfaction of current needs means a (traditional) direct utility effect. The indirect utility effect of consumption consists in an increase in the probability of survival. The importance of this additional consumption effect falls with an increase in the standard of living.

The individual considered exists for two periods (presence and future) and solely receives income in the first period. This income ($y$) is divided up between current consumption ($c_1$) and savings ($s$); future consumption ($c_2$) equals current savings multiplied by an interest rate factor ($R$):

$$y = c_1 + s, \tag{2.1}$$

$$c_2 = R \cdot s. \tag{2.2}$$

[22] Empirical investigations of the saving behaviour in DCs usually test the validity of the absolute-income-, the permanent-income- and the life-cycle hypothesis; see Mikesell and Zinser (1973). For more current investigations see Aghevli et. al. (1990) and Reichel (1993).

[23] However, Gersovitz does not speak of productive consumption but, for example, of the *"physiological consequences of poor nutrition associated with low income"*, Gersovitz (1983), p. 842.

[24] Such a relation already results from the keynesian absolute-income hypothesis which is, however, based on the ad hoc assumptions of an autonomous consumption and a constant marginal rate of consumption.

The probability of surviving the first period ($\pi$) increases with the level of consumption during the first period, furthermore a concave and twice continuously differentiable "survival-probability-function" is assumed:

$$\pi = \pi(c_1), \qquad \text{with } \pi' > 0 \text{ and } \pi'' \leq 0 . \tag{2.3}$$[25]

The individual chooses $c_1$ and $c_2$, in order to maximise the expected lifetime utility ($Eu$),

$$Eu = u(c_1) + \pi(c_1) \cdot u(c_2) , \tag{2.4}$$

with respect to (2.1) and (2.2).[26] The instantaneous utility function [$u(c)$] is also assumed to be concave and twice continuously differentiable. Application of the Lagrangian method yields the first-order conditions for an interior solution ($c_1, c_2 > 0$):

$$u'(c_1) + \pi'(c_1) \cdot u(c_2) = R \cdot \pi(c_1) \cdot u'(c_2) . \tag{2.5}$$

For an optimal solution, the marginal gain in welfare due to an increase in current consumption equals the marginal gain in welfare due to an increase in future consumption. The left-hand side of (2.5) shows the marginal increase in welfare resulting from current consumption, which consists of two components. A (marginal) increase in current consumption causes the expected lifetime utility to rise according to the marginal utility of current consumption [$u'(c_1)$] and, additionally, according to a rise in the expected future utility, resulting from an increase in the probability of survival [$\pi'(c_1) \cdot u(c_2)$].[27] The right-hand side of (2.5) shows the marginal increase in welfare due to an increase in future consumption. The probability of experiencing the future is, of course, considered for the calculation of the expected value.

Gersovitz discusses two threshold effects: Below a subsistence level of consumption ($\tilde{c}$), survival is impossible [$\pi(c_1 < \tilde{c}) = 0$]. If income does not exceed this value,

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[25] Gersovitz justifies the form of this function merely with reference to the plausibility of this assumption; see Gersovitz (1983), p. 844.

[26] Discounting future utility by means of a time preference rate is not considered in (2.4); future utility is nonetheless "discounted" by means of multiplication with the probability of survival which is by assumption smaller than one.

saving is zero, $c_2 = s = 0$. A second threshold effect appears as soon as the initial level of consumption exceeds a value above which no further influence on the probability of survival exists, $\pi(c_1 > \hat{c}_1) = \overline{\pi}$. Without this "survival effect" of consumption ($\pi \equiv 1$), condition (2.5) turns into the usual optimum condition:

$$u'(c_1) = R \cdot u'(c_2)\,. \tag{2.6}$$

As long as the utility function is isoelastic and the interest rate equals zero, a positive relation between the average saving rate and income exists, provided that:

$$\gamma + \lambda \cdot (\eta - 1) > 0\,, \qquad \text{where } \lambda \equiv \frac{c_2}{c_1} \text{ and } \eta \equiv -\frac{\pi''}{\pi'} c_1\,, \tag{2.7}$$

and $\gamma$ denotes the elasticity of utility. Condition (2.7) is fulfilled whenever the elasticity of the marginal probability of survival with respect to consumption ($\eta$) is greater than one.[28] Provided that the marginal probability of survival declines sufficiently fast in response to current consumption, the individual is willing to increase future consumption more than proportionately as income rises, thereby increasing the saving rate.[29]

### 2.2.3 Consumption and labour productivity

As mentioned above, the traditional efficiency wage theory assumes a positive impact of the individual wage rate on the efficiency of labour. In this context, a higher wage rate is implicitly assumed to induce an increase in the level of consumption. Due to a "physiological-technological" relationship a higher productivity per man-hour results. By means of a second model, Gersovitz (1983) analyses the resulting implications for the individual saving behaviour. He describes the productive consumption effect and its possible implication for the saving behaviour as follows:

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[27] A sufficient condition for a maximum is a concave curved "survival-probability-function"; the validity of this assumption is presupposed by Gersovitz (1983).

[28] The elasticity of the marginal probability of survival with respect to consumption is defined as $\eta \equiv -\frac{d\pi'}{dc} \cdot \frac{c}{\pi'} = -\frac{\pi'' \cdot c}{\pi'}$.

[29] For further interpretations see Gersovitz (1983), p. 845.

*"Greater current consumption adds to utility directly and indirectly by increasing income, thereby creating a bias against saving"*.[30]

The crucial hypothesis of consumption ($c_1$) enhancing the efficiency of labour ($h$) is represented by a concave and twice continuously differentiable "effort-function". Thus, it is supposed in accordance with efficiency wage literature, that consumption increases the efficiency of labour without any delay:[31]

$$h = h(c_1), \qquad \text{with } h' \geq 0. \tag{2.8}$$

The individual considered exists for two periods, the entire income is received exclusively during the first period and experience of the second period is – in contrast to the former model – certain. Current and future consumption are chosen in order to maximise total utility,

$$V = u(c_1) + u(c_2), \tag{2.9}$$

subject to the constraints,

$$R \cdot s = c_2, \tag{2.10}$$

$$c_1 + s = y = w \cdot h(c_1) + \alpha. \tag{2.11}$$

In this case $w$ denotes the wage rate per efficiency unit of labour [i.e. the wage rate per man-hour in relation to one unit of efficient labour ($w_0 / h$)], $h(c_1)$ the efficiency of labour, so that $w \cdot h(c_1)$ represents the wage income[32] and $\alpha$ all components of non-wage income. The first-order condition for an interior solution reads:

$$u'(c_1) = -R \cdot u'(c_2) \cdot (w \cdot h' - 1). \tag{2.12}$$

Taking into consideration the presumed positive marginal utility, condition (2.12) can only be fulfilled if the following inequality holds:

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[30] Gersovitz (1983), p. 848.

[31] Gersovitz uses an effort-function $h(c)$ which possesses a mixed convex-concave form. Within the efficiency wage literature, convex-concave effort-functions as well as globally concave effort-functions are employed; see Rosenzweig (1988), pp. 720.

[32] In this case it must be supposed that the individual inelastically supplies one unit of labour or that $w_0$ is the wage payment for the whole working time.

$$(w \cdot h' - 1) < 0 \quad \text{or} \quad w \cdot h' < 1. \tag{2.13}$$

The interpretation of condition (2.13) is as follows: Saving necessarily means a reduction in current consumption. Consequently, the efficiency of labour and, therefore, the wage income decreases in accordance with the effort-function. The condition $wh' < 1$ means that further saving (renunciation of current consumption) by one unit can only be reasonable if the induced fall in income turns out to be smaller. The bias toward current consumption in the case of low incomes becomes clear if (2.12) is transformed to:

$$u'(c_1) = R \cdot u'(c_2) - R \cdot u'(c_2) \cdot w \cdot h'. \tag{2.12a}$$

For comparably low incomes and, consequently, ceteris paribus low consumption levels, $h'$ is relatively high, and the value of the right-hand side of (2.12 a) is relatively small. Hence, a low marginal utility of consumption in the first period (left-hand side) and, taking into account the concavity of the utility function, a comparably high level of current consumption results. This effect disappears with a rise in income and for $h' = 0$ (2.12 a) turns into the usual optimum condition.

The average saving rate rises with income provided that the following condition holds:[33]

$$(1 + \lambda) \cdot (\varepsilon - 1) + wh' - \mu\varepsilon > 0,$$

$$\text{with } \varepsilon \equiv \frac{-h''}{h'} c_1, \ \mu \equiv \frac{\alpha}{c_1} \text{ and as before } \lambda \equiv \frac{c_2}{c_1}. \tag{2.14}$$

Provided that the individual has no non-wage income ($\alpha = 0$), $\varepsilon > 1$ is a sufficient condition for the saving rate to increase with income. Accordingly, the marginal attractiveness of current consumption as a result of the efficiency and wage increasing effect must fall sufficiently fast.[34]

[33] In addition, an isoelastic instantaneous utility function and an interest rate equal to zero is assumed as before; see Gersovitz (1983), p. 850.

