Inequality-Driven Growth: Unveiling Aggregation Effects in Growth Equations

Pedro H. Albuquerque

Department of Accounting, Economics and Finance
Texas A&M International University

pedrohalbuquerque@yahoo.com
http://go.to/pedroalbuquerque

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Abstract

It is well known from nonlinear aggregation theory that distributions play a central role in the determination of aggregate relations. This paper establishes a bridge between the aggregation and the inequality and growth literature by applying a log-linear aggregation method to a simple heterogeneous AK growth model. The aggregation effect is explicitly captured in the growth equation by the changes of the mean logarithmic deviation (MLD or Theil's second measure) of the income, implying that increases in income inequality may be unambiguously associated with temporary increases in a country's growth rate, in agreement with the empirical findings of Forbes (AER, 2000). Consequently, empirical studies of the long-run effects of income inequality may suffer from aggregation bias if the temporary effects of the MLD changes are not considered. The accelerated growth episodes observed in Brazil and China demonstrate that the increase in income inequality may have resulted in substantial temporary increases in the aggregate growth rates experienced by those countries.

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Corresponding author: Pedro H. Albuquerque, Department of Accounting, Economics and Finance, 5201 University Blvd., Laredo, TX, 78041-1920. Phone: 956-326-2510. Fax: 956-326-2494. E-mail: pedrohalbuquerque@yahoo.com.

"The cake has to grow in order to be cut." Delfim Netto, Brazilian Minister of Finance, justifying the increase in income inequality during the Brazilian "economic miracle" period. Inequality did not decrease afterwards.

"Draw a cake to satisfy one's hunger." Chinese proverb.

1 Introduction

A large body of literature addressing the relationship between inequality and growth has been developed across the years. Yet, despite all the theoretical and empirical developments since the pioneering work of Kuznets (1955), much theoretical and empirical disagreement remains. Barro (2000) summarizes the lack of consensus by stating that "Many nice theories exist for assessing the effects of inequality on investment and economic growth. The problem is that these theories tend to have offsetting effects and that the net effects of inequality on investment and growth are ambiguous. The theoretical ambiguities do, in a sense, accord with empirical findings, which tend not to be robust."

This paper will explore an effect that, to a certain extent, has been disregarded both theoretically and empirically. It originates from the nonlinear aggregation literature, and explains why fast increases in income inequality may be associated with abnormal aggregate growth rates. Episodes like such are defined here as *inequality-driven growth*. The effect should be prevalent, even if presenting varying magnitude, since it does not arise from structural assumptions but from nonlinear aggregation properties.

It is well known from the aggregation theory that distributions may play a role in the determination of the values of aggregate variables. As Stoker (1986) puts it, "individual differences, or more general behavioral nonlinearities, must coincide with the presence of distributional effects in macroeconomic equations."[1] Despite that, only Ravallion (1998) explicitly considered effects of nonlinear aggregation in the inequality and growth context.

This paper will address the aggregation effects by applying a log-linear aggregation method developed in Albuquerque (2003) to a simple version of a Cass-Koopmans-Ramsey AK growth model based on heterogeneous households. The household production function will incorporate spillovers from public capital, which will work as a simple redistribution mechanism.

The combination of heterogeneous productivities with the redistribution mechanism, under given conditions, will generate balanced growth income trajectories defined by an equilibrium distribution of income and a unique income growth rate common to all households. A log-linearized Euler equation will be aggregated and, as a result, the aggregate growth rate will be decomposed into four parts, which represent a structural constant, changes of the average savings rate, a long-run distributional effect, and the inequality-driven effect – the latter representing the main contribution of this paper.

These findings should not be confounded with previous literature results that depend on structural assumptions. The inequality-driven growth effect presented here does not follow from any special assumption regarding the household behavior, except for heterogeneity and nonlinearity at the household level. Since almost any devisable growth model based on intertemporal optimization and heterogeneity fits this description, the aggregation effect should be prevalent.

Notice that the structural model used in this paper was chosen with tractability issues in mind. Other growth models should present the same aggregation effect, however in less tractable forms. The form of the effect presented here therefore may serve as a first approximation. Moreover, the

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[1] See also Lewbel (1992), Ravallion (1998), Garderen et al. (2000) and Albuquerque (2003).

specification presented in this paper is particularly well suited to the log-linear models commonly used in empirical studies.

