The Golden Growth Law in Economic Process

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Abstract

Based on the partial distribution[1] and the developower (development power)[2], this paper puts forward the golden growth law in economic process for the first time. The law describes the optimal relation between the economic investment and the economic growth, and could be taken as a basis to distinguish that the economic process is higher in developing efficiency or not. A series of important constants in economy are obtained on the golden growth law, like the coefficient of golden growth and the increment contribution of developower in economic growth. These coefficients can reflect some of key number relations among the economic growth. Also in this paper, the programming and managing models for economic growth are given on the economic structure. We can use them as the tools to analyze and control the macroeconomic growth in analytic way. Finally, by the empirical researches, the golden growth law is explained to be existent and effective, the programming model for economic structure are proved to be useful to make decision in macroeconomic management.

Key Words partial distribution, developower, economic growth, golden growth law, economic structure

1 Introduction

There are many of the important studies in the economic theory, such as the business cycle theory (Lucas, 1981), the real business cycle theory (Kydland and Prescott, 1982, and Long and Plosser, 1983), and new growth theory (Romer, 1986), etc. These theories has availably propeled forward the economic development of world. In recent years, the economists pay their more attention to make the efficiency of economic development be higher (Collier and Dollar, 2004), evaluate the government's economic aid to nation or district and their actual results (Guillaumont and Chauvet, 2001), insure the economic growth by establishing and choosing the policies (Graziella, 2001) and perfecting the system of economic education (Graziella, 2004), etc.

It is worthy to advert to that economic development is a large scale system itself, and we perhaps need a new and different theory for economic development and the systemic and structural models on it. According to

1 The partial distribution (F. Dai, 2001). Though the univariate partial distribution (UPD) is the univariate left-truncated normal (Gaussian) distribution with truncation below zero, an approximate closed forms of integral $\int_0^x e^{-\frac{t^2}{2}}dt$ and $\int_0^x e^{-\frac{(t-\mu)^2}{2\sigma^2}}dt$, which are given in discussions on UPD, have not been given in the discussions on the univariate truncated normal distribution (Johnson, Kotz and Balakrishnan, 1994), and the multivariate partial distribution (Dai, Liu, and Wang, 2005) is different in its definition from the current multivariate truncated normal distribution (Kotz, Johnson and Balakrishnan, 2000, Horrace, 2005). So the partial distribution is still called.

2 The basic meaning of developower is the development power (Dai, Sun and Sun, 2004), but developower should include more contents than development power. So the technical term "developower" is given here.

the basic characters of economic growth and economic structure, here, author try to make some of discussions as following based on partial distribution (Dai, 2001) and developower:

● The developower theory is perfected. It will be further explained that the developower, which can impel the economy to grow and advance the productivity to develop, comes into being from the changes in economic environment, and the key problem of economic development is the interrelation and interaction between productivity and developower.

● The golden growth law is given. Describing the analytic relation between the economic level and developower, the optimal correlation model is established on the correlation between the economic growth rate and the developower rate, which is called the golden growth law.

● The economic programming model is established on developower. The economic system is divided in two kinds of patterns, the producing type (economy is divided into the producing domains) and functional type (economy is divided into the functional realms), and then, the structural model of economic system is built. By the structural model, we could analyze, plan and control the economic growth.

● The empirical researches are put up. Some of empirical researches on the results in this paper are proceeded on the US GDP data (chained price index and in billions of dollars) from 1940 to 2004, in order to illuminate that the golden growth law is existed in the process of economic growth, and the economic programming model is useful to make decisions on economic development.

On the other hand, a series of important constants in economy are obtained on the average growth and golden growth in economic process, like the coefficients of average growth and golden growth, the contribution quantity of developower in economic average growth and economic golden growth, and the stock coefficients of developower. These coefficients can reflect some of key number relations among the economic growth.

2 The Economic Level and the Economic Developower

2.1 The basic concepts and assumptions

The economic level is the state of economic development, also the evaluated value of economic status. The economic level includes the basic level of economy and the real level of economy. The basic level of economy, basic level for short, is the evaluated value of basic establishment for economy and the due ability in economy; the real level of economy, real level for short, is the real ability presented in economic production. In order to describe the determinacy (the current basic level) and the randomicity (the future real level) in economic process, some of assumptions about the economic level are given here.

Assumption 1 Suppose the real level is a stochastic variable which has a determinate background, and this determinate background is based on the basic level which is a constant in a period of time.

1) The values of basic level and real level are non-negative, i.e. the minimum values of basic level and real level are zero.

2) Both the basic level and the real level are always fluctuated, and the fluctuation ranges are positive, i.e. the minimum values of the fluctuation ranges are zero.

3) The real level changes around the basic level, and the more the real level is apart from the basic level, the less the probability of real level appearing is.

Definition 1 The economic developower, developower for short, is the motivity to push economy to progress. The economic development rate is the relative intensity of developower on unit of economic level.

Developower is a kind of the invisible and potential force, like policy and system, science and technology, knowledge and education, market system, economic management, law and regulation, cultural background, public idea, consumed desires, etc., which exist widely in economic field, development rate could be valuated by the developower on unit of economic level. developower comes into being from the changes of the economic environment. The economic environment said here is composed of the environments of production, resource, investment, policy, science and technology, education, management, sale, etc. So developower reflects the uncertainty in economic process.

Based on the discussion above, we see that developower includes the developower in production which is caused by the changes of environment in economic production) and the developower in nonproduction which is caused by the changes of environment in economic policies, science and technology, management, education, market system, etc.

There is the duality in the asset. The asset, which is physical (like something in kind) and called the real assets, is the productivity if it has some kind of abilities to produce, and the asset, which is nonphysical (like money) and called the nonphysical asset, is the developower if it is the potential motivity to push economy to progress. For the financial asset, it could be taken as productivity if relating to real asset and it could be taken as developower if relating to nonphysical asset.

In any stage of economic production, the basic level could represent the original ability to produce, and the real level represents the closed ability to produce. In this stage, an investing asset as is the developower can not influence the basic level, but the investing asset can influence the real level and will be the basic level in the next stage. So we have

Assumption 2 Any of nonphysical asset will influence economy as a developower before it become a real ability to produce. Developower can be strengthened by adding nonphysical asset.

2.2 The measurement for developower and development rate

The economy is composed of producing economy and nonproducting economy. The producing economy is a series of economic activities to provide directly the commodities, products and serves for society and related realms. The nonproducting economy is a series of activities to ensure the producing economy to be running normally.