[34] For further interpretations see Gersovitz (1983), p. 850.

# 3 The importance of productive consumption for growth

## 3.1 A linear growth model with productive consumption

### 3.1.1 The human-capital-enhancement-function

The above-reviewed empirical studies investigating the relationship between labour productivity and output growth on the one hand and nutrition, health, and education on the other hand clarify two essential points:

(a) A rise in the level of nutrition, health, and education increases the productive potential of labour, and

(b) with an increasing level of nutrition, health, and education, the importance of the marginal effect on the productive potential of labour decreases.[35]

Nutrition and health expenditures are clearly made in order to satisfy current needs and can be classified as consumption; in the case of education, this does not follow unambiguously. In fact, a considerable part of the educational activities may not be considered as pure pleasure and is probably conceived as a traditional investment activity.[36]

To analyse the implications of productive consumption in the context of growth, the productive consumption effect is interpreted as enhancing the stock of human capital. This central hypothesis is specified in the form of a *human-capital-enhancement-function*. In its intensive form, this concave and twice continuously differentiable function reads:

$$\dot{h}(t) = \phi[c(t)] - (n+\delta)\cdot h(t), \qquad \text{with } \phi'(c) > 0 \text{ and } \phi''(c) < 0. \tag{3.1}$$

In this case $h(t)$ denotes the stock of human capital per capita at time $t$, $c(t)$ consumption per capita, $\delta$ the depreciation rate of human capital, and $n$ the population

[35] Both propositions must be qualified in the case of education; for this see section 2.1.

[36] Recent growth models, which intend to endogenise labour supply, indirectly interpret education as satisfying needs. In addition to consumption, the utility function contains "leisure in efficiency units" as the product of leisure and the individual stock of human capital as an argument; see Ladrón-de-Guevara, Ortigueira, and Santos (1997), pp.130. Moreover, see Lazear (1977), who theoretically and empirically analyses the question whether education should be regarded as a production or consumption process.

growth rate, respectively.[37] Equation (3.1) represents the equation of motion for the average stock of human capital. As a result of productive consumption activities, the stock of human capital per capita increases according to the function $\phi[c(t)]$, while it decreases due to depreciation and population growth. Consequently, $\phi[c(t)]$ can be designated as the gross human-capital-enhancement-function. The positive, but decreasing marginal human-capital-enhancement-effect of consumption $[\phi'(c) > 0, \phi''(c) < 0]$ appears justified by the empirical evidence. The "smooth" shape may not be reasonable at an individual level. However, this assumption hardly appears problematic at an aggregate level, that is if (3.1) is interpreted in the sense of an average human-capital-enhancement-function.

On account of its static character, the traditional efficiency wage theory was forced to assume that consumption increases the efficiency of labour without any delay. In contrast, equation (3.1) indicates a human-capital-enhancing-effect of consumption for the next period. The above-stated formulation of the human-capital-enhancement-function means that human capital is formed exclusively as a result of productive consumption. Consequently, formal education as far as it represents an investment decision and learning-by-doing effects are ignored. The representation of productive consumption effects together with, for example, formal education is naturally possible within the framework of a comprehensive production function for human capital:

$$\dot{h}(t) = B \cdot [1 - u(t)] \cdot h(t) + \phi[c(t)] - (\delta + n) \cdot h(t) . \tag{3.2}$$

The first term on the right-hand side equals the production function for human capital corresponding to the Uzawa-Lucas model.[38]

Several authors have recently incorporated a subsistence level of consumption into different growth models by means of Stone-Geary preferences [Rebelo (1992), Ben-David (1994), Easterly (1994), Sarel (1994)]. The concept of a subsistence level of consumption can be related to the human-capital-enhancement-function. The subsistence level of consumption can be interpreted to denote the income level below

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[37] To the best of my knowledge, the only model which postulates consumption to be productive in the sense of increasing the stock of human capital is Becker and Murphy (1988), though they call it "consumption capital".

[38] See Lucas (1988), p. 18.

which survival is impossible, that is the physical minimum of existence.[39] The subsistence level of consumption can be considered to represent the minimum gross enhancement of human capital to cover depreciation.[40] The differences between the concept of subsistence consumption and the human-capital-enhancement-function are the following: (a) the former represents a modification of the standard preference formulation while the later represents a modification of the technological opportunities, and (b) the human-capital-enhancement-function continuously accounts for productive consumption effects beyond the subsistence level.

The hypothesis of productive consumption in the form of equation (3.1) or (3.2) is not an assumption which serves primarily for abstraction, that is reducing the complexity of the real world. It rather constitutes a crucial assumption for the growth model presented in the next section, which focuses on important aspects of capital accumulation and growth in DCs.

### 3.1.2 The model

The linear growth model [Romer (1986), Barro (1990), Jones and Manuelli (1990), Rebelo (1991); in addition see Barro and Sala-i-Martin (1995), chapter 4] is a fairly simple endogenous growth model. Due to its simple structure, it shows very clearly the conditions for permanent growth as well as the main implications of the endogenous growth theory. Permanent growth is guaranteed by constant returns to scale in the factors that can be accumulated as well as a sufficiently high marginal productivity of these factors.[41] The steady-state growth rate is determined by technology and preference parameters, so that internationally different parameter values can explain internationally different growth rates. A special feature of the linear growth model is that the steady-state is realised at any point in time; consequently, transitional dynamics do not exist. In

[39] The subsistence level of consumption is sometimes denoted as the minimum absolute poverty line; for details and empirical estimates see World Development Report (1990).

[40] Fogel (1994) reports estimates of the daily calorie requirements to cover the baseline maintenance (the energy to keep the body functioning, that is basal metabolism plus energy for digestion of food and vital hygiene) for American males in the 18th century; see Fogel (1994), p.372.

[41] For a derivation of the general conditions of permanent growth see Jones and Manuelli (1997).

the following, a modified linear growth model extended by productive consumption is presented in a general form using continuous time notation.[42]

Every representative household has access to a one-sector production technology with capital [ $k(t)$ ] as the only input, which is employed to produce an output good [ $y(t)$ ] that can be used universally for investment or consumption purposes. The production function in its intensive form reads:

$$y(t) = f[k(t)] \quad \text{with} \quad f(0) = 0 \quad \text{and} \quad f'(k) = A. \tag{3.3}$$

The production function is linear homogeneous and, hence, the marginal productivity of capital ( $A$ ) is constant. The absence of diminishing returns is crucial for the generation of endogenous growth and can be justified mainly by two interpretations: First, capital is thought to exhibit positive spill-over effects or, second, capital can be interpreted broadly. Following this second interpretation, capital is defined to encompass physical as well as human capital. In addition, both types of capital can be additively aggregated if they are assumed to be perfect substitutes in production:

$$k(t) = k_p(t) + k_h(t). \tag{3.4}$$

Physical capital per capita [ $k_p(t)$ ] increases as a consequence of the renunciation of consumption, taking into account the rate of depreciation ( $\delta$ ) as well as the rate of population growth ( $n$ ):

$$\dot{k}_p(t) = f[k(t)] - (n+\delta) \cdot k_p(t) - c(t). \tag{3.5}$$

Human capital per capita [ $k_h(t)$ ] is exclusively formed by productive consumption; the equation of motion for human capital equals the human-capital-enhancement-function:

$$\dot{k}_h(t) = \phi[c(t)] - (n+\delta) \cdot k_h(t). \tag{3.6}$$

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[42] The linear growth model is used as a framework primarily because of analytical reasons. The model implies a constant saving rate and constant growth rates. Consequently, any pattern of a changing saving rate and the occurrence of transitional dynamics can directly be assigned to the productive consumption hypothesis.