Two results of this paper lend support to the empirical findings of Forbes (2000): the presence of an inequality-driven growth component in the aggregate growth equation, and the need to distinguish between long-run and short-run effects of inequality on growth. As summarized by Forbes, "(empirical) results suggest that, in the short and medium term, an increase in a country's level of income inequality has a significant positive relationship with subsequent economic growth. This relationship is highly robust across samples, variable definitions, and model specifications." The magnitude of the positive effect of inequality on growth found in Forbes (2000) is comparable with the theoretical magnitude of the aggregation effect found in this paper.

Finally, it will be shown that the aggregation effect can be indeed very significant, and consequently able to bias empirical results that do not take it in consideration. Two inequality-driven growth cases will be used for that: the Brazilian "economic miracle" high-growth period and the recent Chinese high-growth episode. In the two cases, the inequality changes may have had substantial temporary impacts on the aggregate growth rates.

2 A Simple AK Growth Model with Productivity Heterogeneity

2.1 The AK Model

The endogenous growth model that is presented here is based on a Cass-Koopmans-Ramsey framework with an AK production function.[2]

[2] See, for example, Aghion and Howitt (1998) or Jones and Manuelli (1997).

Consider therefore an economy with a large number of households $(N \to \infty)$. Each infinitely lived household maximizes the following CRRA utility function:

$$U_n = \sum_{t=0}^{\infty} e^{-\rho t} \frac{C_{nt}^{1-\theta}}{1-\theta},$$

where

$$\rho > 0, \quad \text{and} \quad 0 < \theta < 1.$$

To keep the representation as simple as possible, and without loss of generality, it is assumed, as in other AK models, that capital is represented by a single variable that encompasses all production factors. The production function depends not only on the household's private capital but also on spillovers from public capital.

The production function accordingly is defined as

$$Y_{nt} = f_n\left(\widetilde{K}_t, K_{nt}\right) = A_n \widetilde{K}_t^{\gamma} K_{nt}^{1-\gamma}, \tag{2.1}$$

where

$$0 < \gamma < 1, \quad A_n > 0, \quad \widetilde{K}_t = \prod_{n=1}^{N} K_{nt}^{1/N}$$

is a proxy for public capital, and K_{nt} is the household's private capital. The household's production level depends on the productivity parameter A_n (which is heterogeneous across households), on the private capital level K_{nt}, and on the spillovers from the public capital $\widetilde{K}_t$ (the geometric average of all private capital levels in the economy).

In the simple production function above, government policies are assumed to affect the redistribution parameter γ. The redistribution parameter also represents possible spillovers that are not related to public capital, such as: government-enforced income transfers, donations, charity, crime, epidemics, riots, specialization, trade, or any other positive or negative externality originating from private capital. Given the unbalanced nature of those externalities, a zero-sum restriction on the redistributive transfers will typically not hold.

Finally, notice that households must observe the budget constraint

$$Y_{nt} \geq C_{nt} + \Delta K_{nt+1} . \tag{2.2}$$

2.2 First-Order Conditions

Assume now that all necessary conditions for the existence of an interior solution hold. The first-order conditions are thereafter given by:

$$\lambda_{nt} = e^{-\rho t} C_{nt}^{-\theta} ,$$

and

$$\frac{\lambda_t}{\lambda_{t+1}} = 1 + (1-\gamma) A_n \left(\frac{\widetilde{K}_{t+1}}{K_{nt+1}} \right)^{\gamma} ,$$

which, when combined, lead to the Euler equation

$$\frac{C_{nt+1}}{C_{nt}} = \left\{ e^{-\rho} \left[1 + (1-\gamma) A_n \left(\frac{\widetilde{K}_{t+1}}{K_{nt+1}} \right)^{\gamma} \right] \right\}^{\frac{1}{\theta}} .$$

Now, assume that $(1-\gamma) A_n \left(\widetilde{K}_t / K_{nt} \right)^{\gamma} << 1$ (an acceptable hypothesis for growth rates observed in real economies), and take the logarithm of the equation above to find

$$\Delta c_{nt} \approx \frac{1}{\theta} \left[-\rho + (1-\gamma) A_n \left(\frac{\widetilde{K}_t}{K_{nt}} \right)^{\gamma} \right], \tag{2.3}$$

where $c_{nt} = \ln C_{nt}$. Due to the spillovers, the Euler equation describes the household consumption growth rate as an increasing function of the household productivity A_n and a decreasing function of the relative wealth level $K_{nt} / \widetilde{K}_t$.