According to the different cases, economic level could be measured by the total level of productivity (the quantity of all the effective assets in economic society), or the total quantity of input (GDP). The level of producing economy (the level of productivity) could be measured by the total quantity of productive assets, and the level of nonproducting economy could be measured by the total quantity of nonproductive assets.

Because the developower is come from by the changes of economic environment, developower could be measured by the fluctuating range of economic level. Further more, the development rate could be measured by the ratio of developower to economic level, i.e.

$$\text{development rate} = \frac{\text{developower}}{\text{the economic level}} .$$

In the view of intuition, the development rate means developower contained in unit of economic level ; and in intrinsic, the development rate means the function of economic environment to advance in economic level. Development rate can describe the vitality of economic development.

3 The Partial Distribution and the Related Results

3.1 The Univariate Partial Distribution

Definition 1 (univariate partial distribution, UPD for short) Let X be a non-negative stochastic variable, and it follows the distribution of density

$$f(x)=\begin{cases} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \Big/ \int_0^{\infty} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\,dx & x\geq 0 \\ 0 & x<0 \end{cases} \tag{1}$$

where $\mu\geq 0$ and $\sigma>0$. Then, X is called to follow a univariate partial distribution, and note as $X\in P(\mu, \sigma^2)$. If all the μ, σ, and X are time-variant, then note as $X(t)\in P(\mu(t), v^2(t))$.

According to references (Dai, F. and G. Ji, 2001, Dai, F., Xu, W..X., Liu, H. and H. Xu, 2003), we have two basic results [3] about UPD as follow:

Theorem 1 For any $x\in[0,\infty)$, the following formulas are correct approximately:

1) $\int_0^x e^{-\frac{t^2}{2}}\,dt=\sqrt{\frac{\pi}{2}(1-e^{-\frac{2}{\pi}x^2})}$;

2) $\int_0^x e^{-\frac{(x-\mu)^2}{2\sigma^2}}\,dx=\sqrt{\frac{\pi}{2}}\sigma\left((\sqrt{1-e^{-\frac{2}{\pi}(\frac{\mu}{\sigma})^2}}+\mathrm{sgn}(x-\mu)\sqrt{1-e^{-\frac{2}{\pi}(\frac{x-\mu}{\sigma})^2}}\right)$, where, $\mathrm{sgn}(x)=\begin{cases}1 & x>0\\ 0 & x=0\\ -1 & x<0\end{cases}$.

Theorem 2 Let X follow the partial distribution $P(\mu, \sigma^2)$, thus

1) The expected value $E(X)$ is as follows

$$E(X)=\mu+\sqrt{\frac{2}{\pi}}\frac{\sigma e^{-\frac{\mu^2}{2\sigma^2}}}{\sqrt{1-e^{-\frac{2}{\pi}(\frac{\mu}{\sigma})^2}}+1} \tag{2}$$

2) The variance $D(X)$ is as follows

$$D(X)=\sigma^2+E(X)[\mu-E(X)] \tag{3}$$

Definition 2 (Rightward Partial Distribution, RPD for short) If X is a non-negative stochastic variable, and it has the probability density function as follows

$$f(x)=\begin{cases} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \Big/ \int_a^{\infty} e^{-\frac{(u-\mu)^2}{2\sigma^2}}\,du & x\geq a \\ 0 & x<a \end{cases}$$

where, the constant $a>0$, then X is called to follow the rightward partial distribution, and note as $X\in P_a(\mu, \sigma^2)$. When $\mu>\sigma$ and $a=\mu-\sigma$, X is called to follow the standard rightward partial distribution, SRPD for short, and note as $X\in F_a(\mu, \sigma^2)$.

Corollary 1 For any $x\in[a,\infty]$, a, μ and σ are constant, a, $\mu\geq 0$, $\sigma>0$, then the following equations are correct approximately:

$$\int_a^x e^{-\frac{(u-\mu)^2}{2\sigma^2}}\,du=\sqrt{\frac{\pi}{2}}\sigma\left(\mathrm{sgn}(x-\mu)\sqrt{1-e^{-\frac{2}{\pi}\left(\frac{x-\mu}{\sigma}\right)^2}}-\mathrm{sgn}(a-\mu)\sqrt{1-e^{-\frac{2}{\pi}\left(\frac{a-\mu}{\sigma}\right)^2}}\right)$$

where, sgn(x) is the same as in theorem 1.

3 These results above have ever not appeared in the discussions on the univariate truncated normal distribution.

If $X\in P_a(\mu, \sigma^2)$, and $a<\mu$, thus

$$E(X)=\mu+\sqrt{\frac{2}{\pi}}\sigma\frac{e^{-\frac{(\mu-a)^2}{2\sigma^2}}}{1+\sqrt{1-e^{-\frac{2}{\pi}\frac{(\mu-a)^2}{\sigma^2}}}} \tag{4}$$

$$D(X)=\sigma^2-[E(X)-a][E(X)-\mu]=\sigma^2-[E(X)-a]R(X) \tag{5}$$

3.2 The Multivariate Partial Distribution

Definition 3 (Multivariate Partial Distribution, MPD for short) if $X_1, \cdots, X_n$ ($n\geq 2$) are all the non-negative stochastic variables, and they have the multivariate probability density function as follow

$$f(x_1,\cdots,x_n)=\begin{cases}\dfrac{e^{-\frac{1}{2|M|}\left[\sum_{i=1}^{n}|M_{ii}|(x_i-\mu_i)^2+\sum_{i,j=1,i\neq j}^{n}|M_{ij}|\left(\sigma_i\left(x_j-\mu_j\right)\right)\right]}}{\int_0^\infty\cdots\int_0^\infty e^{-\frac{1}{2|M|}\left[\sum_{i=1}^{n}|M_{ii}|(x_i-\mu_i)^2+\sum_{i,j=1,i\neq j}^{n}\sigma_i|M_{ij}|\left(x_j-\mu_j\right)\right]}dx_1\cdots dx_n} & 0\leq x_1,\cdots,x_n<\infty \\ 0 & \text{other cases}\end{cases} \tag{6}$$