The gross human-capital-enhancement-function $\phi[c(t)]$ is assumed to be strictly concave and twice continuously differentiable with asymptotically vanishing first and second derivative:

$$\phi'(c) > 0\,,\ \phi''(c) < 0\,,\ \lim_{c\to\infty} \phi'(c) = 0 \ \text{ and, hence, } \ \lim_{c\to\infty} \phi''(c) = 0\,. \qquad (3.7)$$

Using the simplifying assumption of identical depreciation rates, the equation of motion for the whole stock of capital per capita reads according to (3.4) as follows:

$$\dot{k}(t) = f[k(t)] - (n+\delta)\cdot k(t) - \psi[c(t)] \quad \text{with} \quad \psi[c(t)] \equiv c(t) - \phi[c(t)]. \quad (3.8)$$

Usually, consumption is costly because it fully reduces net investments, that is the accumulation of capital. In the present context, consumption partially contributes to the accumulation of human capital. . Consequently, $\psi(c)$ can be designated as the *net cost of consumption* (*ncc*) which consists in consumption less the human-capital-enhancement effect of consumption.[43]

The representative household is assumed to maximise its dynastic lifetime utility. The corresponding dynamic optimisation problem is a concave, infinite time problem of optimal control with one control [ $c(t)$ ] and one state variable [ $k(t)$ ] as well as a bounded control set:

$$\max_{\{c(t)\}} \int_0^\infty u[c(t)]\cdot e^{-(\rho-n)\cdot t} dt$$

$$\dot{k}(t) = f[k(t)] - (\delta+n)\cdot k(t) - \psi[c(t)]$$

$$k(0) = k_0\,,\ k(t) \geq 0$$

$$0 \leq c(t) \leq f[k(t)], \qquad (3.9)$$

where $c(t)$ denotes per capita consumption at time $t$, $u[c(t)]$ the instantaneous utility function, $\rho$ the individual time preference rate, and $n$ the constant growth rate of population, respectively. The instantaneous utility function is assumed to be strictly concave [ $u'(c) > 0$ and $u''(c) < 0$ ] and to possess a constant elasticity of marginal

[43] I thank Karl-Josef Koch for suggesting the clarifying expression "net cost of consumption".

utility with respect to consumption ($\sigma \equiv -\frac{u''(c)\cdot c}{u'(c)}$), that is a constant intertemporal elasticity of substitution ($1/\sigma$).

The marginal *ncc* [$\psi'(c) = 1 - \phi'(c)$] is negative as long as the marginal human-capital-enhancement effect of consumption is greater than one. In this case, it clearly makes no sense to refrain from consumption and, therefore, saving must be zero. Moreover, rational individuals would try to dissave whenever this possibility arises. However, since only human capital has been accumulated in the past and the transformation of human capital into consumption goods seems to be impossible, the (state-dependent) inequality constraint on the control [$c \leq f(k)$] must be imposed and will turn out to be effectively binding at early stages of economic development.

The Lagrangian and the current-value Hamiltonian for the maximisation problem (3.9) read as follows:[44]

$$L(c,k,\lambda,v_1,v_2) = H(c,k,\lambda) + v_1 \cdot [f(k) - c] + v_2 \cdot c\,, \tag{3.10}$$

$$H(c,k,\lambda) = u(c) + \lambda \cdot [f(k) - (\delta + n)\cdot k - \psi(c)]\,. \tag{3.11}$$

The Hamiltonian (3.11), which represents the contribution of current output allocated to consumption and investment to the overall benefit, illustrates the "dual welfare-effect" of (productive) consumption. The first part shows the direct contribution to utility [$u(c)$]. As already mentioned, productive consumption reduces the *ncc* [$\psi(c)$] according to the human-capital-enhancement-effect of consumption [$\phi(c)$]. The second part of the Hamiltonian, representing the net increase in the capital stock evaluated at the current-value shadow price ($\lambda$), captures this mechanism.[45]

[44] For a presentation of optimal control theory with bounded control sets, see Kamien and Schwartz (1990), section 10. Especially for optimal control theory with state-dependent inequality constraints on the control variable, see Feichtinger and Hartl (1986), chapter 6. In order to simplify the notation, the time index will be suppressed in the following.

[45] For an economic interpretation of the maximum principle and the Hamiltonian see Dorfman (1969).

The application of the maximum principle leads to the first-order conditions, where $\nu_1$ and $\nu_2$ denote the dynamic Lagrangian multipliers associated with each of the inequality constraints stated in (3.9):[46]

$$\frac{\partial L}{\partial \lambda} = \dot{k} = f(k) - (\delta + n) \cdot k - \psi(c), \tag{3.12}$$

$$\dot{\lambda} = \lambda \cdot (\rho - n) - \frac{\partial L}{\partial k} = \lambda \cdot [\rho + \delta - f'(k)] - \nu_1 \cdot f'(k), \tag{3.13}$$

$$\frac{\partial L}{\partial c} = u'(c) - \lambda \cdot \psi'(c) - \nu_1 + \nu_2 = 0, \tag{3.14}$$

$$\nu_1 \geq 0, \qquad \nu_1 \cdot [f(k) - c] = 0, \tag{3.15}$$

$$\nu_2 \geq 0, \qquad \nu_2 \cdot c = 0. \tag{3.16}$$

### 3.1.3 Implications

The first-order conditions stated above indicate that boundary solutions have to be distinguished from interior solutions. Furthermore, within interior solutions, the transition process can be distinguished from the asymptotic balanced growth equilibrium. In order to illustrate the implications of the linear growth model with productive consumption, three ranges are distinguished: The *no-saving range*, the *transition range* and the *asymptotic range*.

#### No-saving range

The first-order conditions (3.14), (3.15), and (3.16) imply a boundary solution with $c = f(k)$ if the following weak inequality holds:

$$u'(c) \geq \lambda \cdot \psi'(c). \tag{3.17}$$

[46] Because the Hamiltonian is concave in the control and the state, the necessary conditions are also sufficient for a maximum. In addition to the first-order conditions an optimal trajectory must satisfy the transversality condition: $\lim_{t \to \infty} e^{-(\rho - n) \cdot t} \cdot \lambda(t) \cdot k_p(t) = 0$. Note that the transversality condition only applies to capital which results from the renunciation of consumption, that is physical capital.

As long as the marginal utility of consumption exceeds the marginal *ncc* measured in units of utility, an increase in consumption approaching the upper limit of the control set [ $k \geq 0$, $0 \leq c \leq f(k)$ ] seems to be rational and, consequently, saving is zero.

Within the no-saving range the evolution of the economy is determined by the equation of motion for capital (3.8), taking into account that saving is zero:

$$\dot{k} = \phi[f(k)] - (\delta + n) \cdot k . \qquad (3.18)$$

Note that an increase in capital per capita is possible in the present model although the saving rate is zero. As a consequence of the properties of the human-capital-enhancement-function, the differential equation in the capital stock (3.18) possesses a stable equilibrium point whenever the sum of the depreciation rate and the population growth rate is positive. If the stationary value of capital per capita lies inside the no-saving range, the economy runs into a *poverty-trap*: Individuals do not save during this early stage of economic development and no physical capital is accumulated. However, human capital is created as a consequence of productive consumption activities. Because of the diminishing marginal human-capital-enhancement-effect of consumption, the net human capital accumulation decreases and may approach zero before the end of the no-saving range is reached and the accumulation of physical capital sets in.[47]

The growth rates of consumption per capita and output per capita are equal to the growth rate of capital per capita:

$$\frac{\dot{c}}{c} = \frac{\dot{y}}{y} = \frac{\dot{k}}{k} = \frac{\phi[f(k)]}{k} - (\delta + n) . \qquad (3.19)$$

The growth rate of consumption per capita in (3.19) is independent of preferences. That is, the intertemporal elasticity of substitution (IES) is effectively zero within the no-saving range.

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[47] Surprisingly, the possibility of a poverty-trap even opens-up within the framework of an endogenous growth model. One can expect that productive consumption within a neoclassical model increases the probability of a poverty-trap.

## Transition range

The first-order conditions (3.14), (3.15), and (3.16) imply an interior solution ($0 < c < y$) if the following equality holds: [48]

$$u'(c) = \lambda \cdot \psi'(c). \tag{3.20}$$

As soon as the marginal utility of consumption equals the marginal *ncc* measured in units of utility, the optimal trajectory leaves the boundary and runs into the interior of the open control set [$k \geq 0,\ 0 \leq c \leq f(k)$]; provided that the poverty-trap did not become binding. Along the optimal trajectory, equality (3.20) holds as a necessary optimum condition. Taking into account the definition of the marginal *ncc* [$\psi'(c) = 1 - \phi'(c)$] as well as the concavity of the utility function and comparing (3.20) to the usual optimum condition [$u'(c) = \lambda$]**,** it becomes clear that the level of consumption is higher compared to a situation without productive consumption effects, as one would expect.