2.3 The Log-Linearized Euler Equation

In order to easily aggregate equation (2.3), a log-linearized version will be derived. From equation (2.1),

$$\frac{\widetilde{K}_t}{K_{nt}} = \left(\frac{A_n}{\widetilde{A}} \frac{\widetilde{Y}_t}{Y_{nt}} \right)^{\frac{1}{1-\gamma}} . \tag{2.4}$$

Applying the equation above to equation (2.3) leads to

$$\Delta c_{nt} \approx \frac{1}{\theta} \left[-\rho + (1-\gamma) A_n \left(\frac{A_n}{\widetilde{A}} H_{nt} \right)^{\psi} \right], \tag{2.5}$$

where $H_{nt} = \widetilde{Y}_t / Y_{nt}$ represents the relative income gap of the household, and

$$\psi = \frac{\gamma}{1-\gamma} .$$

Log-linearizing equation (2.5) for H_{nt} results in the following approximation:

$$\Delta c_{nt} \approx \alpha_n + \beta_n h_{nt} , \tag{2.6}$$

where

$$\alpha_n \approx \frac{1}{\theta} \left[-\rho + (1-\gamma) A_n \left(\frac{A_n}{\widetilde{A}} \right)^{\psi} \right], \quad \beta_n = \frac{\gamma}{\theta} A_n \left(\frac{A_n}{\widetilde{A}} \right)^{\psi} ,$$

and

$$h_{nt} = \ln H_{nt} = \overline{y}_t - y_{nt}$$

is the logarithmic income gap.

2.4 The Aggregation Method

Consider now the log-linear aggregation method presented in Albuquerque (2003). Take I+1 vectors representing the values of I+1 variables for N households at time t,

$$\mathbf{Y}_t = \begin{bmatrix} Y_{1t} & \cdots & Y_{nt} & \cdots & Y_{Nt} \end{bmatrix}' , \qquad \mathbf{X}_{it} = \begin{bmatrix} X_{i1t} & \cdots & X_{int} & \cdots & X_{iNt} \end{bmatrix}' ,$$

where

$$Y_{nt} > 0, \quad X_{int} > 0, \quad i = 1,\ldots,I, \quad n = 1,\ldots,N, \quad \forall t,$$

and I parameter vectors

$$\mathbf{a}_i = \begin{bmatrix} a_{i1} & \cdots & a_{in} & \cdots & a_{iN} \end{bmatrix}'.$$

If a log-linear functional form with heterogeneous parameters across units

$$Y_{nt} = X_{1nt}^{a_{1n}} X_{2nt}^{a_{2n}} \cdots X_{Int}^{a_{In}} \tag{2.7}$$

describes each household relationship in the economy, then the relationship among the aggregate variables $\overline{Y}_t$ and $\overline{X}_{it}$ will be given by

$$\overline{Y}_t = \overline{X}_{1t}^{\overline{a}_1} \overline{X}_{2t}^{\overline{a}_2} \cdots \overline{X}_{It}^{\overline{a}_I} D_t, \tag{2.8}$$

where the aggregate variables are defined as per capita values

$$\overline{Y}_t = \frac{1}{N}\sum_{n=1}^{N} Y_{nt}, \; \overline{X}_{it} = \frac{1}{N}\sum_{n=1}^{N} X_{int}, \; \overline{a}_i = \frac{1}{N}\sum_{n=1}^{N} a_{in},$$

and the term

$$D_t = D(\mathbf{Y}_t, \mathbf{X}_{1t}, \ldots, \mathbf{X}_{Nt}) = \exp\left\{L(\mathbf{Y}_t) - \sum_{i=1}^{I}\left[\overline{a}_i L(\mathbf{X}_{it}) - \operatorname{cov}(\mathbf{x}_{it}, \mathbf{a}_i)\right]\right\} \tag{2.9}$$

represents scale-independent distributional effects, where

$$L(\mathbf{Y}_t) = \frac{1}{N}\sum_{n=1}^{N} \ln\left(\frac{\overline{Y}_t}{Y_{nt}}\right)$$

is the sample analog of the *mean logarithmic deviation* (MLD), also known as the *Theil's second measure* of $\mathbf{Y}_t$, a measure of income inequality,[3]