where, $\boldsymbol{M}=(\sigma_{ij})_{n\times n}$, $\sigma_{ii}=\sigma_i^2, \sigma_{ij}=r_{ij}\sigma_i\sigma_i$ ($i\neq j$), $\sigma_i>0$, $|r_{ij}|\leq 1$, $i, j=1, \cdots, n$. Then $X_1, \cdots, X_n$ is called to follow multivariate partial distribution[4] (MPD), and note as $\boldsymbol{X}\in P(\boldsymbol{\mu}, \boldsymbol{\sigma}^{\mathrm{T}}\boldsymbol{\sigma}, \boldsymbol{R})$. where, $\boldsymbol{X}=(X_1, \cdots, X_n)^{\mathrm{T}}$, $\boldsymbol{\mu}=(\mu_1, \cdots, \mu_n)^{\mathrm{T}}\geq\boldsymbol{0}$, $\boldsymbol{\sigma}=(\sigma_1, \cdots, \sigma_n)^{\mathrm{T}}>\boldsymbol{0}$, $\boldsymbol{R}=(r_{ij})_{n\times n}$, r_{ij} is called the correlation coefficient between X_i and X_j, $r_{ii}=1$, $i, j=1, \cdots, n$.

As a special example of MPD, if the non-negative stochastic variables X and Y follow the bivariate distribution:

$$f(x,y)=\begin{cases}\dfrac{e^{-\frac{1}{2(1-r^2)}\left[\left(\frac{x-\mu_1}{\sigma_1}\right)^2-2r\left(\left(\frac{x-\mu_1}{\sigma_1}\right)+\left(\frac{y-\mu_2}{\sigma_2}\right)\right)+\left(\frac{y-\mu_2}{\sigma_2}\right)^2\right]}}{\int_0^\infty\int_0^\infty e^{-\frac{1}{2(1-r^2)}\left[\left(\frac{x-\mu_1}{\sigma_1}\right)^2-2r\left(\left(\frac{x-\mu_1}{\sigma_1}\right)+\left(\frac{y-\mu_2}{\sigma_2}\right)\right)+\left(\frac{y-\mu_2}{\sigma_2}\right)^2\right]}dxdy}, & 0\leq x,y<\infty \\ 0 & x<0 \text{ or } y<0\end{cases} \tag{7}$$

then, (X, Y) is said to have the bivariate partial distribution, and note as $(X, Y)\in P(\mu_1,\mu_2,\sigma_1^2,\sigma_2^2,r)$, where, the constants $\mu_1, \mu_2\geq 0$, $\sigma_1, \sigma_2>0$, $-1<r<1$.

When $|r|=1$, we know, according to reference (Fisz, 1978), X is correlating with Y in linearity and on probability 1, i.e., the probability $P(Y=dX+h)=1$,where, both d and h are constant, $d>0$ if $r=1$, and $d<0$ if $r=-1$.

Theorem 3 If both X_1 and X_2 are stochastic variables and follow bivariate partial distribution, i.e., $(X_1, X_2)\in P(\mu_1,\mu_2,\sigma_1^2,\sigma_2^2,r)$, thus

1) $\int_0^\infty\int_0^\infty e^{-\frac{1}{2(1-r^2)}\left[\left(\frac{u-\mu_1}{\sigma_1}\right)^2-2r\left(\left(\frac{u-\mu_1}{\sigma_1}\right)+\left(\frac{v-\mu_2}{\sigma_2}\right)\right)+\left(\frac{v-\mu_2}{\sigma_2}\right)^2\right]}dudv$

4 We see that the definition of multivariate partial distribution is different from the current multivariate truncated normal distribution, see references (Kotz, Johnson and Balakrishnan, 2000, Horrace, 2005).

$$=\frac{\pi}{2}\sigma_1\sigma_2(1-r^2)e^{\frac{r^2}{1-r^2}}\left(1+\sqrt{1-e^{-\frac{2}{\pi}\left(\frac{\mu_1+r\sigma_1}{\sigma_1\sqrt{1-r^2}}\right)^2}}\right)\left(1+\sqrt{1-e^{-\frac{2}{\pi}\left(\frac{\mu_2+r\sigma_2}{\sigma_2\sqrt{1-r^2}}\right)^2}}\right)$$

2) $\int_0^{x_1}\int_0^{x_2} f(u,v)dudv = \dfrac{A_1(x_1)A_2(x_2)}{\left(1+\sqrt{1-e^{-\frac{2}{\pi}\left(\frac{\mu_1+r\sigma_1}{\sigma_1\sqrt{1-r^2}}\right)^2}}\right)\left(1+\sqrt{1-e^{-\frac{2}{\pi}\left(\frac{\mu_2+r\sigma_2}{\sigma_2\sqrt{1-r^2}}\right)^2}}\right)}$, $0\le x_1, x_2<\infty$

where, $A_i(t)=\sqrt{1-e^{-\frac{2}{\pi}\left(\frac{\mu_i+r\sigma_i}{\sigma_i\sqrt{1-r^2}}\right)^2}}+\mathrm{sgn}(t-\mu_i)\sqrt{1-e^{-\frac{2}{\pi}\left(\frac{t-(\mu_i+r\sigma_i)}{\sigma_i\sqrt{(1-r^2)}}\right)^2}}$, sgn($x$) is the same as in theorem 1, i=1, 2.

Denoting:

$$f_{1r}(x)=\int_0^\infty f(x,y)dy=\begin{cases} e^{-\frac{[x-(\mu_1+r\sigma_1)]^2}{2\sigma_1^2(1-r^2)}}\Big/\int_0^\infty e^{-\frac{[u-(\mu_1+r\sigma_1)]^2}{2\sigma_1^2(1-r^2)}}du & x\ge 0\\ 0 & x<0\end{cases}$$

and $$f_{2r}(y)=\int_0^\infty f(x,y)dx=\begin{cases} e^{-\frac{[y-(\mu_2+r\sigma_2)]^2}{2\sigma_2^2(1-r^2)}}\Big/\int_0^\infty e^{-\frac{[u-(\mu_2+r\sigma_2)]^2}{2\sigma_2^2(1-r^2)}}du & y\ge 0\\ 0 & y<0\end{cases}$$

Theorem 4 If both X_1 and X_2 are stochastic variables and follow the bivariate Partial Distribution, i.e., (X_1, X_2) $\in P(\mu_1,\mu_2,\sigma_1^2,\sigma_2^2,r)$, thus

1) The expected values of each stochastic variable are

$$E_r(X_i)=\int_0^\infty xf_{ir}(x)dx=\mu_i+r\sigma_i+\sqrt{\frac{2}{\pi}}\frac{\sigma_i\sqrt{1-r^2}\,e^{-\frac{1}{2}\left(\frac{\mu_i+r\sigma_i}{\sigma_i\sqrt{1-r^2}}\right)^2}}{1+\sqrt{1-e^{-\frac{2}{\pi}\left(\frac{\mu_i+r\sigma_i}{\sigma_i\sqrt{1-r^2}}\right)^2}}}$$, where i=1, 2.

