1. **Introduction.**

Recent work on international trade theory has established a clear strategic role for industrial policy in oligopolistic industries. Specifically, Spencer and Brander (1983) have shown that a subsidy to domestic Research and Development (R&D) enables the domestic firm to commit to "aggressive" output and investment strategies, thereby shifting profits from the foreign to the domestic firm. In other words, countries have a "strategic incentive" to subsidize domestic R&D. In their set up[1], the incentive to subsidize domestic R&D arises from two properties of their model: (1) the R&D reaction functions are negative sloped and (2) increases in foreign R&D lower domestic profits.

There is however evidence that industrial policy is used by a number of countries to advance various development objectives. Renewed attention has been focused on the links between trade, foreign investment, technology and growth (see Grossman and Helpman (1991) for an excellent survey) and there is evidence on the existence of technological spillovers from foreign firms (see Blomstrom and Persson(1983)). For many developing countries, an important policy issue is how best to take advantage of the benefits of a superior foreign technology when the transfer of technological knowledge is not automatic but involves real resource costs (e.g., Teece, 1976).

Our objective is to analyze the nature of appropriate domestic R&D policy in an imperfectly competitive world, where both the R&D rivalry among firms and the presence of technological spillovers from a superior foreign technology play a crucial role.

We use the basic Spencer and Brander (1983) model with three important modifications. The first departure from their setup is the introduction of R&D dynamics. We consider both an initial R&D investment and a subsequent improvement after the initial choices are known. There are two periods and two stages in each of them (an output stage and an R&D stage). The second modification is that there is an asymmetry between the two firms: the foreign firm is more advanced -so it has to invest fewer resources to achieve a given technological level. The third departure is the introduction of technological spillovers between firms. We assume that both firms produce side by side in the domestic country and that technological spillovers have the following form: initial foreign R&D lowers the costs of subsequent technology improvements by the home firm. In other words, it is cheaper to improve your technology after seeing the technology of your more advanced competitor ("imitation" is cheaper than "discovery"). We can then think of the home firm as engaging in R&D activities in the first period and in imitation activities in period two.

We consider two types of R&D policies: a subsidy/tax to initial domestic R&D and a subsidy/tax to the subsequent domestic improvement (or imitation). The nature of the optimal R&D policy depends again on two properties of the model: (1) the slope of the first period R&D reaction curves, (2) the effect of initial foreign R&D on the present value of domestic profits. These properties are the natural analogs of the ones that were relevant in the Spencer and Brander (1983) setup.

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[1] They have two exporting countries, each with one firm and a single importing country. Product market competition occurs after R&D investments result in lower production costs.

We show that since in the standard case first period R&D reaction functions are negative sloped, the optimal policy depends on whether greater initial foreign R&D diminishes or increases the present value of domestic profits. Because of the technological spillovers and the two period structure, an increase in first period foreign R&D has two opposite effects on domestic profits. First period domestic profits decrease because of the strategic interactions between the two firms (this is the only effect present in Spencer and Brander (1983)). On the other hand, second period domestic profits increase for two reasons: (i) R&D improvements are cheaper, and (ii) foreign output in period two drops since due to the spillovers the decrease in foreign improvements outweighs the initial R&D increase. The overall effect thus depends on the balance of these two effects. We call the first effect the "strategic" effect and the second effect the "spillover" effect.

We find that the appropriate R&D policy balances the strategic incentive to induce a reduction in foreign initial R&D with the spillover incentive to induce the foreign firm to invest more. If initial foreign R&D increases the present value of domestic profits (i.e., the spillover effect dominates), the home government should encourage the foreign firm to invest more in initial R&D. We show that either a tax to first period domestic R&D or a subsidy to imitation by the home firm can achieve this goal.

Consider now the case where improvements in first period foreign technology have a negative effect on the present value of domestic profits (i.e., the strategic effect dominates). Then the government should subsidize first period domestic R&D in order to induce the foreign firm to decrease its initial R&D investment. However, the nature of the optimal policy on imitation will depend on the relative importance of first and second period effects. If the first period strategic effect (through changes in foreign R&D) is relatively small, a subsidy to imitation is still optimal. If on the other hand, second period effects are relatively small, the optimal policy is a tax on domestic imitation.

Our work is related to the literature on R&D policies in oligopolistic industries, technology transfers and spillovers. Bagwell and Staiger (1994) and Cheng (1987) extend Spencer and Brander's (1983) work on optimal R&D policies in oligopolistic industries in different dimensions. Bagwell and Staiger (1994) analyze the extent to which the desirability of an R&D subsidy depends upon the nature of product market competition, the number of domestic firms and the existence and form of uncertainty associated with R&D investments. With a single firm in each country they show that the strategic role of an R&D subsidy is independent of the nature of product market competition. Specifically, using standard models of product market competition they find that the investment reaction functions have a negative slope whether firms compete in prices or quantities. If instead there is more than one domestic firm engaged in R&D, a corrective incentive emerges for an R&D tax. In addition they show that the optimal R&D policy is highly sensitive to the nature of uncertainty in the R&D process itself. Cheng (1987) incorporates continual technological innovation and domestic consumption into the Spencer and Brander framework.

The literature on technology transfers is quite extensive and different authors have taken alternative routes to analyzing this process. Rivera-Batiz and Romer (1991a,b) look at it as simply the purchase of blueprints that automatically give access to the technological improvement. Others think of the process as one of spillovers or externalities resulting from trade and foreign investment (e.g., Findlay (1978), Koizumi and Kopecky (1977), Das (1987), and Wang (1990)): here too, the access to the better technology is essentially automatic. Our work instead emphasizes the fact that the transfer of technological knowledge is costly and results from conscious decisions by firms. This is also the approach taken by Grossman and Helpman (1991). Wang and Blomstrom (1992) also look at technology transfers which involve conscious choices and resource costs when there are technological spillovers and strategic interactions between a multinational and a domestic firm. Our treatment is different to that of Wang and Blomstrom (1992) in two main dimensions. The first difference is that they assume positive sloped R&D reaction functions while we work with negative sloped functions. The second difference is that while they look at an open loop equilibrium, which is not subgame perfect, our model analyzes subgame perfect equilibria. This also distinguishes our model from Cheng (1987).

There are several papers that focus on the role of spillovers in oligopolistic industries. D'Aspremont and Jacquemin (1988) examine R&D in a linear duopoly with spillovers. In their model, some benefits of a firm's R&D flow without payment to other firms. These spillover benefits are assumed to be symmetric in their model. Henriques (1990) establishes the stability conditions for the d'Aspremont-Jacquemin model: she shows that stability requires that the spillovers not be too small. More recent extensions of d'Aspremont and Jacquemin include Kamien, Muller, and Zang (1992), Suzumura (1992) and Dutta and Suzumura (1993). These models all focus on comparing the outcomes of cooperative and non-cooperative R&D. None of them have the sharp asymmetry of the externality effect that is the center of our model, which is focused on the transfer of foreign technology, rather than on how to manage R&D in a domestic industry. Clearly the two foci are complementary.

The remainder of the paper is organized as follows. In section 2, we describe the basic model and assumptions. We choose our assumptions to facilitate comparison with previous work on R&D rivalry in particular Spencer and Brander (1983). A key novelty that is introduced in our model is the treatment of technological spillovers in the second period. Given the complexity of the model, however, we begin our analysis in section 3 with the case of no spillovers. This provides some insight into the structure of the model and some benchmark characterization of outcomes. In section 4 we return to the spillover case, which is our prime focus. While in the no-spillover situation there was no incentive for R&D investments in the second period, this is not the case when there is are spillovers, and our goal is to provide some understanding of when the home and foreign firms will want to wait till the second period for their R&D investments. We derive necessary conditions, which essentially state that when the home firm waits for some of its investment, the spillover has to be sufficiently large to justify this; and when the foreign firm waits to invest, it does so to avoid imitation and a large loss in second period profits.

Section 5 of the paper turns to the policy implications, which are our ultimate concern. We first consider a subsidy/tax to first period domestic R&D. Next, we consider a subsidy/tax on imitation by the home firm, announced at the beginning of the game, and with complete commitment by the government. We derive conditions for when a subsidy or a tax will be optimal. The concluding comments are in Section 6.

2. The Model

Many features of the model and notation are chosen to facilitate comparison with Spencer and Brander (1983), henceforth SB. Key differences include: (1) both firms operate in the same country, i.e., the foreign firm is a multinational; (2) there are technological spillovers, as well as initial asymmetries in R&D capabilities which create an additional role for policy beyond the "strategic " incentive; (3) the model has a two period structure, with two stages of decision-making within each period, the latter feature being the same as SB. A similarity is that we assume that all output is exported. This facilitates welfare and policy analysis, and captures LDC situations we have in mind, e.g., a domestic firm and a foreign firm may both be engaged in producing some consumer electronics in a typical South East Asian country where the domestic market is very small, so that the bulk of output is for export.

2.1. First Period

There are two firms, denoted by superscripts "H" and "F" for "home" and "foreign" respectively. Each firm i produces output y^i at variable cost C^i, which includes all costs except R&D, and earns revenue R^i. The R&D level of firm i in period one is denoted by x^i and costs v^i per unit. The variable x^i can also be interpreted as firm i's technology level in period one. The costs and revenue are assumed to be the same in both periods. We will discuss below how x^i and v^i change in the second period.

The profit π^i of firm i in period one is:

$$\pi^i(y^H, y^F, x^i) = R^i(y^H, y^F) - C^i(y^i, x^i) - v^i x^i \qquad i = H, F. \tag{1}$$

This is identical to equation (1) of SB. We introduce the asymmetry in R&D capabilities by assuming that $v^H > v^F$. This assumption will turn out to imply that the foreign firm will invest more in first period R&D than the home firm (i.e., $x^F > x^H$).