[3] It is illustrative to reproduce here the properties of this measure according to Bourguignon (1979): "That the inequality measure L has seldom been used in applied works on income distribution is somewhat surprising because it has very much to commend it. Besides the fact that it is decomposable … and satisfies the basic properties of an inequality measure, L lends itself to a very simple interpretation in terms of social welfare. In the utilitarian framework, the social welfare function is the sum of identical concave individual utility function. If we choose the logarithm form for those utility functions, L is simply the difference between the maximum social welfare for a given total income, which corresponds to the equalitarian distribution, and the actual social welfare." This paper results provide additional reasons for the use of this measure in empirical studies.

$$\operatorname{cov}(\mathbf{x}_{it},\mathbf{a}_i)=\frac{1}{N}\sum_{n=1}^{N}(x_{int}-\bar{x}_{it})(a_{in}-\bar{a}_i),\quad \mathbf{x}_{it}=\begin{bmatrix}x_{i1t}\\ \vdots \\ x_{iNt}\end{bmatrix}=\begin{bmatrix}\ln(X_{i1t})\\ \vdots \\ \ln(X_{iNt})\end{bmatrix},\quad \bar{x}_{it}=\frac{1}{N}\sum_{n=1}^{N}x_{int}\,.$$

Note that all components of D_t represent relative measures of inequality, meaning that D_t is scale invariant.

The logarithmic version of (2.8) is

$$y_t=\bar{a}_1x_{1t}+\bar{a}_2x_{2t}+\cdots+\bar{a}_Ix_{It}+d_t\,, \tag{2.10}$$

where

$$y_t=\ln\bar{Y}_t,\quad x_{it}=\ln\bar{X}_{it},\quad \text{and}\quad d_t=\ln D_t\,.$$

2.5 Aggregating the Model

A literal solution to the aggregation problem can now be provided. From equation (2.6):

$$c_{nt}=c_{nt-1}+\alpha_n+\beta_nh_{nt}\,. \tag{2.11}$$

Equation (2.10) can be applied to equation (2.11), resulting in the following per household aggregate consumption growth rate equation:

$$\Delta c_t=\bar{\alpha}+\operatorname{cov}(\boldsymbol{\beta},\mathbf{h}_t)+\Delta L(\mathbf{C}_t), \tag{2.12}$$

where

$$\boldsymbol{\beta}=[\beta_1\ \cdots\ \beta_N]',\ \mathbf{h}_t=[h_{1t}\ \cdots\ h_{Nt}]',$$

and

$$\bar{\alpha}=-\frac{\rho}{\theta}+\frac{1-\gamma}{\theta}\hat{A}\,, \tag{2.13}$$

$$\hat{A}=\frac{1}{N}\sum_{n=1}^{N}A_n\left(\frac{A_n}{\tilde{A}}\right)^{\psi},\quad \text{and}\quad \mathbf{H}_t=[H_{1t}\ \cdots\ H_{nt}\ \cdots\ H_{Nt}]',$$

since

$$L(\mathbf{Y}_t)=L(\mathbf{H}_t),\qquad h_t=\ln\bar{H}_t=\bar{h}_t+L(\mathbf{H}_t)=L(\mathbf{H}_t),$$

and

$$d(\mathbf{C}_t, \mathbf{C}_{t-1}, \mathbf{H}_t) = \Delta L(\mathbf{C}_t) - \overline{\beta} L(\mathbf{H}_t) + \mathrm{cov}(\boldsymbol{\beta}, \mathbf{h}_t).$$

Equation (2.12) can be extended to aggregate output using the following approximation. From (2.2), and since savings represent a relatively small share of the household output and of the economy output at the aggregate level, it follows that

$$\Delta c_t = \Delta y_t + \Delta \ln(1 - s_t) \approx \Delta y_t - \Delta s_t, \tag{2.14}$$

where $s_t = \Delta K_t / Y_t$, and also that

$$L(\mathbf{C}_t) = L(\mathbf{Y}_t * (\mathbf{1} - \mathbf{s}_t)) \approx L(\mathbf{Y}_t * \mathbf{1}) = L(\mathbf{Y}_t), \tag{2.15}$$

since consumption, due to its higher share in income, dominates savings rates in the determination of output growth and income inequality.