2) The variances of each stochastic variable are

$$D_r(X_i)=\int_0^\infty [x-E_r(X_i)]^2 f_{ir}(x)dx$$
$$=\sigma_i^2(1-r^2)+E_r(X_i)[\mu_i+r\sigma_i-E_r(X_i)]$$, where i=1, 2.

We can validate that $D_r(X_1)=D(X_1)$ and $D_r(X_2)=D(X_2)$ if r=0.

3.3 Estimating the parameters in MPD

Here we take the bivariate partial distribution as an example. The samples series of stochastic variable 1 and variable 2 are separately $x_{1,1}, x_{1,2}, \cdots, x_{1n}$ and $x_{21}, x_{22}, \cdots, x_{2n}$ (x_{1i}, x_{2i} >0, i=1,⋯, n).

According to the modified maximum likelihood estimation [10], we can obtain $\hat{\mu}_k$ (the estimate of μ_k) and $\hat{\sigma}_k$ (the estimate of σ_k), k=1, 2. Thus, the correlation coefficient can be estimated as:

$$\hat{r}_{1,2}=\frac{\sum_{i=1}^{n}(x_{1i}-\hat{\mu}_1)(x_{2i}-\hat{\mu}_2)}{\sqrt{\sum_{i=1}^{n}(x_{1i}-\hat{\mu}_1)^2\cdot\sum_{j=1}^{n}(x_{2j}-\hat{\mu}_2)^2}}$$

4 The Model of Golden Growth Law in Economic Process

We will use the following notations:

μ the basic economic level, basic level for short, $\mu\geq 0$.

σ the standard variance of basic level, $\sigma>0$. σ could measure the developower because the standard variance can measure the fluctuation range of basic level.

$v=\sigma/\mu$: the fluctuation rate of basic level, v could measure the development rate.

Z the real economic level, real level for short. Z is a non-negative stochastic variable.

4.1 The basic conclusions

From assumption 1, the real level $Z\in P(\mu, \sigma^2)$, and according to theorem 2, have

1) The expectation of Z, i.e. the average of Z, is

$$E(Z)=\mu+\sqrt{\frac{2}{\pi}}\sigma\frac{e^{-\frac{\mu^2}{2\sigma^2}}}{1+\sqrt{1-e^{-\frac{2}{\pi}(\frac{\mu}{\sigma})^2}}}$$

where, $R(Z)=\sqrt{\frac{2}{\pi}}\sigma\frac{e^{-\frac{\mu^2}{2\sigma^2}}}{1+\sqrt{1-e^{-\frac{2}{\pi}(\frac{\mu}{\sigma})^2}}}$ can valuates the average increment of real level based on basic level, and this increment is caused by the original developower σ.

2) The square of final developower is

$$D(Z)=\sigma^2+E(Z)[\mu-E(Z)]=\sigma^2-E(Z)R(Z)$$

$E(Z)>\mu$ means the economy growing in its average level is a general trend though the economy may slumps in some period of time. $D(Z)<\sigma^2$ means the economic growth needs to be pushed by the continuous release of developower.

In reality, the real level will not be generally apart from the basic level if no serious accident happened. So we could suppose that the real level $Z\in P_a(\mu, \sigma^2)$, specially, the real level follows SRPD, i.e., $Z\in F_a(\mu, \sigma^2)$.

The assumptions above are applicable to producing economy and nonproducting economy.

4.2 The average growth law in economic process

If the real level follows SRPD at a period of time, that is $Z(t)\in F_a(\mu(t), [\sigma(t)]^2)$, $t\in(t_0, T)$, t is omitted thereinafter. According to formula (4), the average of real level at the end of this period is

$$E(Z)=\mu+b\sigma \qquad (8)$$

where $b=\sqrt{\frac{2}{\pi}}\frac{e^{-\frac{1}{2}}}{1+\sqrt{1-e^{-\frac{2}{\pi}}}}$ =0.2869947990 is called the average growth coefficient. Formula (8) can also be expressed as $E(Z)=\mu(1+bv)$, $v=\frac{\sigma}{\mu}$ is the original development rate.

According to formula (5), the square of developower at the end of this period is

$$D(Z)=[1-(1+b)b]\sigma^2=b_0\sigma^2 \tag{9}$$

where $b_0=[1-(1+b)b]=0.6306391865$. $\bar{b}=\sqrt{b_0}=\sqrt{1-(1-b(1+b))}$ =0.7941279409 is the preserved coefficient of developower in average growth.

If the basic level is constant, the real economic growth rate in average is

$$g(v)=\frac{E(Z)}{\mu}-1=bv \tag{10}$$

4.3 The golden growth law in economic process

If the real level $Z\in P(\mu, \sigma^2)$, according to reference (Dai, F., Xu, W..X., Liu, H. and H. Xu, 2003), then the optimal value of real level at the end of this period is

$$Z^*=\frac{\mu+\sqrt{\mu^2+4\sigma^2}}{2}$$

Correspondingly, the square of developower is

$$D^*=D(Z)+[E(Z)-Z^*]^2$$

where, the "optimal" means the product of the Z and its appearing probability reaches maximum.

If the real level $Z\in P_a(\mu, \sigma^2)$, the optimal value of real level at the end of this period is

$$Z^*=a+\frac{(\mu-a)+\sqrt{(\mu-a)^2+4\sigma^2}}{2} \tag{11}$$

Further more, If $Z\in F_a(\mu, \sigma^2)$, i.e. $a=\mu-\sigma$, then

$$Z^*=\mu+c\sigma \tag{12}$$

where, $c=\frac{\sqrt{5}-1}{2}$ =0.618033989, is called the golden growth coefficient. Formula (12) can also be expressed as $Z^*=\mu(1+cv)$, $v=\frac{\sigma}{\mu}$ is the original development rate.