The outputs y^H and y^F are substitutes. We assume that increasing the output of one good decreases the total and marginal revenue of the other. Using subscripts on functions to denote derivatives,

$$R^i_j \ < \ 0, \quad R^i_{ij} \ < 0 \qquad i, j = H, F, \quad i \neq j. \tag{2}$$

The effect of an increase in cost-reducing R&D is to reduce C^i given y^i, and the rate of decreases declines as x^i increases. Marginal cost is positive and also falls as x^i increases. Thus:

$$C^i_x \ < \ 0, \quad C^i_{xx} \ > \ 0, \quad C^i_{yx} \ < \ 0 \quad i = H, F. \tag{3}$$

The Nash Equilibrium in outputs can be characterized by first order conditions involving period one profits only, because we assume that period two profits do not depend on period one outputs. In particular, there are no "learning by doing" effects such as characterize Fudenberg and Tirole (1983)[2]. The first order conditions are:

$$\pi_i^i = R_i^i(y^H, y^F) - C_y^i(y^i, x^i) = 0 \quad i = H, F. \tag{4}$$

The second order conditions are:

$$\pi_{ii}^i = R_{ii}^i - C_{yy}^i \ < \ 0, \quad i = H, F. \tag{5}$$

We further assume that own effects of output on marginal profit dominate cross effects, i.e.:

$$\left|\pi_{ii}^i\right| \ > \ \left|\pi_{ij}^i\right|, \quad i = H, F; i \neq j. \tag{6}$$

Condition (6) ensures uniqueness and stability of the Nash Equilibrium. This implies that the determinant of the second order derivatives is positive, i,e.:

$$A = \pi_{HH}^H \pi_{FF}^F - \pi_{HF}^H \pi_{FH}^F \ > \ 0. \tag{7}$$

The solutions y^H and y^F to (4) depend on the vector of first period R&D investments $\mathbf{x} = (x^H, x^F)$ and can be written as:

$$y^H = Y^H(\mathbf{x}), \quad y^F = Y^F(\mathbf{x}). \tag{8}$$

Using (2) and (3) and differentiating totally (4) with respect to y^i, y^j and x^i we get the standard result that a firm's Nash Equilibrium level of output is increasing in its own R&D and decreasing in the other firm's R&D:

$$Y_i^i = \frac{dy^i}{dx^i} = \frac{C_{yx}^i \pi_{jj}^j}{A} > 0, \qquad Y_i^j = \frac{dy^j}{dx^i} = \frac{C_{yx}^i \pi_{ji}^j}{A} < 0. \tag{9}$$

First period profits as functions of the R&D levels are then:

$$\begin{aligned} \Pi^i(\mathbf{x}) &= \pi^i\left(Y^H(\mathbf{x}), Y^F(\mathbf{x}), x^i, v^i\right) \\ &= R^i\left(Y^H(\mathbf{x}), Y^F(\mathbf{x})\right) - C^i(Y^i(\mathbf{x}), x^i) - v^i x^i \quad i = H, F. \end{aligned} \tag{10}$$

The analysis of the choices of x^H and x^F must be deferred until we have described the second period.

2.2. **Second Period**

We denote second period cost reducing R&D investments by z^H and z^F and assume that first period investment decisions are perfectly durable [3]. As a result second period technology levels (and profits) depend on the R&D choices made in the first period.

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[2] Such learning by doing effects could create implications for policy beyond what we consider here, but our analysis is then complementary to theirs.

[3] This is plausible if there is no depreciation, either physically, or of knowledge. Allowing for depreciation is a straightforward extension.

We will use the "^" notation to denote the second period variables whenever possible. Therefore $\hat{y}^i$, $\hat{x}^i$, $\hat{v}^i$ and $\hat{\pi}^i$ denote second period output, technology level, R&D cost and profits for firm i. Notice that the second period technology level is equal to the sum of R&D investments in both periods (i.e., $\hat{x}^i = x^i + z^i$), and firm i's production costs are $C^i(\hat{y}^i, \hat{x}^i)$.

Our main assumption is that there are positive R&D externalities or spillovers from the foreign to the home firm. In other words, the home firm learns from the more advanced foreign firm and the cost of improving its technology decreases. In some sense the home firm is imitating the foreign firm and the cost of imitation is lower than that of generating the technology. Thus, while in the second period the foreign firm's marginal cost of R&D effort is still the same ($\hat{v}^F = v^F$), for the home firm it becomes:

$$\hat{v}^H = v^H \psi(x^F), \quad \text{with } \psi(0) = 1, \quad \psi(x^F)' < 0, \quad \lim_{x^F \to \infty} \psi(x^F) = \frac{v^F}{v^H}. \tag{11}$$

Notice that in our formulation there is no learning-by-doing in the usual sense, which might be captured by having ψ as a function of x^H. Further, we assume the learning is not automatic, in the sense that x^F does not directly enter the cost function C^H. What our specification implies is that the foreign superiority in performing R&D matters only to the extent that is translated into a superior technological level. Furthermore, the home firm must still invest its own resources to capture the benefits from the foreign technology: learning, adaptation and imitation are costly activities. Thus, if the home firm decides not to invest in technology improvements in period two, there is no benefit from the foreign technology. In this sense the gains are not automatically built in.

With this specification, period two profits for the home and foreign firms are:

$$\begin{aligned} \hat{\pi}^H(\hat{y}^H, \hat{y}^F, \hat{x}^H, z^H, \hat{v}^H) &= R^H(\hat{\mathbf{y}}) - C^H(\hat{y}^H, \hat{x}^H) - v^H \psi(x^F) z^H && (12) \\ \hat{\pi}^F(\hat{y}^H, \hat{y}^F, \hat{x}^F, z^F, \hat{v}^F) &= R^F(\hat{\mathbf{y}}) - C^F(\hat{y}^F, \hat{x}^F) - \hat{v}^F z^F, && (13) \end{aligned}$$

where $\hat{\mathbf{y}} = (\hat{y}^H, \hat{y}^F)$ is the vector of second period outputs.

The first order and second order conditions for the Nash Equilibrium in second period outputs are similar to those for the first period:

$$\hat{\pi}^i_i = 0, \quad \hat{\pi}^i_{ii} < 0, \tag{14}$$

and we assume the stability condition analogous to (6) holds in the second period.

Then the second period Nash Equilibrium outputs are:

$$\hat{y}^H = Y^H(\hat{\mathbf{x}}), \quad \hat{y}^F = Y^F(\hat{\mathbf{x}}), \tag{15}$$

where $\hat{\mathbf{x}} = (\hat{x}^H, \hat{x}^F)$ is the vector of second period technology levels.

Note that the output functions are the same as in period one. The only difference in second period outputs comes from the change in technology levels from x^i to $\hat{x}^i$. In particular, if $z^i = 0, \ i = H, F$ the equilibrium outputs will be the same in each period.

Substituting the functions in (15) into the profit functions, we obtain second period profits as functions of technological investment choices only:

$$\hat{\Pi}^i(\mathbf{z}, \mathbf{x}) = \hat{\pi}^i(Y^H(\hat{\mathbf{x}}), Y^F(\hat{\mathbf{x}}), \hat{x}^i, \hat{v}^i), \quad i = H, F. \tag{16}$$

Comparing (10) with (12) and (13) we can write (16) as:

$$\hat{\Pi}^i(\mathbf{z}, \mathbf{x}) = \Pi^i(\hat{\mathbf{x}}) + v^i \hat{x}^i - \hat{v}^i z^i, \quad i = H, F. \tag{17}$$

2.3 Cost-reducing investments

Firms in period two choose whether to make additional cost-reducing investments or not. The Nash Equilibrium in second period R&D investments is characterized by the first order conditions for each firm:

$$\frac{\partial \hat{\Pi}^i(\mathbf{z}, \mathbf{x})}{\partial z^i} = {\Pi^i}_i(\hat{\mathbf{x}}) + v^i - \hat{v}^i = R^i_j(\hat{\mathbf{y}}) Y^j_i(\hat{\mathbf{x}}) - C^i_x(\hat{y}^i, \hat{x}^i) - \hat{v}^i \leq 0, \; \frac{\partial \hat{\Pi}^i(\mathbf{z}, \mathbf{x})}{\partial z^i} z^i = 0. \tag{18}$$

The second order condition is [4]:

$$\frac{\partial^2 \hat{\Pi}^i(\mathbf{z}, \mathbf{x})}{\partial {z^i}^2} = {\Pi^i}_{ii}(\hat{\mathbf{x}}) = R^i_j(\hat{\mathbf{y}}) Y^j_{ii}(\hat{\mathbf{x}}) - Y^j_i(\hat{\mathbf{x}}) \frac{dR^i_j(\hat{\mathbf{y}})}{dz^i} - C^i_{yx}(\hat{y}^i, \hat{x}^i) - C^i_{xx}(\hat{y}^i, \hat{x}^i) < 0. \tag{19}$$

To ensure uniqueness and stability of the Nash Equilibrium we assume:

$$|{\Pi^i}_{ii}| \; > \; |{\Pi^i}_{ij}|, \tag{20}$$

where

$$\Pi^i_{ij}(\hat{\mathbf{x}}) = \frac{\partial^2 \hat{\Pi}^i(\mathbf{z}, \mathbf{x})}{\partial z^i \partial z^j} = R^i_j(\hat{\mathbf{y}}) Y^j_{ij}(\hat{\mathbf{x}}) + Y^j_i(\hat{\mathbf{x}}) \frac{dR^i_j(\hat{\mathbf{y}})}{dz^j} - C^i_{yx}(\hat{y}^i, \hat{x}^i) Y^i_j(\hat{\mathbf{x}}). \tag{21}$$