Differentiation of equation (3.20) with respect to time, subsequently dividing the result by the original relation (3.20), eliminating the shadow price using equation (3.13), and noting that $v_1$ is zero for an interior solution yields the optimal growth rate of consumption per capita:

$$\frac{\dot{c}}{c} = \frac{1}{\sigma - \eta_{\psi'}(c)}\left(f'(k) - \delta - \rho\right)$$

with $$\eta_{\psi'}(c) \equiv -\frac{c \cdot \psi''(c)}{\psi'(c)} = \frac{c \cdot \phi''(c)}{1 - \phi'(c)} < 0. \tag{3.21}$$ [49]

The evolution of the economy in the case of interior solutions is governed by the differential equations in the state (3.8) and in the control (3.21). The second expression of the denominator on the right-hand side of (3.21) denotes the *elasticity of the marginal*

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[48] The third possibility of $c = 0$ would require $u'(c) \leq \lambda \cdot \psi'(c)$. On account of the characteristics of the utility function and the human-capital-enhancement-function as well as a positive shadow price of capital, the case of $c = 0$ does not emerge.

*ncc* with respect to consumption [ $\eta_{\psi'}(c)$ ]. Corresponding to the nature of the human-capital-enhancement-function, $\eta_{\psi'}(c)$ is negative while its absolute value decreases with an increasing level of per capita consumption and asymptotically approaches zero.

Equation (3.21) is the *modified Keynes-Ramsey rule* of optimal consumption/saving. The ratio on the right-hand side of (3.21) can be designated *effective IES* (*eIES*). It is worth noting that the *eIES* is not exclusively determined by the instantaneous utility function but that it is additionally dependent on the technological possibilities to enhance the stock of human capital as a result of productive consumption. The *eIES* expresses the willingness to substitute consumption intertemporally, taking into account a change in the marginal utility as well as a change in the marginal *ncc*. Along the (infinite) transition to the asymptotic balanced growth equilibrium the *eIES* and the growth rate of consumption per capita increase.

With respect to (3.21), one could argue that productive consumption has no impact on growth rates if one only assumes the human-capital-enhancement-function to imply a constant elasticity of the marginal *ncc*; analogous to the use of constant intertemporal elasticity of substitution (CIES) utility functions. The class of human-capital-enhancement-functions which give rise to a constant elasticity of the marginal *ncc* can be shown to be of the following form (see the appendix):

$$\phi(c) = c - \frac{a_1}{1-\eta} c^{1-\eta} + a_2 , \qquad (3.22)$$

where $a_1$ and $a_2$ are some positive constants and $\eta$ denotes the (constant) elasticity of the marginal *ncc*. However, it can be shown that (3.22) cannot fulfil the requirements of $\phi'(c) > 0$ and $\phi''(c) < 0$, stated in (3.7). In addition, the conventional case of consumption inducing no productive effects at all can be considered as a special case with $a_1 = 1$, $a_2 = 0$ and $\eta = 0$.

In order to give a clear economic interpretation of the *modified Keynes-Ramsey rule* equation (3.21) is slightly reworded to:

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[49] The validity of the so-called growth condition $f'(k) > \rho + \delta$ is assumed. Equation (3.21) is only valid for an interior solution, where the marginal *ncc* must be smaller than one. Consequently, the denominator of $\eta_{\psi'}(c)$ cannot become negative and $\eta_{\psi'}(c)$ is well defined.

$$f'(k) = \rho + \delta - \frac{u''(c) \cdot \dot{c}}{u'(c)} + \frac{\psi''(c) \cdot \dot{c}}{\psi'(c)}. \tag{3.23}$$

The third term on the right-hand side of (3.23) is the percentage time rate of change of the marginal utility which represents "the psychic cost of saving". The last term on the right-hand side is the percentage time rate of change of the marginal *ncc*. Holding an additional unit of capital during a short interval of time causes a rising consumption profile and induces a rise of the marginal *ncc*. Along the optimal path the rate of consumption at each moment must be chosen so that the marginal productivity of capital covers four components: The time preference rate, the depreciation rate, the psychic cost of saving, and in addition the rise in the marginal *ncc*.[50]

The saving rate is zero at the beginning of the transition process and can be shown to converge asymptotically toward a positive constant, which equals the saving rate of the original linear growth model (see the appendix and section 3.2, figure 3).

What about the growth rate of per capita income? Whether it decreases or increases seems not to be unequivocal a priori. Differentiation of the growth rate of per capita income with respect to time yields:

$$\frac{\partial}{\partial t}\frac{\dot{k}}{k} = \frac{\partial}{\partial t}\frac{\phi(c)}{k} - \frac{\partial}{\partial t}\frac{c}{k} \tag{3.24}$$

The first term on the right-hand side of (3.24) represents the time rate of change of the growth rate of human capital per capita while the second term on the right-hand side of (3.24) represents the time rate of change of the growth rate of physical capital per capita (see the appendix). Because no explicit solution can be found for the differential equation system (3.8) and (3.21), the time path of the growth rate of capital per capita and income per capita is analysed by means of a numerical solution (section 3.2).

## Asymptotic range

The linear growth model with productive consumption does not possess a balanced growth equilibrium defined by constant growth rates. However, its asymptotic properties are characterised by the asymptotic balanced growth path ($BGP^{\infty}$). Inside the interior of

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[50] For an economic interpretation of the Keynes-Ramsey rule see Dorfman (1969), p. 825.

the control region, the optimal trajectory asymptotically converges to this $BGP^{\infty}$, along which the growth rates of all endogenous variables are identical and constant. In order to get an idea of the asymptotic behaviour of the extended linear growth model, consider the growth rate of capital per capita as time approaches infinity and consumption per capita as well as capital per capita grow without bounds:

$$\lim_{k\to\infty}\frac{\dot{k}}{k}=\left(f'(k)-\delta-n\right)-\lim_{\substack{k\to\infty\\ c\to\infty}}\frac{c}{k}+\lim_{\substack{k\to\infty\\ c\to\infty}}\frac{\phi(c)}{k} \qquad (3.25).$$

The last term on the right-hand side of (3.25) eventually vanishes [see equation (5.7) in the appendix]. With respect to the relation between the asymptotic growth rate of consumption per capita ($\gamma_c\,*$) and the asymptotic growth rate of capital per capita ($\gamma_k\,*$), three cases can be distinguished in principle: (a) $\gamma_c * < \gamma_k *$, (b) $\gamma_c * > \gamma_k *$, and (c) $\gamma_c * = \gamma_k *$.

In the case of (a), equation (3.25) implies an asymptotic growth rate of capital per capita equal to $f'(k)-\delta-n$. However, this would violate a necessary optimum condition, the transversality condition (stated in footnote 46).[51] In case (b), equation (3.25) formally implies that the growth rate of capital per capita tends to minus infinity. In fact, in this case the trajectory would hit the boundary of the admissible control set and would subsequently run into the poverty-trap. This can obviously not be optimal as well. The only remaining possibility is case (c) with $\gamma_c * = \gamma_k *$, that is the asymptotic growth path is characterised by identical growth rates of consumption per capita and capital per capita. Taking into account the disappearance of the elasticity of the marginal *ncc* as time approaches infinity, equation (3.21) shows the asymptotic growth rate of per capita consumption. The relation $\gamma_c * = \gamma_k *$ together with the production function (3.3)

[51] Because (gross) physical capital accumulation is a linear function in output and (gross) human capital accumulation is a sub-linear function in output, the portion of physical capital to overall capital asymptotically goes to one. Consequently, the asymptotic growth rate of physical capital equals the right-hand side of (3.25), ignoring the last term. The transversality condition forces the stock of physical capital per capita to grow asymptotically with a rate smaller than $f'(k)-\delta-n$. To see this, note that the transversality condition can be written as $\lim_{t\to\infty}\lambda(\tau)\cdot e^{-(f'(k)-\delta-n)\cdot t}\cdot k_p(t)=0$, where $\tau$ denotes the point in time for which an interior solution occurs for the first time; see Barro and Sala-i-Martin (1995), pp. 167-169.

imply that all endogenous variables, that is consumption per capita, capital per capita, and income per capita, asymptotically grow with the same growth rate:[52]

$$\lim_{t\to\infty}\frac{\dot{c}}{c}=\lim_{t\to\infty}\frac{\dot{k}}{k}=\lim_{t\to\infty}\frac{\dot{y}}{y}=\sigma^{-1}\cdot\left(f'(k)-\delta-\rho\right). \tag{3.26}$$

Finally, figure 1 sketches the phase diagram of the extended linear growth model. The two rays starting from the origin represent the production function and the asymptotic balanced growth path ($BGP^{\infty}$), respectively. The curve starting from the origin, running through the second quadrant, crossing the ordinate, and entering the first quadrant represents the $\dot{k}=0$-locus. The horizontal broken line marks the level of per capita consumption for which the marginal human-capital-enhancement-effect of consumption equals one. The region below this line necessarily belongs to the no-saving range and the dynamics of the system is governed by (3.18) and $c=f(k)$. Starting with an initial stock of capital per capita $k_0$, which is chosen sufficiently small to give rise to a boundary solution at the starting point, the corresponding level of consumption per capita $c_0$ is located on the boundary of the control set. The optimal trajectory (equilibrium growth path, $EGP$) moves along the production function north-east and enters into the interior of the control region as soon as the marginal utility of consumption equals the marginal *ncc* measured in units of utility. However, if this "critical point" is located north-east in relation to the poverty-trap coordinates ($\tilde{c},\tilde{k}$), the economy would run into a poverty-trap. Provided that the poverty-trap did not become binding, the $EGP$ enters into the interior of the control region and converges to the $BGP^{\infty}$ (which equals the $BGP$ of the linear model) as time approaches infinity.