Correspondingly, the square of developower is

$$D^*=\{[1-(1+b)b]+[b-c]^2\}\sigma^2=c_1\sigma^2 \tag{13}$$

where, $c_1=[1-(1+b)b]+[b-c]^2=0.7402261316$. $\bar{c}^*=\sqrt{c_1}=\sqrt{1-(1-b(1+b))+(b-c)^2}$ =0.8603639532 is the preserved coefficient of developower in golden growth.

If the basic level is constant, the real economic growth rate in optimization is

$$g(v)=\frac{Z^*}{\mu}-1=cv \tag{14}$$

It is interest that, the growth coefficient $c=\frac{\sqrt{5}-1}{2}$ =0.618033989 in expression (12) is just the golden section in mathematics. So, the economic process is called to follow the golden growth law if its growth rate and development rate satisfies the equation (14).

In the meaning of expression (14), the development rate will be released to real growth rate according to its 61.8% under the golden growth law. Here, the "release" means that the remained development rate does not reduce the same quantity as 61.8% from the original development rate, but $\bar{c}^*$ =86.03639532% of the original development rate has been preserved according to formula (13). Similarly, the equation (10) means that the development rate will be released to real growth rate according to its 28.7% under the average growth law, and $\bar{b}$ =79.41279409% of the original development rate has been preserved according to formula (9). These interpret that in the process of developower pushing economic growth, the release of developower is not simple in linearity, but non-linearity and partly re-usable.

4.4 The increment contribution of developower to economic growth

Let the basic level be constant and the economic process be divided to n stage, $Z_i \in F_a(\mu_i, \sigma_i^2)$ for stage i, i=0, 1, ⋯. According to (8) and (9), denoting $\mu_0=\mu$, $\sigma_0=\sigma$, $\mu_{i+1}=E(Z_i)=\mu_i+b\sigma_i$, $\sigma_{i+1}=\sqrt{D(Z_i)}=\bar{b}\,\sigma_i$, thus

$$E(Z_n)=\mu_n+b\sigma_n=\mu+b\sum_{i=0}^{n}\sigma_i=\mu+b\sigma\sum_{i=0}^{n}\left(\bar{b}\right)^i$$

This means the developower is released in the nonlinear, i.e., total average increment contributions of original developower σ to economic growth on n times are added up as

$$R_n=b\sigma\sum_{i=0}^{n}\left(\bar{b}\right)^i=\frac{b\sigma[1-(\bar{b})^{n+1}]}{1-\bar{b}}$$

Let $n\to\infty$, then the total average increment contributions of developower σ is

$$R_\Sigma=\lim_{n\to\infty}R_n=\frac{b\sigma}{1-\bar{b}}=1.394044438\sigma.$$

Correspondingly, the coefficient of total average increment contributions of developower is

$$\text{I}=1.394044438 \qquad (15)$$

Similarly, by use of (12) and (13), the developower is released also in the nonlinear, i.e. the total optimal increment contributions of original developower σ to economic growth on n times are added up as

$$R_n^*=c\sigma\sum_{i=0}^{n}(\bar{c}^*)^i=\frac{c\sigma[1-(\bar{c}^*)^{n+1}]}{1-\bar{c}^*}$$

Let $n\to\infty$, the total optimal increment contributions of developower σ is

$$R_\Sigma^*=\lim_{n\to\infty}R_n^*=\frac{c\sigma}{1-\bar{c}^*}=4.426034704\sigma.$$

Correspondingly, the coefficient of total optimal increment contributions of developower is

$$\text{I}^*=4.426034704 \qquad (16)$$

From the coefficients of increment contributions in expression (15) and (16), and if the real levels follow SRPD, we know the total average increment contribution and the total optimal increment contribution of unit developower are separately 1.394044438 and 4.426034704.

5 The Models of Programming and Managing for Economic Growth

on Golden Law

Here we need to define two concepts: producing domain and functional realm. The economic producing domains, domain for short, are the domains to provide the products, commodities and serves to society or economic production, like industry, agriculture, business, tourism, etc. The economic functional realms, realm for short, are the different realms in economy which has its function, like production, policy, science and technology, education, management, sale, advertisement, etc. The economy can be divided into the different producing domains, called the producing type of economy, also can be divided into the different functional realms, called the functional type of economy.

5.1 Notation and basic relations

In addition to the notations in section 4, we also use the following notations for i=1, ···, n, j=1, ···, m:

μ_i The basic level for ith producing domain, $\mu_i \geq 0$.

σ_i The standard variance of basic level for ith producing domain, i.e. original development power of ith producing domain, $\sigma_i > 0$.

λ_j The basic level for jth functional realm, $\lambda_j \geq 0$.

κ_j The standard variance of basic level for jth functional realm, i.e. the original development power of jth functional realm, $\kappa_j > 0$.

Z The real economic level, Z is a non-negative stochastic variable.

X_i The real level of ith producing domain, X_i is a non-negative stochastic variable.

Y_j The real level of jth functional realm, Y_j is a non-negative stochastic variable.

Suppose the real economic level is equal to the sum of all the real levels of producing domains, and the sum of all the real levels of functional realms, i.e.

$$Z=\sum_{i=1}^{n} X_i \text{ , } Z=\sum_{j=1}^{m} Y_j$$

thus, when all the producing domains are independent one another, and all the functional realms are independent one another, have

$$E(Z)=\sum_{i=1}^{n} E(X_i)=\sum_{j=1}^{m} E(Y_j) \text{ , } D(Z)=\sum_{i=1}^{n} D(X_i)=\sum_{j=1}^{m} D(Y_j)$$

If giving no special declaration thereinafter, all the producing domains are independent one another, and all the functional realms are independent one another. We will give a discussion on domains and realms in a certain period of time, i.e., the basic levels are their original values at the beginning of the period, and real levels are their final values at the end of the period.

Suppose the basic levels are constant in the discussed period of time thereinafter if we do not give a special declaration.

5.2 The analytic structure of economic growth in golden law

We divide each of producing domains into the corresponding elementary components according to the different functional realm and each of functional realms into the corresponding elementary components according to the different producing domain. Suppose all the producing domains and the functional realms are independent one another, the real economic level $Z \in F_a(\mu, \sigma^2)$, the real level of ith producing domain $X_i \in F_a(\mu_i,$

σ_i^2), real level of jth functional realm $Y_j \in F_a(\lambda_j, \kappa_j^2)$, and real level of every elementary components follow SRPD, i.e., $X_{ij} \in F_a(\mu_{ij}, \sigma_{ij}^2)$, where, $X_i = \sum_{j=1}^{m} X_{ij}$, $Y_j = \sum_{i=1}^{n} X_{ij}$, $Z = \sum_{i=1}^{n}\sum_{j=1}^{m} X_{ij}$ $i=1, \cdots, n, j=1, \cdots, m$.