Condition (20) states that own effects of R&D on marginal profit dominate cross effects and implies that the determinant of the second order derivatives is positive, i,e.:

$$\hat{D} = {\Pi^H}_{HH}(\hat{\mathbf{x}}) {\Pi^F}_{FF}(\hat{\mathbf{x}}) - {\Pi^H}_{HF}(\hat{\mathbf{x}}) {\Pi^F}_{FH}(\hat{\mathbf{x}}) \; > \; 0. \tag{22}$$

The sign of $\Pi^i_{ij}(\hat{\mathbf{x}})$ is crucial for our results. Suppose first period R&D investments are such that the Nash equilibrium levels of z^H and z^F are strictly positive. In this case the slope of the R&D reaction function for firm i is:

$$\frac{\partial z^i}{\partial z^j} = \frac{\partial r^i(x^j + z^j, x^i)}{\partial z^j} = -\frac{\Pi^i_{ij}(\hat{\mathbf{x}})}{\Pi^i_{ii}(\hat{\mathbf{x}})}, \tag{23}$$

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[4] Although not all the terms in (19) are negative we are implicitly assuming that C^i_{xx} is relatively large so that the second order condition for an interior maximum holds.

where $r^i(x^j + z^j, x^i) = argmax_{z^i}\hat{\Pi}^i(\mathbf{z}, \mathbf{x})$. Since $\Pi^i_{ii}(\hat{\mathbf{x}}) < 0$ by (19) the sign of Π^i_{ij} determines the slope of the R&D reaction function and therefore the direction of our comparative static results.

From examination of its terms, Π^i_{ij} would normally be negative, an increase in other firm's R&D normally reduces the effect of own R&D on own profit (this is the case considered by SB[5]).

Although the SB model (as well as ours) considers only the case of Cournot competition in the product market, the presumption of a negative sloped reaction function is robust to the specific form of market competition. Specifically, Bagwell and Staiger (1994) show that using standard models of product market competition, the R&D reaction functions have a negative slope whether firms compete in prices or quantities.

We will then assume:

$$\Pi^i_{ij}(\hat{\mathbf{x}}) = \frac{\partial^2 \hat{\Pi}^i(\mathbf{z}, \mathbf{x})}{\partial z^i \partial z^j} < 0 \quad i, j = H, F, \quad i \neq j. \tag{24}$$

The solutions to (18), i.e., the Nash Equilibrium in R&D investments in period two, depend on $\mathbf{x}$ and can be written as:

$$z^H = Z^H(\mathbf{x}), \quad z^F = Z^F(\mathbf{x}). \tag{25}$$

Substituting (25) into (16) we obtain second period profits as functions of the first period R&D choices:

$$\hat{\Pi}^i(\mathbf{Z}(\mathbf{x}), \mathbf{x}),$$

where $\mathbf{Z}(\mathbf{x}) = (Z^H(\mathbf{x}), Z^F(\mathbf{x}))$.

Finally, the choices of cost-reducing investments in period one can be described. These investments take into account the effects on period two profits as well.

Discounted profits for firm i are denoted by $P^i(\mathbf{x})$ and can be written as:

$$P^i(\mathbf{x}) = \Pi^i(\mathbf{x}) + \beta\hat{\Pi}^i(\mathbf{Z}(\mathbf{x}), \mathbf{x}), \tag{26}$$

where β is the discount factor. The first order conditions for an interior solution are:

$$P^i_i(\mathbf{x}) = \frac{\partial P^i(\mathbf{x})}{\partial x^i} = \frac{\partial \Pi^i{}_i(\mathbf{x})}{\partial x^i} + \beta\frac{d\hat{\Pi}^i(\mathbf{Z}(\mathbf{x}), \mathbf{x})}{dx^i} = 0, \quad i = H, F. \tag{27}$$

We will assume that the second order conditions are satisfied, i.e,:

$$P^i_{ii} < 0, \quad i = H, F, \tag{28}$$

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[5] Following SB, the term $\frac{dR^i_j(\hat{y}^H, \hat{y}^F)}{dz^j} = R^i_{ji}Y^i_j + R^i_{jj}Y^j_j \geq 0$ from (2) and (9) in the normal case in which $R^i_{jj} > 0$ (i.e., an increase in $\hat{y}^j$ reduces R^i but at a diminishing rate). This ensures that the second term of Π^i_{ij} is negative. Since the third term is negative from (3) and (9), Π^i_{ij} is negative if the first term is not too positive. Notice that differentiating (9), Y^j_i is zero in the case of a constant marginal cost and linear demand.

and that:

$$\left|P^i_{ii}\right| \; > \; \left|P^i_{ij}\right|, \tag{29}$$

to ensure uniqueness and stability of the Nash Equilibrium. As a result the determinant of the second order derivatives is positive, i,e.:

$$D = P^H_{HH}P^F_{FF} - P^H_{HF}P^F_{FH} \; > \; 0. \tag{30}$$

In the remainder of the paper, to focus on the differences on technological capabilities, we assume that the cost and revenue functions R^i and C^i are independent of i. Thus, for example there is no difference in quality that would affect the demand functions asymmetrically[6]. For costs, differences are the result only of investment in R&D in either period. Hence, the initial asymmetry between the home and foreign firm is captured entirely in the inequality $v^H > v^F$. We shall refer to these assumptions as the symmetry assumptions.

3. The Case without Spillovers

Analyzing this case is important as a benchmark for the situation of interest, namely that with spillovers. The reason is that the characterization of the equilibrium without spillovers helps to separate out strategic and spillover effects when technological spillovers are present. The first order conditions for firm i given by (18) can be written as:

$$\frac{\partial\hat{\Pi}^i(\mathbf{z},\mathbf{x})}{\partial z^i} = \Pi^i{}_i(\hat{\mathbf{x}}) \leq 0, \quad \frac{\partial\hat{\Pi}^i(\mathbf{z},\mathbf{x})}{\partial z^i} z^i = 0. \tag{31}$$

When spillovers are absent, the following lemma characterizes the second period Nash Equilibrium in cost reducing investments, by relating them to the first period R&D investments. It provides the groundwork for Proposition 1, which follows by showing that in the case without spillovers if firms find R&D investments worthwhile they will do them all in the first period.

Let $z^i_0 = Z^i(0,0), \quad i = H, F$ be the period two Nash Equilibrium when first period R&D investments are zero and $\mathbf{z}_0 = (z^H_0, z^F_0)$ the corresponding vector. We will assume that the parameters are such that $\mathbf{z}_0$ is strictly positive.

Lemma 1. *The Nash Equilibrium for the second period R&D game is characterized as follows:*

1. *if* $(x^H, x^F) \leq (z^H_0, z^F_0)$ *then* $Z^i(\mathbf{x}) = z^i_0 - x^i$*;*
2. *if* $(x^H, x^F) > (z^H_0, z^F_0)$ *then* $Z^i(\mathbf{x}) = 0$*;*
3. *if* $x^i < z^i_0$, $x^j > z^j_0$ *and* $x^i \leq r^i(x^j, 0)$, *then* $Z^i(\mathbf{x}) = r^i(x^j, 0) - x^i$, $Z^j(\mathbf{x}) = 0$*;*
4. *if* $x^i < z^i_0$, $x^j > z^j_0$ *and* $x^i > r^i(x^j, 0)$, *then* $Z^i(\mathbf{x}) = Z^j(\mathbf{x}) = 0$*;*

where $i, j = H, F, \; i \neq j$*.*

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[6] The assumption on the revenue function means that $R^H(a, b) = R^F(b, a)$, where $y^H = a$, $y^F = b$ in the arguments of R^H and $y^H = b$, $y^F = a$ in the arguments of R^F.

Proof. See Appendix.

Figures 1, 2, and 3 give some intuition for the different cases identified in Lemma 1. Figure 1 shows the reaction functions of the second period R&D game when first period R&D investments are zero. The Nash equilibrium is at N_0, where second period R&D investments are z_0^i and z_0^j respectively.

Figures 2 and 3 show the same reaction functions for different levels of initial R&D. If the initial R&D pair is to the southwest of N_0 in Figure 1, then the Nash equilibrium in second period technology levels is at N_0. In this case second period R&D for firm i is the difference between $\hat{x}^i = z^i{}_0$ and x^i (i.e., $Z^i(\mathbf{x}) = z_0^i - x^i$, case (1)). If the initial R&D pair is to the northeast of N_0, there is no R&D investment in period two (i.e. $Z^i(\mathbf{x}) = 0$, case (2)). If the initial pair is at either A or B in Figure 1, one of the firms has zero R&D investment in period two. Figures 2 and 3 show these equilibria in both the $\hat{x}$'s and z's planes (case (3)). If the initial pair is at a point like C in Figure 1, neither firm invests in R&D in period two (case (4)).

It is easy to see that if both $Z^i(\mathbf{x})$, $Z^j(\mathbf{x})$ are positive, Lemma 1 has the following implications:

$$\frac{\partial Z^i(\mathbf{x})}{\partial x^i} = -1, \quad \frac{\partial Z^j(\mathbf{x})}{\partial x^i} = 0, \quad i,j = H,F, \; i \neq j. \tag{32}$$

The first equality states that increases in a firm's initial R&D are matched by identical decreases in second period improvements. As a result, its second period technological level remains unchanged and the is no reason for the other firm to change its second period R&D level. That is why the second equality states that changes in x^i have no effect on z^j.

These comparative statics results will help to identify the effects of introducing technological spillovers in the next section.