[52] The production function is assumed to be sufficiently productive to guarantee permanent growth, and overall utility is assumed to be bounded. This requires: $f'(k)-\delta>\rho>[(1-\sigma)/\sigma]\cdot(f'(k)-\delta-\rho)+n$; see Barro and Sala-i-Martin (1995), p. 142.

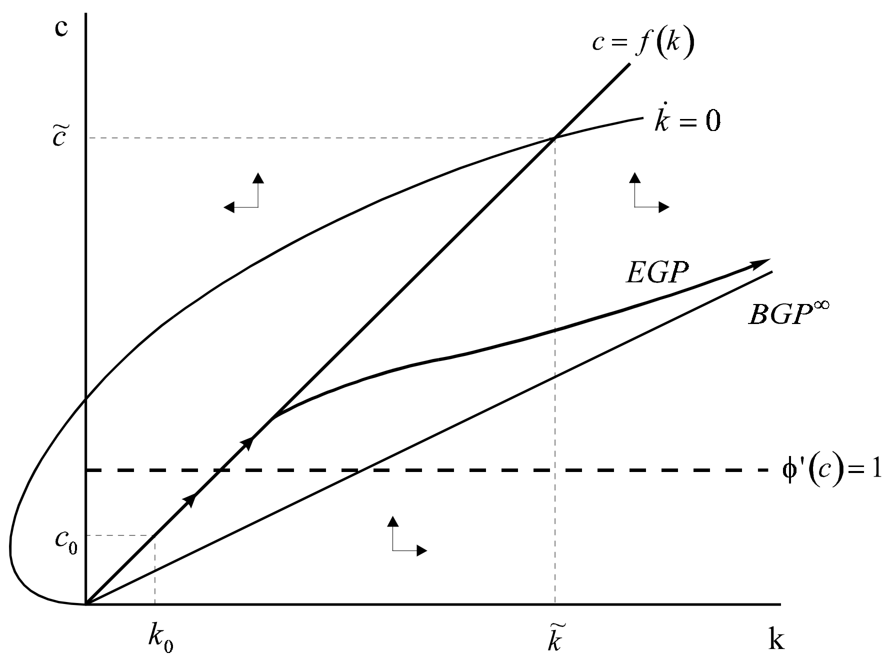
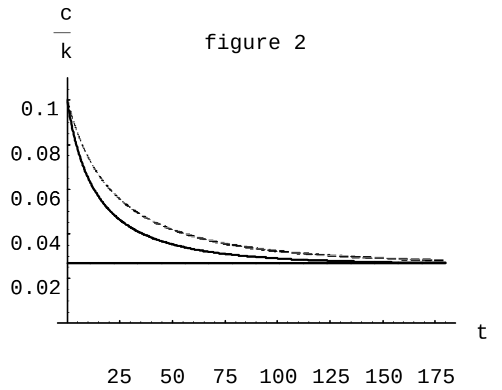


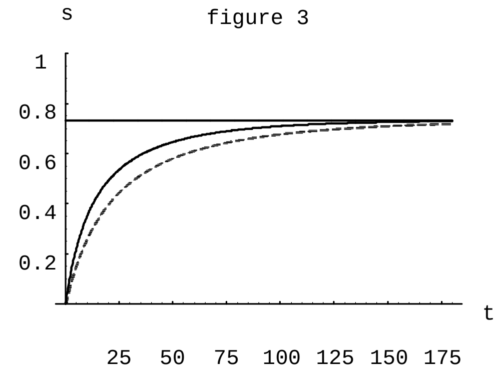

Figure 1: Phase diagram of the linear growth model with productive consumption

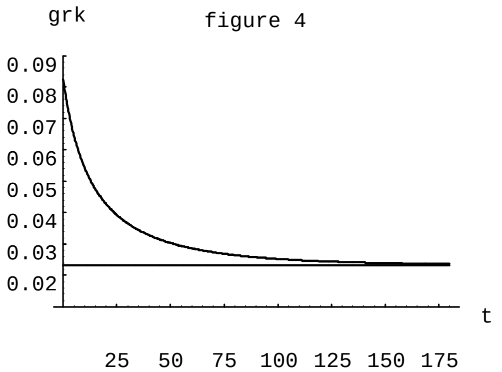
### 3.2 Simulation results

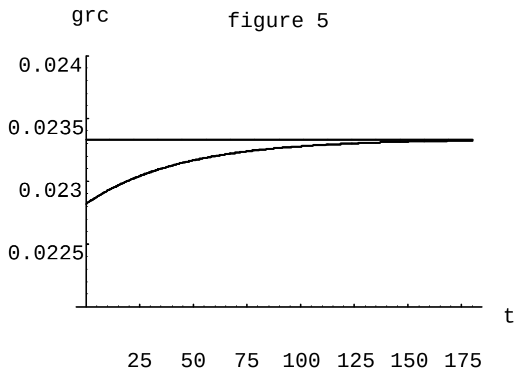
In order to illustrate the dynamics of the model, the transition process for an interior solution is simulated. That is, the system of differential equations (3.8) and (3.21) is approximated numerically by use of the subroutine NDSolve of Mathematica. The following functions and parameter values are employed: $u(c) = \frac{c(t)^{1-\sigma} - 1}{1-\sigma}$, $f(k) = A \cdot k$, $\phi(c) = c^{ß}$, $A = 0.1$, $\sigma = 3$, $\delta = 0.02$, $\rho = 0.01$, $n = 0.03$, and $ß = 0.05$ ( $ß = 0.35$).[53]

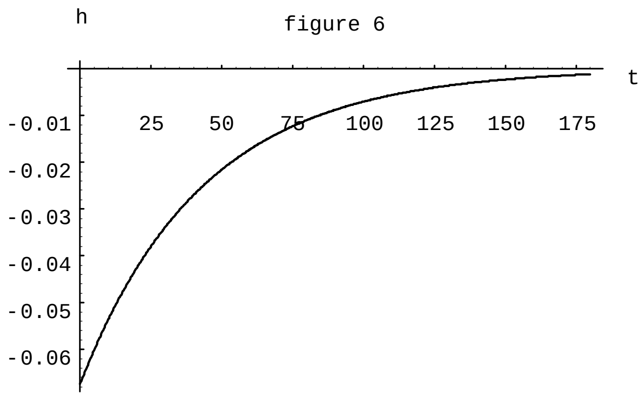
[53] The initial conditions are determined as follows: First, an arbitrarily but sufficiently high value of capital per capita is selected. The value of per capita consumption is chosen to be located on the asymptotic balanced growth path. Second, starting from this point the dynamical system is solved backwards. Third, the resulting trajectory in the c/k-plane passes through the boundary of the control set; this point of intersection is used as the starting point of the forward solution. The initial values of the backward solution are sufficiently high in the following sense: Multiplying these values by two and following the same procedure does not alter the time paths of the figures 2 to 7 significantly.

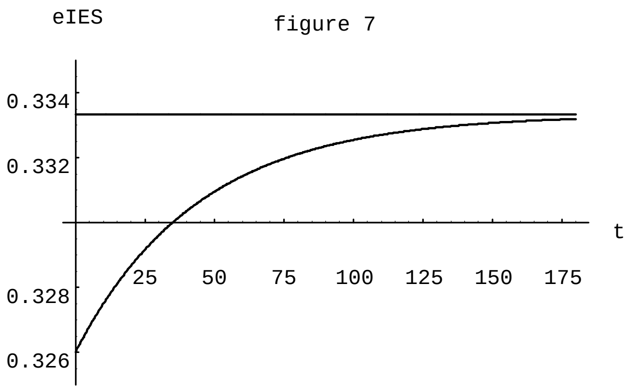



The figures 2 to 7 show the time path of the consumption/capital ratio $(c/k)$, the saving rate $(s)$, the growth rate of capital per capita $(grk)$, the growth rate of consumption per capita $(grc)$, the elasticity of the marginal *ncc* $(\eta)$, and the effective intertemporal elasticity of substitution (*eIES*), respectively. Several points are worth noting:

First of all, all variables monotonically converge to their asymptotic steady-state values. The speed of convergence measured by the use of the consumption/capital ratio according to $b(t) - b^* \cong [b(0) - b^*] \cdot e^{-\lambda \cdot t}$ is $\lambda \cong -\ln(0.5)/15 \cong 0.046$. The broken line in figure 2 shows the time path of the consumption/capital ratio for the case of $ß = 0.35$. The speed of convergence in this case is $\lambda \cong -\ln(0.5)/20 \cong 0.035$. Therefore, the speed of convergence is inversely related to the efficiency of the human-capital-enhancement process. The economic reasoning for this result is as follows: The higher the marginal human-capital-enhancement effect of consumption the stronger is the bias against saving. Consequently, because the marginal human-capital-enhancement effect of consumption is smaller than one for interior solutions the accumulation of (physical and human) capital is smaller on balance.