Substituting (2.14) and (2.15) into (2.12) leads to

$$\Delta y_t \approx \overline{\alpha} + \Delta s_t + \mathrm{cov}(\boldsymbol{\beta}, \mathbf{h}_t) + \Delta L(\mathbf{Y}_t), \tag{2.16}$$

Equation (2.16) determines approximately the aggregate growth rate of the economy.

2.6 Balanced Growth

Assume now that the income distribution converges to some relative income profile under balanced growth such that $\Delta c_n^* = \Delta y_n^*$ and, from equations (2.3) and (2.4),

$$\Delta y_n^* = \frac{1}{\theta}\left[-\rho + (1-\gamma) A_n \left(\frac{A_n}{\widetilde{A}} H_n^* \right)^{\psi} \right],$$

where $H_n^* = \lim_{t \to \infty} \widetilde{Y}_t / Y_{nt}$ represents the balanced growth relative income gap of household n.

Balanced growth is thereafter feasible as long as

$$\frac{\partial \Delta y_n^*}{\partial H_n^*} = \frac{\gamma}{\theta} A_n \left(\frac{A_n}{\widetilde{A}} \right)^{\psi} H_n^{*\frac{2\gamma - 1}{1-\gamma}} > 0,$$

or

$$\gamma > 0, \quad \forall n.$$

This AK model represents an economy that converges to some level of income inequality. The redistribution mechanism, the result of a strictly positive γ, guarantees that the aggregate growth engine works for every household in the long run. The model is consistent therefore with the Hirschman and Rothschild (1973) tunnel effect hypothesis.

From equations (2.6) and (2.16), balanced growth is defined as a set of household income growth trajectories where

$$\Delta y^* = \Delta y_n^* = \overline{\alpha} + \operatorname{cov}(\boldsymbol{\beta}, \mathbf{h}^*), \quad \forall n, \tag{2.17}$$

with household income distributed according to a vector of logarithmic income gaps $\mathbf{h}^* = \begin{bmatrix} h_1^* & \cdots & h_n^* & \cdots & h_N^* \end{bmatrix}'$, where

$$h_n^* = \frac{\overline{\alpha} + \operatorname{cov}(\boldsymbol{\beta}, \mathbf{h}^*) - \alpha_n}{\beta_n}. \tag{2.18}$$

According to equation (2.18), the relative income distribution under balanced growth will depend on the distribution of the household productivity parameter A_n. A higher level of productivity inequality will imply a higher level of income inequality. On the other hand, a higher redistribution parameter γ will imply a lower level of income inequality.

2.7 Growth Rate Decomposition

Equation (2.12) reveals that, after taking the aggregation effect in consideration, the per household income growth rate can be divided into four components: $\overline{\alpha}$ and Δs_t, which are related to mean values, and $\operatorname{cov}(\boldsymbol{\beta}, \mathbf{h}_t)$ and $\Delta L(\mathbf{Y}_t)$, which represent distributional effects.

Component $\overline{\alpha}$ is a fundamental constant originating from the structural model and representing the negative time preference effect and the positive aggregate productivity effect on growth rates. Component Δs_t represents the temporary contribution of changes in savings rates to the aggregate growth rate. Those two components are typically found in growth models based on intertemporal optimization. Component $\operatorname{cov}(\boldsymbol{\beta},\mathbf{h}_t)$, on the other hand, represents a distributional effect of income inequality on long-run growth rates. Finally, component $\Delta L(\mathbf{Y}_t)$, represents a transitory effect of inequality changes on growth rates – the *inequality-driven growth effect*. The last two components originate from nonlinear aggregation.

These results can be summarized by the following proposition:

Proposition 1: *Under the assumption of a simple AK growth model, the aggregate growth rate of an economy can be decomposed into four additive terms: the constant* $\overline{\alpha}$ *that represent the negative time preference effect and the positive aggregate productivity effect, the term* Δs_t *that represents transitory effects due to changes of savings rates, the term* $\operatorname{cov}(\boldsymbol{\beta},\mathbf{h}_t)$ that represents permanent distributional effects on growth*, and the inequality-driven effect term* $\Delta L(\mathbf{Y}_t)$ *that represents the temporary effect of income inequality changes.*

Proposition 1 reveals a component of aggregate growth rates that unambiguously depends on inequality. This component can be explicitly measured through MLD (Theil's second measure) changes, and is mostly disregarded in the current inequality and growth literature. The distributional component should appear in any aggregated log-linear growth model based on heterogeneous households, since it arises not at the structural level, but at the aggregation procedure level.[4]