Combining the growth model in golden law in (12) with the formula for computing developower on golden law in (13), we can design the structural table for economic level, see table 1.

Table 1 The analytic structure table for economic level based on golden growth law

Realms / Domains	Y_1	…	Y_m	The sum of real optimal levels of domains based on golden growth law	The sum of domains developower based on golden growth law
X_1	X_{11}	…	X_{1m}	$X_1^* = \sum_{j=1}^{m} \mu_{1j}(1+cv_{1j})$	$\sqrt{D^*(X_1)} = \sqrt{c_1 \sum_{j=1}^{m} \sigma_{1j}^2}$
⋮	⋮	⋮	⋮	⋮	⋮
X_n	X_{n1}	…	X_{nm}	$X_n^* = \sum_{j=1}^{m} \mu_{nj}(1+cv_{nj})$	$\sqrt{D^*(X_n)} = \sqrt{c_1 \sum_{j=1}^{m} \sigma_{nj}^2}$
The sum of realms contribution to real optimal levels based on golden growth law	$Y_j^* = \sum_{i=1}^{n} \mu_{ij}(1+cv_{ij})$ $j=1, \cdots, m$			$Z^* = \sum_{i=1}^{n}\sum_{j=1}^{m} \mu_{ij}(1+cv_{ij})$	
The sum of realms developower based on golden growth law	$\sqrt{D^*(Y_j)} = \sqrt{c_1 \sum_{i=1}^{n} \sigma_{ij}^2}$ $j=1, \cdots, m$				$\sqrt{D^*} = \sqrt{c_1 \sum_{i=1}^{n}\sum_{j=1}^{m} \sigma_{ij}^2}$
Explanations	Real levels of elementary components $X_{ij} \in P(\mu_{ij}, \sigma_{ij}^2)$, $X_{ij}^* = \mu_{ij}(1+cv_{ij})$, $i=1, \cdots, n, j=1, \cdots, m$. $D^* = \sqrt{\sum_{i=1}^{n} D^*(X_i)} = \sqrt{\sum_{J=1}^{m} D^*(Y_j)}$. c, c_1 are given separately in formula (13) and (14), c=0.618033989, c_1=0.7402261316. $v_{ij} = \sigma_{ij}/\mu_{ij}$ is the development rate. The real levels can be valuated by productive values.				

The real levels can be valuated by real productive values in table 1, like this, we can

- computing the real economic productive value and developower, the real productive value and developower of producing domains, and functional realms contribution to the real economic productive value and their developowers on the golden law.
- comparing the realistic productive values in national economy with those computed in table 1, and judging that the realistic productive values are higher or lower.
- comparing the realistic economic growth rates with those computed by the formula in (14), and judging that the real growth rates are higher or lower.
- analyzing the realistic economic structure is of equilibrium or not, and giving a necessary regulation to realistic economic structure if needed.

By the discussion above, we can give a similar analysis for average economic growth.

5.3 The model of controlling economic growth on the golden law

Let the real level $Z \in F_a(\mu, \sigma^2)$. From the expression (14), the golden growth rate at the end of period discussed is

$$g(v) = \frac{Z^*}{\mu} - 1 = cv$$

The growth rate depends on the development rate v in above formula. If want the realistic growth rate to reach an expected value *e*, we need giving a proper development rate. Let $g(v_e)=e$, then the proper development rate is

$$v_e = \frac{e}{c} \tag{17}$$

The corresponding developower is $\sigma_e = \mu \cdot v_e$. At the end of this period, the real value of development rate is

$$v(Z^*) = \frac{\sqrt{D^*}}{\mu}$$, D^* is determined by formula (13)

Thus, the development rate needs to have an increment on its real value, i.e., the increment should be

$$\hat{v}^* = \sqrt{v_e^2 - v^2(Z^*)}$$

or the increment of developower should be

$$\hat{\sigma}^* = \mu\sqrt{v_e^2 - v^2(Z^*)} \tag{18}$$

Here we suppose the nonphysical asset investing for increasing developower has been done at the moment on the beginning of this stage, for the asset developower to be in full play.

If the basic level changes in the discussed period of time, the real value of development rate at the end of this period need to be modified as $v(Z^*) = \frac{\sqrt{D^*}}{Z^*}$, and the increment of developower should be $\hat{\sigma}^* = Z^*\sqrt{v_e^2 - v^2(Z^*)}$.

Based on the formula (10) and the method above, we can get a similar discussion on average growth rate.

5.4 The economic programming model based on developower

If knowing the value of $\hat{\sigma}^*$ determined by expression (18), we have separately the programming methods for producing domains and functional realms in economy as follow.

5.4.1 The programming methods for producing domains. The real economic level $Z \in F_a(\mu, \sigma^2)$, the economy includes *n* producing domains, their real level $X_i \in F_a(\mu_i, \sigma_i^2)$, and each domain is divided into the elementary components corresponding to different functional realms, their real level $X_{ij} \in F_a(\mu_{ij}, \sigma_{ij}^2)$.

(1) Programming economic developower according to domains. Let $\hat{\sigma}^* = \sqrt{\sum_{i=1}^{n} \beta_i \left(\hat{\sigma}_i^*\right)^2}$ and $\beta_i = \frac{\sigma_i}{\sigma}$, the increments of developower for each domain [5], $\hat{\sigma}_i^*$, is given as following

$$\hat{\sigma}_i^* = \frac{\mu_i}{\beta_i} \frac{\hat{\sigma}^*}{\sqrt{\sum_{i=1}^{n} \left(\frac{\mu_i^2}{\beta_i}\right)}} \qquad (i=1, \cdots, n) \tag{19}$$

The expression (19) means the domain potential is larger if its basic level is higher and original developower is smaller, this kind of domain should be given more of investment.