Second, Rebelo (1992) argues that an important shortcoming of a "broad class of endogenous growth models" is that they cannot explain different growth experiences in the presence of international capital markets. Provided that some symmetry with respect to technology and preferences holds, all economies face the same real rate of return and, consequently, exhibit the same rate of growth of per capita income. After discussing several extensions of the basic linear growth model, Rebelo concludes: *"In summary, with the exception of taxation under the worldwide system, the mechanisms described [...] do not survive as sources of growth differentials in the presence of international capital markets."*[54] The linear growth model with productive consumption shows transitional dynamics which survive international capital markets and identical real rates of return.

Third, as figure 3 demonstrates the (gross) saving rate increases from zero at the beginning of the transition and converges to its asymptotic balanced growth value. The solid line is based on $ß = 0.05$ while the broken line is based on $ß = 0.35$. The rising time path of the saving rate in turn implies a positive correlation between the saving rate and per capita income as figure 8 demonstrates; based on $ß = 0.05$:

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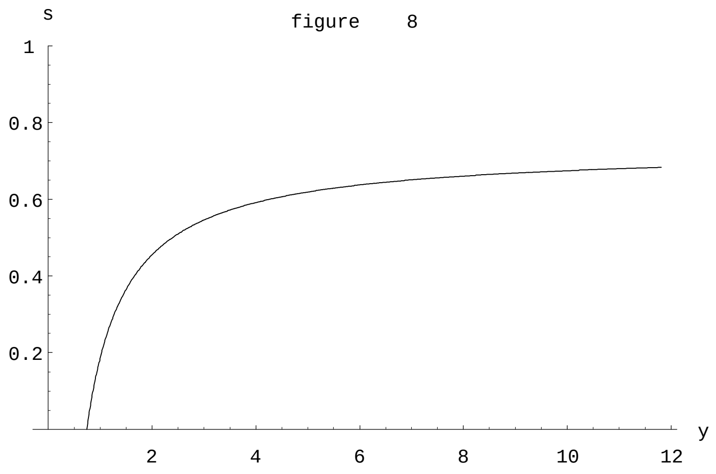
[54] Rebelo (1992), p. 27. Rebelo suggests a modification of the standard CIES preferences into Stone-Geary preferences to solve this theoretical problem.




The empirical evidence for a positive correlation between the saving rate and the level of per capita income is overwhelming.[55] As an illustration of the cross-country result consider the following table:

| *Group of Countries (number of countries)* | *GNP per equivalent adult in 1985 $; 1980-87 average* | *Personal Saving as a percent of GDP [a]* |
|---|---|---|
| Low-income-countries (16) | 1,324.4 | 11.2 |
| Lower middle-income countries (16) | 2,805.8 | 17.1 |
| Upper middle-income countries (11) | 6,165.5 | 19.5 |
| High-income countries (15) | 12,292.9 | 21.1 |

Table 1: Personal saving rates and GNP per capita; classification of economies according to World Bank (1994).
Source: Ogaki, Ostry, and Reinhart (1996), p. 44/45.
[a] With a few exceptions 1985-1993 averages; for details see Ogaki, Ostry, and Reinhart (1996).

Table 1 shows a clearly positive relation between the saving rate and the level of per capita income, with the largest increase in the saving rate occurring with the transition from low-income to lower middle-income countries. Obviously, the relation between the

[55] See Thirlwall (1974), chapter 7, for a review of the older and Reichel (1993) for a review of the more recent literature. In addition, see Rebelo (1992) and Ogaki, Ostry, and Reinhart (1996).

saving rate and the level of per capita income is non-linear which is in line with figure 8.[56]

Fourth, the growth rate of capital per capita and thus income per capita decreases during the transition process (figure 4). This result is somewhat surprising. Intuitively, a rising saving rate induces two effects on the growth rate of capital: (a) As time proceeds a rising portion of output per capita (saving rate) is used for gross physical capital investment, and (b) a falling portion of output per capita (consumption rate) can be used for gross human-capital-enhancements due to productive consumption. As the marginal human-capital-enhancement-effect of consumption is smaller than one for interior solutions, one would at first glance expect that effect (a) dominates effect (b). However, as (3.24) indicates the crucial point is whether the time rate of change of the human capital component exceeds the time rate of change of the physical capital component.[57] Obviously, figure 4 shows that in the present case the time rate of change of the human capital component dominates the time rate of change of the physical capital component. That is, the model implies (conditional) ß-convergence as well as a rising saving rate.

Fifth, figure 6 shows the elasticity of the marginal *ncc*, which is negative and asymptotically converges to zero. As a consequence, the *eIES* is low at early stages of economic development, increases and asymptotically approaches a constant (figure 7). Note that for boundary solutions the *IES* is effectively zero [see equation (3.19)]. Several authors have reported empirical evidence in favour of *IES*-values which do not significantly differ from zero in the case of low-income countries [Giovannini (1985)] as well as empirical evidence in favour of a positive relation between the *IES* and per capita income [Atkeson and Ogaki (1996) and Ogaki, Ostry, and Reinhart (1996)].[58] The present model implies a rising *eIES* during the transition due to the technological possibilities to enhance the stock of human capital as a result of productive

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[56] A positive correlation between the saving rate and the level of per capita income even results from the absolute-income hypothesis as well as the explicit consideration of a subsistence level of consumption within the instantaneous utility function (Stone-Geary preferences); for this see Rebelo (1992), Easterly (1994), and Ogaki, Ostry, and Reinhart (1996).

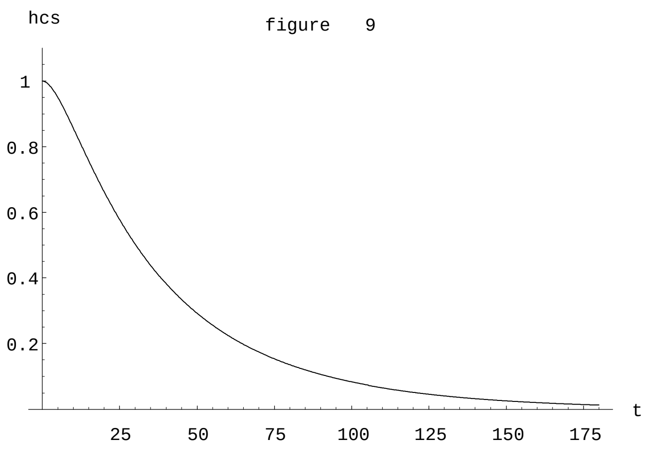
[57] The appendix clarifies the relations between the two components of (3.24) and the two components of the growth rate of overall capital per capita.

[58] The finding of a very low interest rate sensitivity of saving in a number of DCs has led several authors to consider a number of hypotheses that could explain this result. Mainly two hypotheses were employed: liquidity constraints and Stone-Geary preferences [Rebelo (1992), Easterly (1994), and Ogaki, Ostry, and Reinhart (1996)].

consumption. From this one can explain, for example, the negligible effects of financial liberalisation measures with the objective to increase the real interest rate, encourage savings and foster economic growth in the case of low-income countries.[59]

Sixth, King and Rebelo (1993) express very clearly a well-known quantitative problem of the neoclassical growth model: "*Generally, when one tries to explain sustained economic growth with transitional dynamics, there are extremely counterfactual implications. These result from the fact that implied marginal products are extraordinarily high in the early stages of development".*[60] The extended linear growth model with productive consumption does not bear this implication. Sustained economic growth with transitional dynamics is compatible with a constant marginal product of the reproducible factors.[61]

Finally, the subsequent figure 9 shows the share of human capital to overall capital, labelled the human capital share (*hcs*).