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[4] To understand the effect captured by this growth component, a parallel can be made with the case of a locomotive pulling a caboose by means of an elastic cable. The locomotive represents high-productivity households, while the caboose represents low-productivity households. The elastic cable represents the redistribution mechanism. Even if no structural parameter is changing (productivity levels, time preference, risk aversion coefficients – the power sources and the frictions), once the cable is made more elastic, the result is a temporary acceleration of any reference point near the locomotive (the equivalent of the per capita income), at the cost of permanently higher inequality levels (the cable will stretch further). Finally, when the cable is again fully stretched, the locomotive will fall down to the previous speed and acceleration, since it will be subject once more to the deadweight and additional friction of the caboose.

For example, in Albuquerque (2003) a simple nonstructural heterogeneous log-linear growth model presenting asymmetric productivity shocks for skilled and unskilled households is used to explain some features the American "new economy" accelerated productivity growth episode in the nineties. In that model, an increase in productivity inequality is what causes the inequality-driven effect. The inequality-driven effect may be better interpreted therefore as the result of an aggregation "growth identity," obtained from equation (2.10), rather than the result of particular structural model hypotheses.

3 Implications to Empirical Studies

The inequality-driven effect represented by component $\Delta L(\mathbf{Y}_t)$ in equation (2.16) lends support to the empirical findings of Forbes (2000). In that paper, different panel data methods are applied to data representing 45 countries and 180 observations. The main innovations in Forbes' study are the use of the higher-quality data compiled by Deininger and Squire (1997) with shorter time intervals (five years) and the use of panel data methods that capture the effects of inequality changes across time. As summarized by Forbes, "results suggest that, in the short and medium term, an increase in a country's level of income inequality has a significant positive relationship with subsequent economic growth. This relationship is highly robust across samples, variable definitions, and model specifications."

The presence of an inequality change component in equation (2.16) may help to explain Forbes' empirical results. A simplified representation of the regression used in Forbes is[5]

$$\Delta y_t = b_0 + b_1 y_{t-1} + b_2 G_{t-1} + \varepsilon_t \tag{2.19}$$

[5] The regressions in Forbes also include data for male and female education and market distortions.

where t represents periods of five years and the independent variables (the logarithm of per capita real GDP, y, and the Gini coefficient, G) are measured in the year immediately before the five-year period. For example, the average yearly growth rate between 1966 and 1970 is assumed to depend on the values of the income and of the Gini coefficient observed in 1965.

Notice that equation (2.19) follows a stable error-correction model (ECM) when $b_1 < 0$. For example, if b_2 is positive, an increase in the Gini coefficient will generate a temporary sequence of increased income growth rates. The growth rate boost will disappear once the income reaches a new level that cancels out the effect of the inequality increase.

The simplified version of Forbes' empirical model represented by equation (2.19) should not be directly compared with the model given by equation (2.16), since real world dynamics tend to be more complex then the dynamics of parsimonious theoretical models. However, the relevant predictions can be compared. Equation (2.16) predicts that an MLD increase of one point (0.01) should lead to a 1% (0.01) temporarily increase in the income growth rate, everything else constant. How does this prediction compares with the empirical results obtained in Forbes' paper?

First, MLD changes have to be converted to Gini coefficient changes. The relationship between these two inequality measures is highly nonlinear, and depends on the income distribution profiles. To solve this problem without much ado, a simple log-linear regression is employed, where the yearly data is the same used by Forbes: 45 countries ranging from 1966 to 1994. Since the Deininger and Square data do not include MLD measures, they were calculated from the distribution quintiles, when available, resulting in 401 observations. The Gini coefficients were also recalculated from the same distribution quintiles for consistency. A simple unbalanced panel data LS regression with country-specific fixed effects was used. The functional form and the estimated parameters are

$$\ln \text{MLD} = \underset{(0.114)}{0.333} + \underset{(0.058)}{1.945} \ln \text{Gini} - \underset{(0.122)}{0.453} \ln(1 - \text{Gini}), \quad \overline{R}^2 = 0.996, \quad (2.20)$$

where the values between parentheses represent standard errors. The estimated parameters do not change significantly when other panel data estimation methods are employed.