We turn now to first period R&D investments. The first order conditions for these will be useful for proving Proposition 1. Using (17) and (26) the discounted profits for firm i can be written as:

$$P^i(\mathbf{x}) = \Pi^i(\mathbf{x}) + \beta\left\{\Pi^i\left(\hat{\mathbf{X}}(\mathbf{x})\right) + v^i x^i\right\} \quad i = H,F,$$

where $\hat{\mathbf{X}}(\mathbf{x}) = \mathbf{x} + \mathbf{Z}(\mathbf{x})$. The first order conditions for an interior solution are then:

$$\begin{aligned} P_i^i(\mathbf{x}) \;\; &= \;\; \Pi^i{}_i(\mathbf{x}) + \beta\Big\{\Pi^i{}_i\left(\hat{\mathbf{X}}(\mathbf{x})\right)\left[1 + \frac{\partial Z^i(\mathbf{x})}{\partial x^i}\right] \\ &\qquad + \Pi^i{}_j\left(\hat{\mathbf{X}}(\mathbf{x})\right)\frac{\partial Z^j(\mathbf{x})}{\partial x^i} + v^i\Big\} = 0. \end{aligned} \tag{33}$$

The next proposition shows that if there are no spillovers in period two, all cost reducing investment occurs in period one. The intuition is that without spillovers it pays to make all such investments up front and reap the benefits for longer, while on the other hand there is no benefit to waiting.

Proposition 1. *Suppose the Subgame Perfect Nash Equilibrium of the entire game is characterized by positive first period R&D investments for both firms, then:*

$$Z^i(\mathbf{x}^*) = 0, \quad i = H, F,$$

where $\mathbf{x}^*$ *is the vector of first period R&D investments.*

Proof. See Appendix.

The next result shows how first period investments respond to changes in R&D costs. The proof of this result builds on the previous two results (Lemma 1 and Proposition 1). A chief aim of Proposition 2 is its corollary, which establishes that with symmetry in all other respects, the firm with higher R&D cost has a lower first period investment in R&D. This result establishes a benchmark characterization of the no-spillover case[7].

Proposition 2. *A firm's Nash Equilibrium level of first period R&D is decreasing in its own R&D cost and increasing in the other firm's R&D cost.*

Proof. See Appendix.

Corollary 2.1 If $v^H > v^F$ and there is symmetry in all other respects, $x^H < x^F$.

Proof. If $v^H = v^F$, then the unique Nash Equilibrium must be symmetric, i.e.:$x^H = x^F$. Hence, if $v^H > v^F$, by Proposition 2, $x^H < x^F$.

4. Equilibrium with Spillovers

In this section, we conduct our main analysis. The focus is on the equilibrium levels of R&D in the two periods, so we work with profit functions where equilibrium outputs are substituted in. As before, we must proceed by working backwards, starting with period two and then analyzing period one. Using (17), which was a rewriting of the original profit functions, we can write the second period profits for each firm as:

$$\hat{\Pi}^H(\mathbf{z},\mathbf{x}) = \Pi^H(\hat{\mathbf{x}}) + v^H \hat{x}^H - v^H \psi(x^F) z^H \tag{34}$$

$$\hat{\Pi}^F(\mathbf{z},\mathbf{x}) = \Pi^F(\hat{\mathbf{x}}) + v^F x^F, \tag{35}$$

and the first derivatives as:

$$\frac{\partial \hat{\Pi}^H(\mathbf{z},\mathbf{x})}{\partial z^H} = {\Pi^H}_H(\hat{\mathbf{x}}) + v^H[1 - \psi(x^F)] \tag{36}$$

$$\frac{\partial \hat{\Pi}^F(\mathbf{z},\mathbf{x})}{\partial z^F} = {\Pi^F}_F(\hat{\mathbf{x}}) \tag{37}$$

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[7] It is also critical for an alternative, more complex formulation of the spillover effect that we have not pursued in this paper, where the second period spillover depends on the first period technology gap, rather than just on first period investment by the foreign firm. In that case, the sign of the gap is, of course, vital. Our analysis can be extended to this alternative formulation of the spillover: some additional terms are added to the algebra, but qualitative results are not substantively altered. Essentially, now first period investment by the home firm also reduces the spillover by reducing the technology gap, ceteris paribus.

4.1. Second Period

We next characterize the Nash Equilibrium for the second period R&D game. Define $\tilde{Z}^i(x^H, x^F, v^H\psi(a), v^F)$ to be the Nash Equilibrium for the second period R&D game when the first period R&D investments are x^H and x^F and the second period R&D costs are $v^H\psi(a)$ and v^F. Then, let $z_0^i(a) = \tilde{Z}^i(0, 0, v^H\psi(a), v^F)$.

The introduction of the parameter a allows the second period investments to vary independently of the first period ones: of course in equilibrium it must be that $a = x^F$. Notice also that $\psi(a)$ is decreasing in the parameter a so that as a increases firm H's reaction function shifts to the right (this is equivalent to a reduction in the R&D cost). At the new Nash equilibrium domestic R&D is higher and foreign R&D lower (see Figure 4).

The next lemma relates the second period investments to the first period investments. It is analogous to Lemma 1 for the no-spillover case.

Lemma 2. *The Nash Equilibrium for the second period R&D game is characterized as follows:*

1. *if* $(x^H, x^F) \le (z_0^H(x^F), z_0^F(x^F))$ *then* $Z^i(\mathbf{x}) = z_0^i(x^F) - x^i,\ i = H, F;$
2. *if* $(x^H, x^F) > (z_0^H(x^F), z_0^F(x^F))$ *then* $Z^i(\mathbf{x}) = 0,\ i = H, F;$
3. *if* $x^i < z_0^i(x^F)$, $x^j > z_0^j(x^F)$ *and* $x^i \le r^i(x^j, 0)$, *then* $Z^i(\mathbf{x}) = r^i(x^j, 0) - x^i$, $Z^j(\mathbf{x}) = 0$ $i, j = H, F,\ i \neq j;$
4. *if* $x^i < z_0^i(x^F)$, $x^j > z_0^j(x^F)$ *and* $x^i > r^i(x^j, 0)$, *then* $Z^i(\mathbf{x}) = Z^j(\mathbf{x}) = 0,\ i, j = H, F,\ i \neq j.$

Proof. Similar to the proof of Lemma 1.

Lemma 2 is a partial characterization, since it describes the second period investments conditional on those in the first period. In particular, second period investments need not be positive. The next lemma assumes that second period investments are positive, and proceeds to describe how they then change in response to first period R&D investments.

Lemma 3. *Suppose second period R&D investments are positive for both firms then:*

$$\frac{\partial Z^H(\mathbf{x})}{\partial x^H} = -1, \qquad \frac{\partial Z^F(\mathbf{x})}{\partial x^H} = 0, \tag{38}$$

$$\frac{\partial Z^H(\mathbf{x})}{\partial x^F} = \frac{v^H \Pi_{FF}^F \psi'(x^F)}{\hat{D}} > 0, \tag{39}$$

$$\frac{\partial Z^F(\mathbf{x})}{\partial x^F} = -1 - \frac{v^H \Pi_{FH}^F \psi'(x^F)}{\hat{D}} < -1. \tag{40}$$

Proof. See Appendix.

Note that the inequalities in (39) and (40) are based not only on the negative sign of $\psi'(x^F)$, but also $\hat{D} > 0$ (stability condition (22)), the second order condition (19), and downward sloping reaction functions ((24) and (19)). However, given all these, the spillover effect is what weights in as decisive (i.e., $\psi'(x^F)$ needs to be different from zero).

It may be seen that how z^H and z^F respond to changes in x^H is the same as in the case without spillovers (compare (38) and (32). On the other hand the responses to changes in x^F are different when the spillovers are present (compare (39) and (40) with (32)).

The intuition is that without spillovers increases in x^F: (i) were exactly compensated by decreases in z^F leaving $\hat{x}^F$ unchanged and, (ii) had no direct effect on z^H. When spillovers are present, increases in x^F decrease the cost of second period R&D for the home firm and z^H goes up (and so does $\hat{x}^H$). Since R&D reaction functions are negative sloped, $\hat{x}^F$ has to decrease. For this to happen the decrease in z^F has to exceed the initial increase in x^F.

These comparative statics results will provide useful intuition in our results below.

Next, staying with the case of Lemma 3, where second period R&D investments are positive for both firms, we analyze the properties of the two-period present value profits for the two firms, as functions of first period investments. We examine how each firm's first period investment reacts to changes in the other's first period investment. We further examine the slopes of the reaction functions in this case.

When second period investments are positive for both firms, (36) and (37) are equal to zero and therefore:

$$\begin{aligned} \Pi_H^H(\hat{\mathbf{x}}) &= -v^H[1-\psi(x^F)] && (41)\\ \Pi_F^F(\hat{\mathbf{x}}) &= 0. && (42) \end{aligned}$$

Using equations (34) and (35) the discounted profits for both firms (given in equation (26)) can be written as:

$$\begin{aligned} P^H(\mathbf{x}) &= \Pi^H(\mathbf{x}) + \beta\left\{\Pi^H(\hat{\mathbf{x}}) + v^H[x^H + Z^H(\mathbf{x})] - v^H\psi(x^F)Z^H(\mathbf{x})\right\} && (43)\\ P^F(\mathbf{x}) &= \Pi^F(\mathbf{x}) + \beta\left\{\Pi^F(\hat{\mathbf{x}}) + v^F x^F\right\}. && (44) \end{aligned}$$

The first order conditions for an interior solution are:

$$\begin{aligned} P_H^H(\mathbf{x}) &= \Pi_H^H(\mathbf{x}) + \beta\left\{\Pi_H^H(\hat{\mathbf{x}})\left[1 + \frac{\partial Z^H(.)}{\partial x^H}\right]\right.\\ &\quad \left. + \Pi_F^H(\hat{\mathbf{x}})\frac{\partial Z^F(.)}{\partial x^H} + v^H + v^H[1-\psi(x^F)]\frac{\partial Z^H(.)}{\partial x^H}\right\} = 0, && (45)\\ P_F^F(\mathbf{x}) &= \Pi_F^F(\mathbf{x}) + \beta\left\{\Pi_F^F(\hat{\mathbf{x}})\left[1 + \frac{\partial Z^F(.)}{\partial x^F}\right]\right.\\ &\quad \left. + \Pi_H^F(\hat{\mathbf{x}})\frac{\partial Z^H(.)}{\partial x^F} + v^F\right\} = 0. && (46) \end{aligned}$$