The declining time path of the *hcs* illustrates the major importance of human capital which is exclusively accumulated as a result of productive consumption at early stages

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[59] See Ogaki, Ostry, and Reinhart (1996).

[60] King and Rebelo (1993), p. 908.

[61] The Jones/Manuelli model can explain both transitional dynamics and sustained growth. But this model suffers in the same way from the unrealistically high marginal products of reproducible factors. See Jones and Manuelli (1990) and Barro and Sala-i-Martin (1995), pp. 161.

of development. On the other hand, the relative importance of human capital eventually vanishes as the economy develops.

## 4 Summary and Conclusion

Empirical evidence clearly indicates that productive inputs are not exclusively accumulated as a result of the renunciation of consumption. Especially at early stages of economic development, consumption can in a specific sense be regarded as a source of the accumulation of a productive input (human capital) and thus output growth. The model presented above sheds some light on the importance and implications of productive consumption, capital accumulation, and growth in DCs. Specifically, the incorporation of the productive consumption hypothesis into a simple endogenous growth model reveals the following implications:

(a) The "harsh" intertemporal consumption trade-off traditionally assumed is modified. More concrete, the time rate of change of the *marginal net cost of consumption* has to be taken into account for selecting the optimal consumption path. The optimal rule of consumption turns into a *modified Keynes-Ramsey rule*.

(b) The *intertemporal elasticity of substitution* is no longer exclusively based on the instantaneous utility function. The technological opportunities for enhancing the stock of human capital as a result of productive consumption additionally determine the *effective IES* (*eIES*). Consequently, the *eIES* consists in the marginal elasticity of utility as well as the *elasticity of the marginal net cost of consumption*.

(c) As Gersovitz (1983) has demonstrated within the framework of discrete two-period models, the saving rate increases with income if consumption is productive. In contrast to Gersovitz, this is shown within the framework of a continuous growth model and no special parameter restrictions are necessary for this result.

(d) Different growth experiences are explained as a result of transitional dynamics to an *asymptotic balanced growth equilibrium*. The speed of convergence is inversely related to the efficiency of the human-capital-enhancement process. The model does not imply unrealistically high values of the marginal product of reproducible factors for low incomes as all versions of the neoclassical growth model [see King and Rebelo (1993)]. Neither are identical real rates of return across economies due to international capital markets inconsistent with diverging growth rates for different countries [see Rebelo (1992)].

Generally, there are two theoretical possibilities to explain different growth experiences. According to the first strand of models, the empirical data are interpreted to represent a balanced growth phenomenon and, consequently, these models show the possibilities of multiple balanced growth paths [e.g. Azariadis and Drazen (1990)]. The second strand of models interprets the empirical picture as representing a transition phenomenon and emphasises the importance of transition processes [e.g. Romer (1986)]. The model presented in this paper is assigned to the second direction. In addition, with the exception of the poverty-trap it does not possess a balanced growth equilibrium. However, it possesses an asymptotic balanced growth equilibrium [see Jones and Manuelli (1990)].

# 5 Appendix

Human-capital-enhancement-functions with constant $\eta$

The elasticity of the marginal *ncc* is defined as follows:

$$\eta_{\psi'}(c) \equiv -\frac{c \cdot \psi''(c)}{\psi'(c)} = \frac{c \cdot \phi''(c)}{1-\phi'(c)}. \tag{5.1}$$

The class of human-capital-enhancement-functions which imply a constant elasticity of the marginal *ncc* is derived as follows:

$$\frac{d}{dc}\ln[1-\phi'(c)] = -\frac{\eta}{c} \tag{5.2}$$

$$\ln[1-\phi'(c)] = -\eta \cdot \ln(c) + a_0 \tag{5.3}$$

$$1-\phi'(c) = c^{-\eta} \cdot e^{a_0} \tag{5.4}$$

$$\phi(c) = c - \frac{a_1}{1-\eta} c^{1-\eta} + a_2 \quad \text{where } a_1 = e^{a_0} \tag{5.5}$$

## Asymptotic saving rate

The gross saving rate as traditionally defined reads:

$$\begin{aligned} s &\equiv \frac{\dot{k}_p + (\delta + n)\cdot k_p}{f(k)} = \frac{\dot{k} + (\delta + n)\cdot k - \phi(c)}{f(k)} \\ &= \frac{1}{f'(k)}\cdot\left(\frac{\dot{k}}{k} + (\delta + n) - \frac{\phi(c)}{k}\right) \end{aligned}. \tag{5.6}$$

From the properties of the human-capital-enhancement-function and L'Hôpital's rule, it follows that the last term on the right-hand side asymptotically vanishes:

$$\lim_{t\to\infty}\frac{\phi[c(t)]}{k(t)} \le \lim_{t\to\infty}\frac{\phi[f(k(t))]}{k(t)} = \lim_{t\to\infty}\frac{\phi'[f(k)]\cdot f'(k)\cdot \dot{k}(t)}{\dot{k}(t)} = 0. \tag{5.7}$$

Both $\phi[c(t)]$ and $k(t)$ must be positive and, consequently, (5.7) implies that their ratio vanishes as time approaches infinity. Equation (3.26) shows that the growth rate of capital per capita and consumption per capita are asymptotically identical and constant. Given the set of equations (5.6), (5.7), and (3.26) it can be shown that the asymptotic saving rate equals [see Barro and Sala-i-Martin (1995), S. 142/143]:

$$\begin{aligned} \lim_{t\to\infty} s &= \frac{1}{f'(k)}\cdot\left(\gamma_k * + \delta + n\right) = \frac{1}{f'(k)}\cdot\left(\frac{1}{\sigma}\left(f'(k) - \delta - \rho\right) + \delta + n\right) \\ &= \frac{f'(k) - \rho + \sigma\cdot n + (\sigma - 1)\cdot\delta}{\sigma\cdot f'(k)} \end{aligned}. \tag{5.8}$$

The steady-state saving rate of the linear growth model with productive consumption is constant and equals the saving rate of the original linear growth model.

## Growth rate of capital per capita

The growth rate of capital per capita according to (3.8) is

$$\frac{\dot{k}}{k} = (A - \delta - n) - \frac{c}{k} + \frac{\phi(c)}{k}, \tag{5.9}$$

and its time derivative reads

$$\frac{\partial}{\partial t}\frac{\dot{k}}{k} = \frac{\partial}{\partial t}\frac{\phi(c)}{k} - \frac{\partial}{\partial t}\frac{c}{k}. \tag{5.10}$$

The growth rate of capital per capita can likewise be expressed as a weighted sum of the growth rates of physical and human capital:

$$\frac{\dot{k}}{k} = \frac{\dot{k}_p}{k_p} \cdot \frac{k_p}{k} + \frac{\dot{k}_h}{k_h} \cdot \frac{k_h}{k} \,. \tag{5.11}$$

The first term in (5.10) is closely related to the time rate of change of human capital

$$\frac{\phi(c)}{k} = \frac{\dot{k}_h}{k_h} \cdot \frac{k_h}{k} + (n+\delta)\frac{k_h}{k}, \tag{5.12}$$

while the second term in (5.10) is closely related to the time rate of change of physical capital

$$\frac{c}{k} = f'(k) - \left( \frac{\dot{k}_p}{k_p} \cdot \frac{k_p}{k} - (n+\delta)\frac{k_p}{k} \right). \tag{5.13}$$

# 6 References

**Aghevli**, Bijan B., James M. **Boughton**, Peter J. **Montiel**, Delano **Villanueva** and George **Woglom** (1990), The Role of National Saving in the World Economy - Recent Trends and Prospects, IMF Occasional Paper No. 67, Washington.

**Atkeson**, Andrew and Masao **Ogaki** (1996), Wealth-Varying Intertemporal Elasticities of Substitution: Evidence from Panel and Aggregate Data, Journal of Monetary Economics, Vol. 38, 507-534.

**Azariadis**, Costas (1996), The Economics of Poverty Traps, Part One: Complete Markets, Journal of Economic Growth, December, 449-486.

**Azariadis**, Costas and Allan **Drazen** (1990), Threshold Externalities in Economic Development, Quarterly Journal of Economics 105 (May), 501-526.

**Barro**, Robert. J. (1990), Government Spending in a Simple Model of Endogenous Growth, Journal of Political Economy, Vol. 98, No. 5, Pt. 2, S103-S125.

**Barro**, Robert J. (1991), Economic Growth in a Cross Section of Countries, Quarterly Journal of Economics, 106, 2 (May), 407-443.

**Barro**, Robert J. and Xavier **Sala-i-Martin** (1995), Economic Growth, McGraw-Hill, New York.

**Becker**, Gary S. and Kevin M. **Murphy** (1988), A Theory of Rational Adiction, Journal of Political Economy, Vol. 96, 675-700.