The mean value of the Gini coefficient in Forbes is 0.386. Equation (2.20) predicts that when the Gini coefficient increases from 0.38 to 0.39 (a change of 0.01) the LMD increases by 0.016. According to the theoretical predictions in this paper, and as a rough approximation that takes in consideration Forbes' mean Gini levels, a Gini coefficient increase of one point (0.01) should lead to a temporarily increase in the income growth rate of 1.6% (0.016).

How does this theoretical prediction (1.6%) compares with the values estimated in Forbes? The estimated parameters of equation (2.19), according to Forbes' preferred Arellano and Bond method, are b_1 = -0.047 (standard error of 0.008) and b_2 = 0.13 (standard error of 0.06). Therefore, the short-term effect (the temporary growth rate increase accumulated in the next five years) of a Gini coefficient increase of one point (0.01) is $5 \times 0.13 \times 0.01$ = 0.65%. The accumulated effect (the temporary growth rate increase accumulated in all future years) is $-b_2/b_1 \times 0.01$ = 2.77%. The theoretical prediction of this paper (1.6%) is therefore somewhere in between the estimated short-term and accumulated predictions of Forbes' paper. Given all the uncertainties, approximations, and specification problems involved in this estimation, the result may be considered quite surprising.

Finally, notice that the absence of the nonlinear aggregation bias term in current empirical growth research may lead to omitted variable bias and to functional form misspecification, as pointed out in Ravallion (1998). Suppose, for example, that a researcher uses the following statistical model to estimate the relation between inequality and growth

$$\Delta y_t = f(\mathbf{X}_t, G(\mathbf{Y}_t)) + \varepsilon_t, \quad (2.21)$$

where $\mathbf{X}_t$ is a set of control variables, and only the levels of the Gini coefficient (not the changes) are used among the independent variables. This model is misspecified for two reasons. First, because it uses the Gini coefficient instead of the correctly specified MLD, resulting in functional form misspecification. Second, because a correctly specified model, according to equation (2.16), should be written as

$$\Delta y_t - \Delta s_t - \Delta L(\mathbf{Y}_t) = f(\mathbf{X}_t, L(\mathbf{Y}_t)) + \varepsilon_t\,, \tag{2.22}$$

or as an extended dynamic version of the same specification.

Specification problems due to aggregation are well known in the literature – see for example Stoker (1986), Lewbel (1992), Ravallion (1998), Garderen et al. (2000), and Albuquerque (2003). A natural extension of this paper is therefore the reproduction of previous empirical studies using correctly specified models.

In the next section, two empirical examples will demonstrate that the aggregation effect represented by the changes of the MLD may have significant magnitude, impacting aggregate growth rates.

4 Inequality-Driven Growth Episodes

4.1 A Cake Yet to Be Cut: The Brazilian "Economic Miracle"

From 1968 to 1973, Brazil experimented a period of high growth rates that came to be known as the Brazilian "economic miracle" period. This period, according to the usual interpretation, was the result, among other things, of high levels of foreign savings, mostly based on government borrowing in foreign capital markets, of an increase in mandated domestic savings, of the achievement, during the previous years, of fiscal discipline, and of central-planned measures that ranged from managed trade policies to an omnipresent system of subsidies and government credit. See, for example, Fishlow (1972), Sjaastad (1974), Fields (1977), Ahluwalia et al. (1980), Beckerman and Coes (1980), Fields (1980), Fishlow (1980), and Fox (1983) for additional details.

The period was marked by exceptionally high yearly growth rates and substantial increases in income inequality and savings rates, as shown in the following table:

Period	Real GDP per worker, yearly logarithmic growth (A)	Savings rate, average yearly change (B)	MLD, average yearly change (C)	"Treated" yearly logarithmic growth, (D)=(A)-(B)-(C)
1961-1967	0.029	-0.008	0.003	0.034
1968-1973	0.073	0.012	0.017	0.045
1974-1980	0.032	-0.003	-0.010	0.045
1981-1989	0.011	-0.003	0.015	0.000

The real GDP per worker and the savings rate data in this table come from Heston et al. (2002), and the data for income inequality comes from the "high quality" WIID databank based on Deininger and Squire (1997). The MLD values were calculated using the databank income distribution quintiles.[6]