Using Lemma 3, (41) and (42) we can write (45) and (46) as follows:

$$\begin{aligned} P_H^H(\mathbf{x}) &= \Pi_H^H(\mathbf{x}) + \beta v^H\psi(x^F) = 0 && (47)\\ P_F^F(\mathbf{x}) &= \Pi_F^F(\mathbf{x}) + \beta\left\{v^F + \Pi_H^F(\hat{\mathbf{x}})\frac{\partial Z^H(\mathbf{x})}{\partial x^F}\right\} = 0. && (48) \end{aligned}$$

Differentiating (47) and (48) and using Lemma 3 to eliminate terms, the second cross derivatives can be written as:

$$P^H_{HF} = \Pi^H_{HF}(\mathbf{x}) + \beta v^H \psi'(x^F) < 0, \tag{49}$$

$$P^F_{FH} = \Pi^F_{FH}(\mathbf{x}) + \beta \frac{\partial Z^H(\mathbf{x})}{\partial x^F} \left\{ \Pi^F_{HH}[1 + \frac{\partial Z^H(\mathbf{x})}{\partial x^H}] + \Pi^F_{HF} \frac{\partial Z^F(\mathbf{x})}{\partial x^H} \right\} = \Pi^F_{FH}(\mathbf{x}) < 0. \tag{50}$$

The second equality in (50) follows from equation (38) in Lemma 3. Notice that the sign of $P^i_{ij}, \; i,j = H,F \; i \neq j$ determines the slope of firm i's reaction function for the entire game. If the cross derivative is negative, the relevant reaction function slopes downward, so the previous two equations imply that both firms have negatively sloped reaction functions. These reaction function slopes, as is typically the case in such strategic models, will be crucial for determining policy effects that are taken up in the next section.

The intuition behind the above result on the reaction functions is as follows. Let us consider the home firm first. We can divide the effect of changes in x^F on x^H in two parts: the strategic effect and the spillover effect. If x^F increases, x^H will decrease for two reasons. The first reason is a strategic one: since x^F is higher, profit maximization for firm H in period one requires a smaller x^H (the single period R&D reaction function is negatively sloped). The second reason has to do with the spillover. Since a higher x^F results in lower second period R&D costs for the home firm, the firm shifts R&D from the first to the second period.

The foreign firm will also lower its R&D investment in response to an increase in its competitor's investment. We can divide the effect of changes in x^H on x^F again in two parts: the strategic and the spillover effect. What equation (50) shows is that the spillover effect dissapears and only the strategic effect matters. The strategic effect works in the same way as the one for the home firm: since the single period R&D reaction function is negative sloped, the best response to an increase in x^H is to decrease x^F. To understand the spillover effect the key question to ask is: should the foreign firm do anything else in preparation for period two? In this case the answer is no because a higher x^H will not affect second period output levels. Specifically, output levels depend on $x^H + z^H$ and $x^F + z^F$, increases in x^H are perfectly matched by decreases in z^H, and changes in x^H have no effect on z^F.

In Lemma 3 and the subsequent analysis of the reaction functions we have assumed that the second period R&D investments are positive. Building on the characterization of this case, however, we are able, in the next lemma, to provide necessary conditions for a Nash Equilibrium with positive second period R&D. The first part of the Lemma states that the home firm will have positive second period R&D when the spillovers from the foreign firm are sufficiently large to justify waiting. The second part of the Lemma states that the foreign firm will invest a positive amount when the loss in second period profits (due to imitation by the home firm) is sufficiently large to justify waiting.

Lemma 4. *If the Nash Equilibrium for the entire game is characterized by positive R&D for each firm in both periods then:*

1. $\psi(x^F) < \frac{1}{1+\beta}$ *and,*
2. $v^F < -\frac{\Pi^F_H(\hat{\mathbf{x}})\ \Pi^F_{FF}(\hat{\mathbf{x}})\ v^H\psi'(x^F)}{\hat{D}}$.

Proof. See Appendix.

The intuition for the inequalities in the lemma is fairly clear. For example, if the spillover is greater, then $\psi(x^F)$ is smaller for any given x^F , so waiting is more likely to be beneficial for the home firm. As another example, the terms on the right-hand side of the second inequality include the imitation response of the home firm to increases in first period R&D investment by the foreign firm. If this response is greater, the foreign firm is more likely to postpone investment.

In the following section, we trace out some policy implications for the case where second period R&D investments are positive.

5. Policy Implications

We are now in a position to discuss R&D policies. In this section we introduce the domestic country government as the first player and extend the basic game in two ways. First the domestic government is allowed to make a prior commitment to subsidize/tax first period domestic R&D. Second, the domestic government can credibly commit at the beginning of the game to subsidize/tax second period domestic R&D (i.e., imitation). The assumption that this precommitment is feasible is essential to the following analysis. In order to focus on the rent seeking rationale for these R&D policies, we assume that all output is for export to other countries. Furthermore, we assume that any government policy we derive will not affect the foreign firm's profits adversely enough to cause its exit from the home country[8].

5.1. Subsidy to First Period Domestic R&D

The first type of policy we consider is a subsidy (or tax if it is negative), denoted s, to first period domestic R&D. First period profits for the home firm are:

$$\Pi^H(\mathbf{x}, s) = R^i(Y^H(\mathbf{x}), Y^F(\mathbf{x})) - C^H(Y^H(\mathbf{x}), x^H) - (v^H - s)x^H, \tag{51}$$

and the profits for the entire game:

$$\begin{aligned} P^H(\mathbf{x}, s) &= \Pi^H(\mathbf{x}, s) + \beta\hat{\Pi}^H(\mathbf{z}, \mathbf{x}) \\ &= \Pi^H(\mathbf{x}, s) + \beta\{\Pi^H(\hat{\mathbf{x}}) + v^H\hat{x}^H - v^H\psi(x^F)z^H\}. \end{aligned} \tag{52}$$

We first look at the effects of the subsidy on first period R&D levels. Differentiating the first order conditions for the entire game with respect to the subsidy s we get:

$$P^H_{HH}x^H_s + P^H_{HF}x^F_s + 1 = 0, \tag{53}$$

$$P^F_{FH}x^H_s + P^F_{FF}x^F_s = 0. \tag{54}$$

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[8] Note that foreign firms may have alternative profit opportunities and may exit if profits in this particular market fall below a threshold. Since domestic R&D subsidies affect the profits of the foreign firm we have to worry about this possibility.

This implies :

$$x_s^H = \frac{-P_{FF}^F}{D} > 0, \qquad x_s^F = \frac{P_{FH}^F}{D} < 0, \tag{55}$$

since $P_{FF}^F < 0$ by (28) and $P_{FH}^F < 0$ by (50). We then have the following Lemma:

Lemma 5. *A first period domestic R&D subsidy increases first period domestic R&D and reduces first period foreign R&D.*

We now focus on the optimal first period domestic R&D subsidy. The optimal subsidy is found by maximizing net domestic benefit:

$$B^H(s) = P^H(\mathbf{x}, s) - sx^H = \Pi^H(\mathbf{x}, s) + \beta\hat{\Pi}^H(\mathbf{z}, \mathbf{x}) - sx^H.$$

The first order condition for the welfare maximizing subsidy is:

$$\frac{dB^H}{ds} = P_H^H(\mathbf{x}, s)x_s^H + P_F^H(\mathbf{x}, s)x_s^F + x^H - x^H - sx_s^H = 0. \tag{56}$$

Since $P_H^H = 0$ by the first order conditions, we obtain

$$s = P_F^H \frac{x_s^F}{x_s^H}\ . \tag{57}$$

By Lemma 5, $x_s^H > 0$ and $x_s^F < 0$ so the optimal s will be positive or negative depending on the sign of P_F^H.

Since P_F^H plays such a crucial role in determining whether a country should tax or subsidize domestic R&D, we will write it explicitly and provide some intuition about its sign.

Differentiating (52) with respect to x^F we obtain:

$$P_F^H = \frac{dP^H}{dx^F} = \Pi_F^H(\mathbf{x}, s) + \beta\left[\frac{\partial\hat{\Pi}^H}{\partial x^F} + \frac{\partial\hat{\Pi}^H}{\partial z^H}\frac{\partial Z^H(.)}{\partial x^F} + \frac{\partial\hat{\Pi}^H}{\partial z^F}\tfrac{\partial Z^F(\mathbf{x})}{\partial x^F}\frac{\partial Z^F(.)}{\partial x^F}\right]. \tag{58}$$

Since $\frac{\partial\hat{\Pi}^H}{\partial z^H} = 0$ at an interior solution, the second term inside the square brackets equals zero. Using (51), and since $\Pi_F^H(\mathbf{x}, s) = \Pi_F^H(\mathbf{x})$ equation (58) reduces to:

$$P_F^H = \frac{dP^H}{dx^F} = \Pi_F^H(\mathbf{x}) + \beta\left\{\Pi_F^H(\hat{\mathbf{x}})\left[1 + \tfrac{\partial Z^F(.)}{\partial x^F}\right] - v^H\psi'(x^F)z^H\right\}. \tag{59}$$

Notice that $\Pi_F^H(\mathbf{x})$ and $\Pi_F^H(\hat{\mathbf{x}})$ are negative since using (1)-(5), (7) and (10) we obtain:

$$\Pi_F^H = \pi_F^H\frac{\partial y^F}{\partial x^F} = R_F^H\frac{\partial y^F}{\partial x^F} < 0. \tag{60}$$

Using (11) we have that $\psi'(x^F) < 0$ and using (40) that $\left[1 + \frac{\partial Z^F(.)}{\partial x^F}\right] < 0$. Thus, the first term in (59) is negative while the second term is positive.