(2) Programming domain developower according to elementary components. If $\hat{\sigma}_i^*$ and $X_{ij} \in P(\mu_{ij}, \sigma_{ij}^2)$ are known, then let $\hat{\sigma}_i^* = \sqrt{\sum_{j=1}^{m} \gamma_{ij} \sigma_{ij}^2}$ be the increment of domain *i*, where $\sqrt{\gamma_{ij}}\sigma_{ij}$ is the increment of elementary components. The increment coefficients [6], γ_{ij}, is given as

5 The method of determining the increments of each domain is given in appendix.

6 The method of determining the increment coefficients of each elementary components is given in appendix.

$$\gamma_{ij}=\frac{\mu_{ij}\left(\hat{\sigma}_i^*\right)^2}{\sigma_{ij}\sum_{j=1}^{m}\sigma_{ij}\mu_{ij}}\quad (\ i=1,\cdots,n;\ j=1,\cdots,m) \tag{20}$$

$\gamma_\sigma=\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{m}\gamma_{ij}\sigma_{ij}^2}$ is called the integrated index of elementary components on domains.

5.4.2 The programming methods for functional realms. The real economic level $Z\in F_a(\mu,\ \sigma^2)$, the economy includes m functional realms, their real level $Y_j\in F_a(\lambda_j,\ \kappa_j^2)$, and each realm is divided into the elementary components corresponding to different producing domains, their real level $X_{ij}\in F_a(\mu_{ij},\sigma_{ij}^2)$.

(1) Programming economic developower according to realms. Let $\hat{\sigma}^*=\sqrt{\sum_{j=1}^{m}\phi_j\left(\hat{\kappa}_j^*\right)^2}$ and $\phi_j=\frac{\kappa_j}{\sigma}$, the increments of developower for each realm [7], $\hat{\kappa}_j^*$, is given as following

$$\hat{\kappa}_j^*=\frac{\lambda_j}{\phi_j}\frac{\hat{\sigma}^*}{\sqrt{\sum_{j=1}^{m}\left(\frac{\lambda_j^2}{\phi_j}\right)}},\ j=1,\cdots,m.$$

The expression above means the realm potential is larger if its basic level is higher and original developower is smaller, this kind of realm should be given more of investment.

(2) Programming realm developower according to elementary components. If $\hat{\kappa}_j$ and $X_{ij}\in P(\mu_{ij},\sigma_{ij}^2)$ are known, then let $\hat{\kappa}_j^*=\sqrt{\sum_{i=1}^{n}\varphi_{ij}\sigma_{ij}^2}$ be the increment of realm j, where $\sqrt{\varphi_{ij}}\sigma_{ij}$ is the increment of elementary components. The increment coefficients [8], φ_{ij}, is given as

$$\varphi_{ij}=\frac{\mu_{ij}\left(\hat{\kappa}_j^*\right)^2}{\sigma_{ij}\sum_{i=1}^{m}\sigma_{ij}\mu_{ij}},\ i=1,\cdots,n;\ j=1,\cdots,m.$$

$\varphi_\sigma=\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{m}\varphi_{ij}\sigma_{ij}^2}$ is called the integrated index of elementary components on realms.

We will see that, from the latter empirical researches, the integrated index of elementary components can be applied to evaluate the efficiency of investment for developower, so they could be taken as the basis to choose the final plan.

The methods above are also the same with the economic programming based on the average growth law.

5.5 The analytic structure of economic growth under the meaning of correlation

If all the domains are correlated and all the realms are correlated, and let the real levels of domains follow the RPD, i.e., $\boldsymbol{X}\in P_a(\boldsymbol{\mu},\ \boldsymbol{\sigma}^T\boldsymbol{\sigma},\ \boldsymbol{R}\ )$, where, $\boldsymbol{X}=(X_1,\cdots,X_n)^T$, $\boldsymbol{\mu}=(\mu_1,\cdots,\mu_n)^T\geq\boldsymbol{0}$, $\boldsymbol{\sigma}=(\sigma_1,\cdots,\sigma_n)^T>\boldsymbol{0}$, $\boldsymbol{a}=(a_1,\cdots,a_n)^T$, $\boldsymbol{R}=(r_{ij})_{n\times n}$, $|r_{ij}|\leq 1$, $r_{ii}=1$, $i\ ,j=1,\cdots,n$. Taking the two domains for example, i.e., $(X_1, X_2)\in P_a(\mu_1,\mu_2,\sigma_1^2,\sigma_2^2,r)$, and let $\boldsymbol{a}=\boldsymbol{\mu}-\boldsymbol{\sigma}$. From the section 3.2, we have the results for average growth law as following

7 The method of determining the increments of each realm is given in appendix..

8 The method of determining the increment coefficients of each elementary components is given in appendix.

$E_r(X_i)=\mu_i+\sigma_i[r+\sqrt{1-r^2}\ b(r)]$ (21)

$D_r(X_i)=[1-(1+b(r))b(r)]\,\sigma_i^2(1-r^2)$

where, $b(r)=\sqrt{\frac{2}{\pi}}\frac{e^{-\frac{1}{2}\left(\frac{1+r}{1-r}\right)}}{1+\sqrt{1-e^{-\frac{2}{\pi}\left(\frac{1+r}{1-r}\right)}}}$, i=1, 2.

For the golden growth law, replacing μ and σ in expression (12) separately by $\mu_i+r\sigma_i$ and $\sqrt{1-r^2}\ \sigma_i$, and let $a_i=\mu_i+r\sigma_i-\sqrt{1-r^2}\ \sigma_i$, thus

$$X_i^*=\mu_i+\sigma_i[r+c\sqrt{1-r^2}] \quad (22)$$

$$D_i^*=\sigma_i^2(1-r^2)\{[1-(1+b(r))b(r)]+[b(r)-c]^2\}$$

where, $c=\frac{\sqrt{5}-1}{2}$, $b(r)$ is the same in (21), i=1, 2.

We can discuss the real levels and developowers on correlated realms in the same way as the those of correlated domains. And we can discuss the economic programming under the meaning of correlation According to section 5.2 - section 5.4.

For the average growth, denoting $g(r)=r+\sqrt{1-r^2}\ b(r)$ in expression (21), the $g(r)$ is monotone increasing and $g(0)=b$ (b is determined by expression (9)). When the different domains or realms develop in the same direction, i.e., $r>0$, $E_r(X_i)$ will be lager as σ_i become larger, this means the real levels of those domains will increase as developower increases. In reverse, When the different domains or realms develop in the reverse direction, i.e., $r<0$, $E_r(X_i)$ will be smaller as σ_i become larger, this means the real levels of those domains will decrease as developower increases. So, the effective developower investment to one domain or realm will be able to push the economic level to rise when the domains or realms in economy are in the same developing direction.