**Ben-David**, Dan (1996), Convergence Clubs and Diverging Economies, CEPR Discussion Papers No. 922.

**Blaug**, Mark (1987), Productive and Unproductive Consumption, The New Palgrave: A Dictionary of Economics, J. Eatwell, M. Milgate, P. Newman (eds.), The Macmillan Press, New York, 1007-1008.

**Bliss**, Christopher and Nicholas **Stern** (1978), Productivity, Wages and Nutrition: Part I: The Theory + Part II: Some Observations, Journal of Development Economics 5, 331-398.

**Cass**, David (1965), Optimum Growth in an Aggregative Model of Capital Accumulation, Review of Economic Studies 32, July, 233-240.

**Deolalikar**, Anil B. (1988), Nutrition and Labor Productivity in Agriculture: Estimates for Rural South India, The Review of Economics & Statistics, 60, 406-413.

**Dorfman**, Robert (1969), An Economic Interpretation of Optimal Control Theory, American Economic Review, Vol. 59, No. 5, 817-831.

**Easterly**, William (1994), Economic Stagnation, Fixed Factors, and Policy Thresholds, Journal of Monetary Economics, Vol. 33, 525-557.

**Feichtinger**, Gustav and Richard F. **Hartl** (1986), Optimale Kontrolle ökonomischer Prozesse: Anwendung des Maximumprinzips in den Wirtschaftswissenschaften, de Gruyter, Berlin, New York.

**Fisher**, Irving (1907), The Rate of Interest, Macmillan, New York.

**Fogel**, Robert W. (1994), Economic Growth, Population Theory, and Physiology: The Bearing of Long-Term Processes on the Making of Economic Policy, American Economic Review, Vol. 84, 369-395.

**Gersovitz**, Mark (1983), Savings and Nutrition at Low Incomes, Journal of Political Economy Vol. 91, No. 5, 841-55.

**Gersovitz**, Mark (1988), Saving and Development, Handbook of Development Economics, Volume I, H. Chenery and T.N. Srinivasan (eds.) Elsevier Science Publishers, 382-424.

**Giovannini**, Alberto (1985), Saving and the Real Interest Rate in LDCs, Journal of Development Economics, Vol. 18, 197-217.

**Hicks,** Norman (1979), Growth vs. Basic Needs: Is there a Trade-Off?, World Development , Vol. 7, Nov./Dec., 985-994.

**Jones**, Larry E. and Rodolfo E. **Manuelli** (1990), A Convex Model of Equilibrium Growth: Theory and Policy Implications, Journal of Political Economy, Vol. 98, No. 5, 1008-1038.

**Jones**, Larry E. and Rodolfo E. **Manuelli** (1997), The Sources of Growth, Journal of Economic Dynamics and Control, Vol. 21, 75-114.

**Kamien**, Morton I. and Nancy L. **Schwartz** (1991), Dynamic Optimization, The Calculus of Variations and Optimal Control in Economics and Management, second edition, Amsterdam, North-Holland.

**King**, Robert G. and Sergio **Rebelo** (1993), Transitional Dynamics and Economic Growth in the Neoclassical Model, American Economic Review, Vol. 83 (4), September, 908-931.

**Koopmans**, Tjalling C. (1965), On the Concept of Optimal Economic Growth, in: The Econometric Approach to Development Planning, North-Holland Publ. Co. and Rand-McNally, 1966 reissue of Pontificiae Academia Scientiarum Scripta Varia, Vol. 28, 225-300.

**Ladrón-de-Guevara**, Antonio, Salvador **Ortigueira**, and Manuel S. **Santos** (1997), Equilibrium Dynamics in Two-Sector Models of Endogenous Growth, Journal of Economic Dynamics and Control, Vol. 21, 115-143.

**Lazear**, Edward (1977), Education: Consumption or Production?, Journal of Political Economy, Vol. 85, No. 3, 569-597.

**Leibenstein**, H. A. (1957), Economic Backwardness and Economic Growth, New York, Wiley.

**Lucas**, Robert E. Jr. (1988), On the Mechanics of Economic Development, Journal of Monetary Economics, Vol. 22, 1988, 3-42.

**Mikesell**, Raymond F. and James E. **Zinser** (1973), The Nature of the Saving Function in Developing Countries: A Survey of the Theoretical and Empirical Literature, Journal of Economic Literature 11, 1-26.

**Nelson**, Richard R. (1956), A Theory of the Low-Level Equilibrium Trap in Underdeveloped Economies, American Economic Review 46, 894-908.

**Nurkse**, Ragnar (1962), Problems of Capital Formation in Underdeveloped Countries, Basil Blackwell, Oxford; first edition: 1953.

**Obstfeld**, Maurice (1990), Intertemporal Dependence, Impatience, and Dynamics, Journal of Monetary Economics 26, 45-75.

**Ogaki**, Masao, Jonathan D. **Ostry**, and Carmen M. **Reinhart** (1996), Saving Behaviour in Low- and Middle-Income Developing Countries, A Comparison, IMF Staff Papers, Vol. 43, No. 1 (March), 38-71.

**Ram**, Rati and Theodore W. **Schultz** (1979), Life Span, Health, Savings, and Productivity, Economic Development and Cultural Change 27, 399-421.

**Ramsey**, Frank P. (1928), A Mathematical Theory of Saving, Economic Journal 38, No. 152 (Dec.), 543-559. Reprinted in: Stiglitz, Joseph E. and Hirofumi Uzawa (eds.), Readings in the Modern Theory of Economic Growth, MIT Press, 1969.

**Rebelo**, Sergio (1991), Long-Run Policy Analysis and Long Run Growth, Journal of Political Economy, Vol. 99, 500-521.

**Rebelo**, Sergio (1992), Growth in Open Economies, Carnegie Rochester Conference Series on Public Policy, Vol. 36 (July), 5-46.

**Reichel**, Richard (1993), Die Sparquote in Entwicklungs- und Schwellenländern, Determinanten und Möglichkeiten der wirtschaftspolitischen Beeinflussung, Verlag Paul Haupt, Bern.

**Romer**, David (1996), Advanced Macroeconomics, McGraw-Hill, New York et. al.

**Romer**, Paul M. (1986), Increasing Returns and Long-Run Growth, Journal of Political Economy, Vol. 94, No. 6, p. 1002-1037.

**Romer**, Paul M. (1990), Endogenous Technological Change, Journal of Political Economy, Vol. 98, 71-102.

**Rosenzweig**, Mark R. (1988), Labor Markets in Low-Income Countries, Handbook of Development Economics, Volume I, H. Chenery and T.N. Srinivasan (eds.), Elsevier Science Publishers, 714-762.

**Sarel**, Michael (1994), On the Dynamics of Economic Growth, IMF Working Paper 138.

**Solow**, Robert M. (1956), A Contribution to the Theory of Economic Growth, Quarterly Journal of Economics, Vol. 70, 65-94.

**Stiglitz**, Joseph (1976), The Efficiency Wage Hypothesis, Surplus Labour, and the Distribution of Income in LDCs, Oxford Economic Papers, July, 28, 185-207.

**Strauss**, John (1986), Does Better Nutrition Raise Farm Productivity?, Journal of Political Economy, Vol. 94, No. 2, 297-320.

**Swan**, Trevor W. (1956), Economic Growth and Capital Accumulation, Economic Record 32, November, 334-361.

**Thirlwall**, A. P. (1974), Inflation, Saving and Growth in Developing Economies, St. Martin's Press, New York.

**Wheeler**, David (1980), Basic Needs Fulfilment and Economic Growth: A Simultaneous Model, Journal of Development Economics 7, 435-451.

**Wichmann**, Thorsten (1995), Food Consumption and Growth in a Two Sector Economy, Discussion Paper 1996/02, Technical University Berlin.

**Winslow**, Charles-Edward A. (1951), The Cost of Sickness and Health, Genf.

**Wolgenmuth**, June C., Michael C. **Latham**, Andrew **Hall**, Andrew **Chesher** and C. W. T. **Crompton** (1982), Worker Productivity and the Nutritional Status of Keynian Road Construction Labourer, American Journal of Clinical Nutrition 36, 68-78.

**World Bank** (1990), World Development Report 1990: Poverty, Oxford University Press, New York.

**World Bank** (1994), World Development Report 1994: Infrastructure for Development, Oxford University Press, New York.

**Yellen**, Janet L. (1984), Efficiency Wage Models of Unemployment, American Economic Review Papers and Proceedings, Vol. 74, No. 2, 200-205.

**Zee**, Howell H. (1994), Endogenous Time Preference and Endogenous Growth, IMF Working Paper.