Intuitively, the first term in (59) captures the strategic effect on first period profits. Specifically, since the two products are substitutes increases in x^F lower first period profits for the home firm. The second term captures the effects on second period home profits. As x^F increases, home profits in period two increase for two different reasons: (1) since z^F decreases by more than the increase in x^F, the technological level of the foreign firm in period two is lower and thus home profits increase; (2) since imitation costs are lower, profits increase in an interior solution. Note that both reasons for the second term to be positive are traceable to the spillover effect. While $\psi'(x^F) < 0$ is directly so, $\frac{\partial Z^F(\mathbf{x})}{\partial x^F} < -1$ because of the spillover effect, as can be seen from (40) and the discussion following Lemma 3.

The next Proposition formally states the result in equation (57).

Proposition 3. *(1) If $P^H_F > 0$ the optimal first period domestic R&D subsidy is negative (i.e., is a tax).*

(2) If $P^H_F < 0$ the optimal first period domestic R&D subsidy is positive.

The intuitive explanation of the result is as follows. If a better foreign technology in period 1 increases overall domestic profits (i.e. $P^H_F > 0$ because of strong spillovers), the home government should encourage foreign firms to bring a better technology. This can be achieved with a tax that forces the local firm to hold its R&D investment. Another way to state the result is that it is only optimal for the host government to encourage better technology from a foreign firm by taxing a domestic firm if the spillover effect is strong enough to outweigh the strategic effect on home profits. This intuition comes about from the decomposition of P^H_F in (59) into two terms, representing the initial strategic effect and the period two (direct and indirect) spillover effect.

If instead improvements in the foreign technology decrease domestic profits, the government role is to discourage the foreign firm from bringing a better technology. A subsidy to first period domestic R&D increases domestic R&D and as a result foreign investment in R&D decreases.

5.2. Subsidy to Imitation

The next policy we consider is a subsidy (or tax) to imitation announced at the beginning of the game and assuming complete commitment by the government. In this case the home firm receives a subsidy (or tax) to second period R&D. Since second period R&D costs for the home firm are now $\hat{v}^H = v^H\psi(x^F) - \hat{s}$, second period profits are:

$$\hat{\Pi}^H(\mathbf{z}, \mathbf{x}, \hat{s}) = \Pi^H(\hat{\mathbf{x}}) + v^H\hat{x}^H - [v^H\psi(x^F) - \hat{s}]z^H. \tag{61}$$

Second period profits for the foreign firm are still given by equation (35). Assuming positive second period R&D for both firms , the first order conditions are:

$$\frac{\partial\hat{\Pi}^H}{\partial z^H} = \Pi^H_H(\hat{\mathbf{x}}) + v^H[1 - \psi(x^F)] + \hat{s} = 0 \tag{62}$$

$$\frac{\partial\hat{\Pi}^F}{\partial z^F} = \Pi^F_F(\hat{\mathbf{x}}) = 0. \tag{63}$$

Notice that in this case the Nash Equilibrium in second period R&D levels depends on the subsidy, i.e.,

$$z^i = Z^i(\mathbf{x}, \hat{s}).$$

Differentiating (62) and (63) with respect to $\hat{s}$ we get:

$$z^H_{\hat{s}} = \frac{-\Pi^F_{FF}(\hat{\mathbf{x}})}{\hat{D}} > 0, \qquad z^F_{\hat{s}} = \frac{\Pi^F_{FH}(\hat{\mathbf{x}})}{\hat{D}} < 0. \tag{64}$$

We have then proved:

Lemma 6. *Given* (x^H, x^F)*, a subsidy to second period domestic R&D increases domestic R&D while decreasing foreign R&D.*

Notice that strategic considerations are responsible for the previous result. Since the R&D reaction functions are negatived sloped and the subsidy increases domestic R&D, the foreign firm's optimal response is to decrease its R&D.

The effects of the imitation subsidy on first period R&D are considered next. We concentrate in the case where both firms engage in first period R&D.

The profit functions for the entire game are given by (44) and

$$\begin{aligned} P^H(\mathbf{x}, \hat{s}) &= \Pi^H(\mathbf{x}) + \beta \hat{\Pi}^H(\mathbf{z}, \mathbf{x}, \hat{s}) \\ &= \Pi^H(\mathbf{x}) + \beta \left\{ \Pi^H(\hat{\mathbf{x}}) + v^H \hat{x}^H - [v^H \psi(x^F) - \hat{s}] z^H \right\}. \end{aligned} \tag{65}$$

The first order conditions for the entire game for the foreign firm are given by equation (46) and for the home firm by:

$$\begin{aligned} P^H_H(\mathbf{x}, \hat{s}) &= \Pi^H_H(\mathbf{x}) + \beta \left\{ \Pi^H_H(\hat{\mathbf{x}}) \left[1 + \tfrac{\partial Z^H(.)}{\partial x^H} \right] + \Pi^H_F(\hat{\mathbf{x}}) \tfrac{\partial Z^F(.)}{\partial x^H} \right. \\ &\qquad \left. + v^H + v^H [1 - \psi(x^F)] \tfrac{\partial Z^H(.)}{\partial x^H} + \hat{s} \tfrac{\partial Z^H(.)}{\partial x^H} \right\} = 0. \end{aligned} \tag{66}$$

Since the equilibrium x's and z's are functions of $\hat{s}$, and $\mathbf{x}$ and $\hat{s}$ respectively we differentiate (66) and (46) with respect to the subsidy $\hat{s}$ to obtain:

$$P^H_{HH} x^H_{\hat{s}} + P^H_{HF} x^F_{\hat{s}} + E^H - \beta = 0, \tag{67}$$

$$P^F_{FH} x^H_{\hat{s}} + P^F_{FF} x^F_{\hat{s}} + E^F = 0; \tag{68}$$

where

$$E^H = \frac{\partial P^H_H}{\partial z^H} z^H_{\hat{s}} + \frac{\partial P^H_H}{\partial z^F} z^F_{\hat{s}}, \tag{69}$$

$$E^F = \frac{\partial P^F_F}{\partial z^H} z^H_{\hat{s}} + \frac{\partial P^F_F}{\partial z^F} z^F_{\hat{s}}. \tag{70}$$

We show in the Appendix (Lemma 8) that $\frac{\partial P^i_i}{\partial z^i} = 0$, $i = H, F$, $E^H = 0$, and $E^F > 0$. Solving (67) and (68), we have:

$$x^H_{\hat{s}} = \frac{\beta P^F_{FF} + E^F P^H_{HF}}{D} < 0, \qquad x^F_{\hat{s}} = -\frac{\beta P^F_{FH} + E^F P^H_{HH}}{D} > 0, \tag{71}$$

since by (49) and (50) the second cross derivatives are negative and by (28) $P^i_{ii} < 0, i = H, J$.

We have then proved:

Lemma 7. *A subsidy to second period domestic R&D -announced at the beginning of the game and under complete commitment- decreases first period home R&D and increases first period foreign R&D.*

To explain intuitively the previous result, it is convenient to write $x^H_{\hat{s}}$ and $x^F_{\hat{s}}$ as functions of one another. Using (67), (68) and since $E^H = 0$, we have that:

$$x^H_{\hat{s}} = \frac{\beta - P^H_{HF} x^F_{\hat{s}}}{P^H_{HH}}, \qquad x^F_{\hat{s}} = -\frac{E^F + P^F_{FH} x^H_{\hat{s}}}{P^F_{FF}}.$$

The subsidy affects first period R&D through two channels: through changes in the competitor's first period R&D and through changes in second period profits (directly and through changes in the z's).

An increase in $\hat{s}$ lowers x^H because :(i) it is the best response to an increased level of x^F , and (ii) it is optimal to shift some R&D from period two to period one since second period R&D costs decrease.

The effect on x^F goes in the opposite direction. An increase in $\hat{s}$ increases x^F because: (i) it is the best reponse to a lower lever of x^H, and (ii) it is optimal to increase x^F as a preemptive measure since second period home R&D will go up as a response to the subsidy.

The optimal imitation subsidy is found by maximizing the net discounted domestic benefit B^H which equals the profit of the domestic firm minus the cost of the subsidy.

$$B^H(\hat{s}) = P^H(\mathbf{x}, \hat{s}) - \beta \hat{s} z^H.$$

The first order condition for the welfare maximizing subsidy is:

$$\begin{aligned}\frac{dB^H}{d\hat{s}} &= P^H_H(\mathbf{x}, \hat{s}) x^H_{\hat{s}} + P^H_F(\mathbf{x}, \hat{s}) x^F_{\hat{s}} + \frac{\partial P^H}{\partial z^H} z^H_{\hat{s}} + \frac{\partial P^H}{\partial z^F} z^F_{\hat{s}} \\ &\quad + \frac{\partial P^H}{\partial \hat{s}} - \beta \left[\hat{s} \frac{dz^H}{d\hat{s}} - z^H \right] = 0. \qquad (72)\end{aligned}$$

We cancel terms and use the first order conditions for both the entire and second period game so (72) reduces to:

$$\frac{dB^H}{d\hat{s}} = \beta \Pi^H_F(\hat{\mathbf{x}}) z^F_{\hat{s}} + P^H_F x^F_{\hat{s}} - \beta \hat{s} \frac{dz^H}{d\hat{s}} = 0$$

or

$$\hat{s} = \frac{\beta \Pi^H_F(\hat{\mathbf{x}}) z^F_{\hat{s}} + P^H_F x^F_{\hat{s}}}{\beta \frac{dz^H}{d\hat{s}}}, \qquad (73)$$

where the expression for P^H_F is given in equation (59).