It is easy to notice from the table above that the "economic miracle" period (shadowed) was exceptional when compared to all others. The yearly real GDP growth rate per worker during the "miracle" period is approximately 4.2% higher than the rates that prevailed during the preceding and succeeding periods. The income inequality, measured by the MLD, increases substantially during this period. This is also the only period during which the savings rates increased, dramatically. Notice, however, that near half of this growth rate boost is explained by changes of income inequality levels. The inequality-driven effect is approximately equal to 1.7%

[6] Notice that inequality levels and changes are underestimated here due to the quintile approximation. It means that the effects reported could be even more significant. Sadly, very few countries report income inequality using the MLD.

per year. Moreover, if the savings rate effect and the inequality-driven growth effect are subtracted from the growth rates (column D), the "treated" growth rate that results ends up following a very simple pattern: significant growth before the debt crisis of 1981, and no growth after the debt crisis.

4.2 Drawing a Cake: High Growth in China

The Chinese high growth episode, although much more protracted than the Brazilian, has in common the same exceptionally high growth rates and income inequality increases. This topic has been extensively described in previous studies, with a few examples represented by Khan and Riskin (1998), Yao (1999), Xu and Zou (2000), Meng (2001), Galbraith and Wang (2002), Park et al. (2002), and Zhang and Harvie (2002).

Unfortunately, the series for China are relatively short. Additionally, it should be noted that there is much dispute about the comparability of Chinese data with data from other countries, as discussed for example in Gibson et al (2001). Yet, the trends are clear, as summarized in the following table, which uses, as in the Brazilian case, data from Heston et al. (2002) and from the "high quality" WIID income inequality databank:

Period	Real GDP per worker, yearly logarithmic growth (A)	Savings rate, average yearly change (B)	MLD, average yearly change (C)	"Treated" yearly logarithmic growth, (D)=(A)-(B)-(C)
1981-1984	0.064	-0.001	-0.012	0.078
1985-1992	0.046	0.004	0.016	0.026

The period between 1981 and 1984 is atypical, with very high growth rates and decreasing inequality levels. However, the years between 1985 and 1992 (shadowed) can be seen as representing another case of inequality-driven growth. From the total yearly growth rate of 4.7%, approximately 1.6% can be attributed to inequality changes. Savings rate changes of approximately 0.4% per year also contributed to increase the growth rates during the period. According to some of the studies cited above, inequality increased even more dramatically after 1992, meaning that the inequality-driven effect may have become more significant in the nineties. Naturally, high growth in China may also be explained by structural changes captured by other growth rate components described in Proposition 1.

The two examples above show that the inequality-driven growth effect can be very significant, and therefore it can be the source of substantial aggregate bias in growth regressions.

5 Conclusions

It is well known from nonlinear aggregation theory that distributions may play a role in the determination of the values of aggregate variables. This paper tried to establish a bridge between the aggregation and the inequality and growth literature by applying a log-linear aggregation method to a simple Cass-Koopmans-Ramsey AK growth model with heterogeneity.

The aggregation effect was explicitly captured in the aggregate growth rate equation by the changes of the mean logarithmic deviation (MLD or Theil's second measure) of the income, implying that increases in income inequality may be unambiguously associated with temporary increases in a country's growth rate. Consequently, empirical studies searching for long-run effects of income inequality may suffer from aggregation biases if the temporary effects of the MLD changes are not considered.

This component of aggregate growth rates may appear in any aggregated log-linear growth model based on heterogeneous households subject to redistribution mechanisms, since the component arises not at the structural level but at the aggregation level. In this sense, the inequality-driven effect should be interpreted as a term from an aggregation growth identity, and not as a structural component resulting from particular model hypotheses.

The inequality-driven effect found in this paper lends support to the empirical findings of Forbes (2000). As summarized by Forbes, "results suggest that, in the short and medium term, an increase in a country's level of income inequality has a significant positive relationship with subsequent economic growth. This relationship is highly robust across samples, variable definitions, and model specifications." Moreover, the magnitude of the effect found in Forbes is comparable with the theoretical magnitude found in this paper.

The accelerated growth episodes observed in Brazil from 1968 to 1973 and in China recently demonstrate that the increase in income inequality, as measured by the MLD changes, may have resulted, through the aggregation effect, in substantial temporary increases in the aggregate growth rates experienced by those countries. Empirical studies that do not correct for this effect may produce misleading results due to the untreated aggregation bias. A natural extension of this paper is therefore the reproduction of previous empirical studies using correctly specified models.

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