Proposition 4. *(1) If $P_F^H > 0$, the optimal imitation subsidy is positive.*

(2) If $P_F^H < 0$, the optimal imitation subsidy is positive if

$$\beta \Pi_F^H(\hat{\mathbf{x}}) z_{\hat{s}}^F > -P_F^H x_{\hat{s}}^F,$$

and negative (i.e. a tax) if the inequality sign holds in the opposite direction.

Proof. Using (64), (71) and Lemma 3 we have:

$$\frac{dz^H}{d\hat{s}} = \frac{\partial z^H}{\partial x^H} x_{\hat{s}}^H + \frac{\partial z^H}{\partial x^F} x_{\hat{s}}^F + z_{\hat{s}}^H > 0.$$

Also $z_{\hat{s}}^F < 0$ by (64), $\frac{\partial x^H}{\partial \hat{s}} < 0$ by (71) and $\Pi_F^H < 0$ by (60). Thus, using (73) the result follows.

The intuition for the previous result is as follows. A subsidy to imitation decreases foreign R&D in period two and this is good news for the domestic firm (strategic reason). On the other hand, first period foreign R&D goes up as a result of the subsidy.

If increases in foreign R&D in period one are good for overall domestic profits (i.e. $P_F^H > 0$ due to a significant spillover effect) both effects go in the same direction and a subsidy to imitation is optimal.

If instead $P_F^H < 0$ (i.e. the first period strategic effect is stronger than the second period effect and a better initial foreign technology is bad for domestic profits) a tax rather than a subsidy to imitation may be necessary. If P_F^H is negative but small, a subsidy will still be optimal. However if P_F^H is sufficiently large, a tax to domestic imitation is the optimal policy.

Propositions 3 and 4 may be compared. Conditions which ensure a tax on first period domestic R&D also guarantee a subsidy is optimal for second period R&D. However, when a subsidy on first period R&D is optimal, it is possible that a subsidy on second period R&D will also be optimal. This happens if the first period strategic effects of the subsidy are not too strong. Note, however, that we have not solved for which policies are simultaneously optimal[9].

6. Concluding Comments

We have found that the optimal policy on domestic R&D and imitation will depend crucially on the overall effect of first period foreign technology on domestic profits. In other words, the appropriate R&D policy balances the strategic and spillover incentives.

If a better foreign technology increases overall profits, the government should encourage foreign firms to bring more advanced technologies. This objective can be achieved by taxing the domestic firm in the first period or by subsidizing imitation.

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[9] This is not hard to do, at the expense of some more algebra. Essentially one uses the domestic benefit function $B^H(s, \hat{s}) = P^H(x, s, \hat{s}) - sx^H - \beta \hat{s} z^H$, and maximizes it simultaneously with respect to s and $\hat{s}$. This adds terms to (57) and (73), the expressions for s and $\hat{s}$ derived earlier. Again in this case the sign of P_F^H will be crucial in determining the sign of s and $\hat{s}$ and we will have several cases depending on the relative strengths of the strategic and spillover effects. The general pattern is that the sign of one of the two subsidies is clearly determined while the sign of the other is ambiguous.

If instead a better foreign technology has a detrimental effect on home profits, first period domestic R&D should be subsidized. The nature of the optimal policy on imitation in this case will depend on the relative importance of direct effects through second period foreign R&D (strategic) and indirect effects through first period foreign R&D (strategic in first period and spillover in the second period).

If the first period strategic effect is relative small, a subsidy to imitation is still optimal. However if the first period strategic effect is strong, the government should tax domestic imitation (this results in a decrease of foreign R&D investment in the initial period).

Our next step, in future research, will be to look at other policies that could be used by the home government. Two natural candidates are taxes (or subsidies) to the foreign firm and changes in the property rights. Other complications such as host country goals that include employment objectives could also be incorporated.

Our conjecture is that a tightening of property rights will reduce the the size of the spillover and some of the effects will be similar those of an ad-valorem tax on imitation. Taxation of foreign firms could be direct or through alternative schemes. One possibility is to require that the foreign firm uses some domestic input (for example a requirement to hire a certain percentage of local scientists) that results in a higher R&D cost.

In examining taxes on foreign firms, it will be important to bear in mind that foreign firms may have alternative profit opportunities, and may exit if profits in this particular market fall below a threshold. In our analysis of subsidies to a domestic firm this could also happen, since subsidies have a profit-shifting effect. Since the location or entry-exit decision of the foreign firm has not been our focus, we have assumed in our analysis that the optimal government policy towards its own firm does not run up against a constraint that determines whether the foreign firm continues operating in the country. Adding such constraints should not qualitatively affect our results.

Appendix

Proof of Lemma 1:

(1) Since by definition $\mathbf{z}_0$ is an interior solution, the relevant first and second order conditions are as follows:

$$\begin{aligned}\frac{\partial\hat{\Pi}^i(\mathbf{z},0)}{\partial z^i} &= \Pi^i{}_i(0+\mathbf{z}_0)=\Pi^i{}_i(\mathbf{z}_0)=0, && \text{(A.1)}\\ \frac{\partial^2\hat{\Pi}^i(\mathbf{z},0)}{\partial {z^i}^2} &= \Pi^i{}_{ii}(0+\mathbf{z}_0)=\Pi^i{}_{ii}(\mathbf{z}_0)<0,\quad i=H,F. && \text{(A.2)}\end{aligned}$$

Then $Z^i(\mathbf{x}) = z^i_0 - x^i,\quad i=H,F$ is a solution since

$$\begin{aligned}\frac{\partial\hat{\Pi}^i(\mathbf{z}_0-\mathbf{x},\mathbf{x})}{\partial z^i} &= \Pi^i{}_i(\hat{\mathbf{x}}_0-\mathbf{x})=\Pi^i{}_i(\mathbf{z}_0)=0, && \text{(A.3)}\\ \frac{\partial^2\hat{\Pi}^i(\mathbf{z}_0-\mathbf{x},\mathbf{x})}{\partial {z^i}^2} &= \Pi^i{}_{ii}(\hat{\mathbf{x}}_0-\mathbf{x})=\Pi^i{}_{ii}(\mathbf{z}_0)<0,\quad i=H,F. && \text{(A.4)}\end{aligned}$$

(2) Since $z^i \geq 0,\quad i=H,F$ we can use our assumptions on the second derivatives (given in (24) and (19)) to get:

$$\hat{\Pi}^i{}_i(\hat{\mathbf{x}}) \leq \hat{\Pi}^i{}_i(\mathbf{x}) < \hat{\Pi}^i{}_i(\mathbf{z}_0) = 0. \tag{A.5}$$

It is easy to see that the second order conditions are satisfied, then using (31) it follows that $Z^i(\mathbf{x})=0,\ i=H,F$.

(3) Without loss of generality, let $i=H,\ j=F$. We will first show that the proposed solution satisfies the first order conditions (31). Using the definition of a reaction function we obtain:

$$\frac{\partial\hat{\Pi}^H((r^H(x^F,0)-x^H,0),(x^H,x^F))}{\partial z^H} = \Pi^H_H(x^H+r^H(x^F,0)-x^H,x^F) = \Pi^H_H(r^H(x^F,0),x^F)=0.$$

Since in this case $x^F > r^F(r^H(x^F,0))$ due to our assumptions on the reaction functions (stability and negatively sloped) we use again the definition of a reaction function to obtain:

$$\frac{\partial\hat{\Pi}^F((r^H(x^F,0)-x^H,0),(x^H,x^F))}{\partial z^F} = \Pi^F_F(r^H(x^F,0),x^F) < \Pi^F_F(r^H(x^F,0),r^F(r^H(x^F,0))=0.$$

Since it is easy to see that the second order conditions are satisfied, the proposed solution is the correct one.

(4) Since $x^i > r^i(x^j,0)$, our assumptions on the reaction functions imply that $\mathbf{x}^j > r^j(x^i,0)$. Without loss of generality let $i=H,\ j=F$. Using (19), we obtain:

$$\begin{aligned}\frac{\partial\hat{\Pi}^H(0,\mathbf{x})}{\partial z^H} &= \Pi^H_H(\mathbf{x})=\Pi^H_H(x^H,x^F)<\Pi^H_H(r^H(x^F,0),x^F)=0 && \text{(A.6)}\\ \frac{\partial\hat{\Pi}^F(0,\mathbf{x})}{\partial z^F} &= \Pi^F_F(\mathbf{x})=\Pi^F_F(x^H,x^F)<\Pi^F_F(x^H,r^F(x^H,0))=0 && \text{(A.7)}\end{aligned}$$

Since it is easy to see that the second order conditions are satisfied, $Z^H(\mathbf{x}) = Z^F(\mathbf{x}) = 0$ is the solution.

Proof of Proposition 1:

Suppose by way of contradiction that $Z^i(\mathbf{x}^*) > 0$ for some i. Then $Z^j(\mathbf{x}^*)$ $j \neq i$ can either be positive or zero. Using Lemma 1 it is easy to see that in either case:

$$\frac{\partial Z^i(\mathbf{x})}{\partial x^i} = -1, \quad \frac{\partial Z^j(\mathbf{x})}{\partial x^i} = 0, \quad i, j = H, F, \ i \neq j. \tag{A.8}$$

Using (A.8), equation (33) can be written as:

$$P^i_i(\mathbf{x}^*) = \Pi^i_i(\mathbf{x}^*) + \beta v^i = 0. \tag{A.9}$$

Without loss of generality, let i=H, j=F so that $Z^H(\mathbf{x}^*) > 0$ and $Z^F(\mathbf{x}^*) \geq 0$. Using (19) and (A.9) we obtain:

$$\Pi^H_H(x^{*H} + Z^H(\mathbf{x}^*), x^{*F}) < \Pi^H_H(x^{*H}, x^{*F}) = -\beta v^i < 0. \tag{A.10}$$

Since $Z^H(\mathbf{x}^*) > 0$, the relevant first order conditions hold with equality, i.e.:

$$\frac{\partial \hat{\Pi}^H(Z(\mathbf{x}^*), \mathbf{x}^*)}{\partial z^H} = \Pi^H_H(\mathbf{x}^* + Z(\mathbf{x}^*)) = 0. \tag{A.11}$$

Using (24) and (A.11) we obtain:

$$0 = \Pi^H_H(x^{*H} + Z^H(\mathbf{x}^*), x^{*F} + Z(\mathbf{x}^*)) \leq \Pi^H_H(x^{*H} + Z^H(\mathbf{x}^*), x^{*F}). \tag{A.12}$$

Using (A.10) and (A.12):

$$0 \leq \Pi^H_H\left(x^{*H} + Z^H(\mathbf{x}^*), x^{*F}\right) < 0,$$

a contradiction.

Proof of Proposition 2:

Since $Z^i(\mathbf{x}) = 0$, $i = H, F$, we can use Lemma 1 to conclude that $\frac{\partial Z^i(\mathbf{x})}{\partial x^i} = 0$, $\frac{\partial Z^j(\mathbf{x})}{\partial x^i} = 0$, $i, j = H, F$, $i \neq j$. Then the expression for $P^i_i(\mathbf{x})$ in equation (33) becomes:

$$P^i_i(\mathbf{x}) = (1 + \beta)\Pi^i_i(\mathbf{x}) + \beta v^i = 0, \quad i = H, F,$$

where $\Pi^i_i(\mathbf{x})$ is the derivative of $\Pi^i(\mathbf{x})$ (defined in equation (10)) with respect to x_i.

Differentiating $P^i_i(\mathbf{x})$ and $P^j_j(\mathbf{x})$ with respect to v^i:

$$\begin{aligned}(1 + \beta)\left[\Pi^i_{ii}(\mathbf{x})\frac{\partial x^i}{\partial v^i} + \Pi^i_{ij}(\mathbf{x})\frac{\partial x^j}{\partial v^i} - 1\right] + \beta &= 0 &\quad \text{(A.13)}\\ (1 + \beta)\left[\Pi^j_{ji}(\mathbf{x})\frac{\partial x^i}{\partial v^i} + \Pi^j_{jj}(\mathbf{x})\frac{\partial x^j}{\partial v^i}\right] &= 0. &\quad \text{(A.14)}\end{aligned}$$

Hence,

$$\frac{\partial x^i}{\partial v^i} = \frac{1}{1+\beta}\,\frac{\Pi^j_{jj}}{\hat{D}} < 0 \quad \text{and} \quad \frac{\partial x^j}{\partial v^i} = -\frac{1}{1+\beta}\,\frac{\Pi^j_{ji}}{\hat{D}} > 0,$$

where $\hat{D}$ was defined in (22) and the signs were determined using (19) and (24).

Proof of Lemma 3

Since second period R&D investments are positive for both firms, (36) and (37) equal zero. Differentiating with respect to x^H we obtain:

$$\begin{aligned} \Pi^H_{HH}(\hat{\mathbf{x}})\left[1+\frac{\partial Z^H(\mathbf{x})}{\partial x^H}\right] + \Pi^H_{HF}(\hat{\mathbf{x}})\frac{\partial Z^F(\mathbf{x})}{\partial x^H} &= 0 \qquad \text{(A.15)}\\ \Pi^F_{FH}(\hat{\mathbf{x}})\left[1+\frac{\partial Z^H(\mathbf{x})}{\partial x^H}\right] + \Pi^F_{FF}(\hat{\mathbf{x}})\frac{\partial Z^F(\mathbf{x})}{\partial x^H} &= 0. \qquad \text{(A.16)} \end{aligned}$$

Hence,

$$\frac{\partial Z^H(\mathbf{x})}{\partial x^H} = -1 \qquad \frac{\partial Z^F(\mathbf{x})}{\partial x^H} = 0.$$

Differentiating now with respect to x^F we obtain:

$$\begin{aligned} \Pi^H_{HH}(\hat{\mathbf{x}})\frac{\partial Z^H(\mathbf{x})}{\partial x^F} + \Pi^H_{HF}(\hat{\mathbf{x}})\left[1+\frac{\partial Z^F(\mathbf{x})}{\partial x^F}\right] - v^H\psi'(x^F) &= 0 \qquad \text{(A.17)}\\ \Pi^F_{FH}(\hat{\mathbf{x}})\frac{\partial Z^H(\mathbf{x})}{\partial x^F} + \Pi^F_{FF}(\hat{\mathbf{x}})\left[1+\frac{\partial Z^F(\mathbf{x})}{\partial x^F}\right] &= 0. \qquad \text{(A.18)} \end{aligned}$$

Hence,

$$\begin{aligned} \frac{\partial Z^H(\mathbf{x})}{\partial x^F} &= \frac{v^H\Pi^F_{FF}(\hat{\mathbf{x}})\psi'(x^F)}{\hat{D}} > 0, \qquad \text{(A.19)}\\ \frac{\partial Z^F(\mathbf{x})}{\partial x^F} &= -1 - \frac{v^H\Pi^F_{FH}(\hat{\mathbf{x}})\psi'(x^F)}{\hat{D}} < -1. \qquad \text{(A.20)} \end{aligned}$$

Proof of Lemma 4

(1) Using our assumptions on the second derivatives of the profit function ((24), (19)), and the first order conditions for both the second period R&D game and the entire game ((41), (47)) we obtain:

$$\begin{aligned} -\beta v^H\psi(x^F) = \Pi^H_H(\mathbf{x}) &> \Pi^H_H(\hat{\mathbf{x}}) = -v^H[1-\psi(x^F)] \qquad \text{(A.21)}\\ \text{or} \qquad \psi(x^F) &< \frac{1}{1+\beta}. \qquad \text{(A.22)} \end{aligned}$$

(2) Using (24), (19) together with (42) and (48) we have:

$$0 = \Pi^F_F(\hat{\mathbf{x}}) < \Pi^F_F(\mathbf{x}) = -\beta\left\{v^F + \Pi^F_H(\hat{\mathbf{X}}(\mathbf{x}))\frac{\partial Z^H(\mathbf{x})}{\partial x^F}\right\}.$$

Therefore,

$$v^F + \Pi^F_H(\hat{\mathbf{X}}(\mathbf{x}))\tfrac{\partial Z^H(\mathbf{x})}{\partial x^F} < 0,$$

or using (39)

$$v^F < -\frac{\Pi^F_H(\hat{\mathbf{x}})\ \Pi^F_{FF}(\hat{\mathbf{x}})\ v^H \psi'(x^F)}{\hat{D}}.$$

Lemma 8. *E^H is equal to zero and E^F is positive .*

Proof: Differentiating (66) and using Lemma 3 we obtain:

$$\begin{aligned}
\frac{\partial P^H_H(\mathbf{x})}{\partial z^H} &= \beta\left\{\Pi^H_{HH}(\hat{\mathbf{x}})\left[1+\frac{\partial z^H}{\partial x^H}\right] + \Pi^H_{FH}(\hat{\mathbf{x}})\frac{\partial z^F}{\partial x^H}\right\} = 0 \qquad &(\text{A.23})\\
\frac{\partial P^H_H(\mathbf{x})}{\partial z^F} &= \beta\left\{\Pi^H_{HF}(\hat{\mathbf{x}})\left[1+\frac{\partial z^H}{\partial x^H}\right] + \Pi^H_{FF}(\hat{\mathbf{x}})\frac{\partial z^F}{\partial x^H}\right\} = 0. \qquad &(\text{A.24})
\end{aligned}$$

Thus $E^H = 0$.

Differentiating (46) with respect to z^H and z^F we obtain:

$$\begin{aligned}
\frac{\partial P^F_F(\mathbf{x})}{\partial z^H} &= \beta\left\{\Pi^F_{FH}(\hat{\mathbf{x}})\left[1+\frac{\partial z^F}{\partial x^F}\right] + \Pi^F_{HH}(\hat{\mathbf{x}})\frac{\partial z^H}{\partial x^F}\right\}, \qquad &(\text{A.25})\\
\frac{\partial P^F_F(\mathbf{x})}{\partial z^F} &= \beta\left\{\Pi^F_{FF}(\hat{\mathbf{x}})\left[1+\frac{\partial z^F}{\partial x^F}\right] + \Pi^F_{HF}(\hat{\mathbf{x}})\frac{\partial z^H}{\partial x^F}\right\}. \qquad &(\text{A.26})
\end{aligned}$$

By (A.18), we have that $\frac{\partial P^F_F(\mathbf{x})}{\partial z^F} = 0$. Then, using (A.25), Lemma 3, and (64) we get:

$$\begin{aligned}
E^F &= \frac{\partial P^F_F}{\partial z^H}\frac{\partial z^H}{\partial \hat{s}} \qquad &(\text{A.27})\\
&= \frac{\beta v^H \psi'(x^F)}{\hat{D}^2}\Pi^F_{FF}(\hat{\mathbf{x}})\{-\Pi^F_{FF}\Pi^F_{HH} + \Pi^F_{HF}\Pi^F_{FH}\}. \qquad &(\text{A.28})
\end{aligned}$$

Note that $\Pi^F_{HF} = \Pi^F_{FH} < 0$ by (24) and $\Pi^F_{FF} < 0$ by (19). In the normal case in which $R^F_{HH} \geq 0$, $\Pi^F_{HH} \geq 0$ and thus $E^F > 0$.

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