For the golden growth and denoting $g(r)=r+c\sqrt{1-r^2}$ in expression (22), $g(r)$ will reaches its maximum when $r=\frac{1}{\sqrt{1+c^2}}$ =0.8506508081. So $g(r)$ is monotone increasing when $r\in[-1, 0.8506508081]$. At this time, if the domains or realms in economy are in the same developing direction, i.e., $r>0$, then the larger the developower is, the higher the real level is; if the domains or realms in economy are in the reverse developing direction, i.e., $r<0$, then the larger the developower is, the lower the real level is. When $r>0.8506508081$, $g(r)$ is monotone decreasing. At this time, if the domains or realms in economy are in the same developing direction, then the larger the developower is, the lower the real level is. This means the domains or realms should be amalgamated if their developing direction is same highly.

6 The Empirical Researches and Analysis

Here we take US GDP (chained) price index and GDP in billions of dollars (Fiscal Year 2000 = 1.000, http://www.whitehouse.gov) in the period of 1940-2004 as the scales to evaluate the economic level and the empirical samples.

6.1 The notations and descriptions

We have the notations and expressions as follow:

$\mu_1(t)$ GDP (Chained) price index of the year t, t=1940, ⋯, 2004.

$\mu_2(t)$ GDP in billions of dollars of the year t, t=1940, ⋯, 2004.

$\sigma_1(t)$ The fluctuation range of GDP (Chained) price index, $\sigma_1(t)=|\mu_1(t)- \mu_1(t-1)|$, t=1941, ⋯ , 2004.

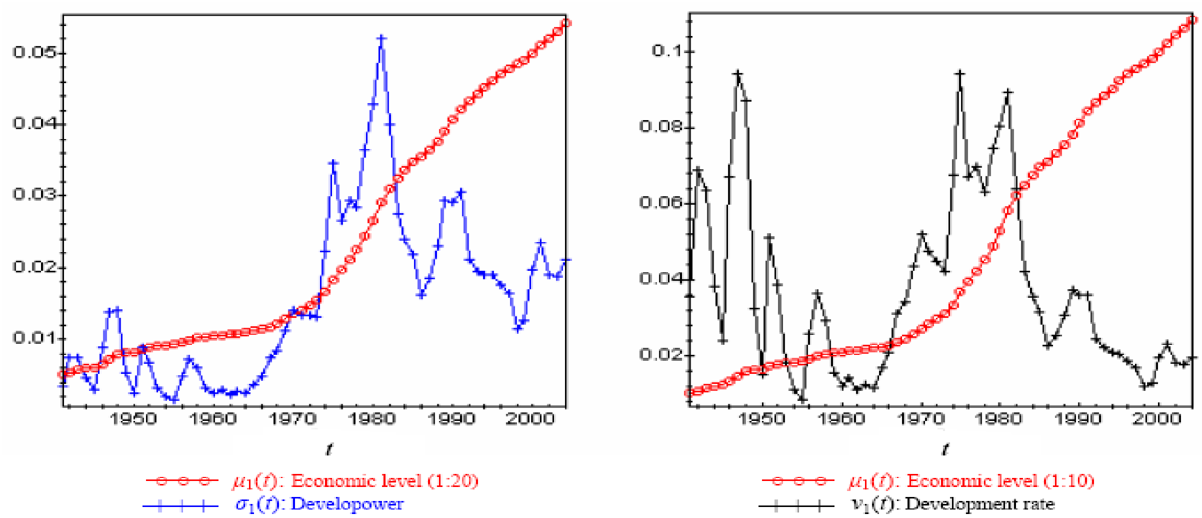
$v_1(t)$ The fluctuation rate of GDP (Chained) price index, $v_1(t)=|\mu_1(t)- \mu_1(t-1)|/\mu(t)$, t=1941, ⋯ , 2004.

$\sigma_2(t)$ The fluctuation range of GDP in billions of dollars, $\sigma_2(t)=|\mu_2(t)- \mu_2(t-1)|$, t=1941, ⋯ , 2004.

$v_2(t)$ The fluctuation rate of GDP in billions of dollars, $v_2(t)=|\mu_2(t)- \mu_2(t-1)|/ \mu_2(t)$, t=1941, ⋯ , 2004.

Here, $\sigma_1(t)$ and $\sigma_2(t)$ are separately the developowers scaled on GDP (Chained) price index and GDP in billions of dollars at year t, and $v_1(t)$ and $v_2(t)$ are separately the development rates scaled on GDP (Chained) price index and GDP in billions of dollars at year t.

The interval of time unit for sampling data GDP is a year, the stability of data is higher, and the difference between GDP of one year and that of last year can nicely describe the economic fluctuation, so we adopt the formulas of $\sigma_1(t)$ and $\sigma_2(t)$ mentioned above to measure the developower, and $v_1(t)$ and $v_2(t)$ to measure the development rate. The curves of developower, development rate and economic level in US economy on GDP (Chained) price index are shown in figure 1. In figure 1(a), there is the curve of developower, and the proportion of the index drawn to the real GDP (Chained) price index $\mu_1(t)$ is 1:20. In figure 1(b), there is the curve of development rate, and the proportion of the index drawn to the real GDP (Chained) price index $\mu_1(t)$ is 1:10.



(a) (b)

Figure 1 The curves of US economic levels $\mu_1(t)$, the developowers $\sigma_1(t)$ and developower rates $v_1(t)$ in the time period from 1941 to 2004. $\mu_1(t)$ is valued by GDP (chained) price index (Fiscal Year 2000=1.000). In (a), the proportion of the drawn indexes of $\mu_1(t)$ to the real indexes is 1:20, and the proportion of the drawn indexes of $\mu_1(t)$ to the real indexes is 1:10 in (b). As we see, both the developower and the developower rate fluctuate always though the economic level $\mu_1(t)$ has been growing.

6.2 The fitness analysis for growth in golden law and average law

Here, we make a fitting on US GDP (Chained) price index according to the expression (12) and (8). And the fitting models are separately as follow

The model on golden law: $X_1^*(t) = \mu_1(t-1) + c\sigma_1(t-1)$

and

The model on average law: $E_1(t) = \mu_1(t-1) + b\sigma_1(t-1)$

where, t=1942, ⋯ , 2004.

The curves of fitting are drawn in figure 2, the fitting of model on golden law is in figure 2(a), and the fitting of model on average law is in figure 2(b).

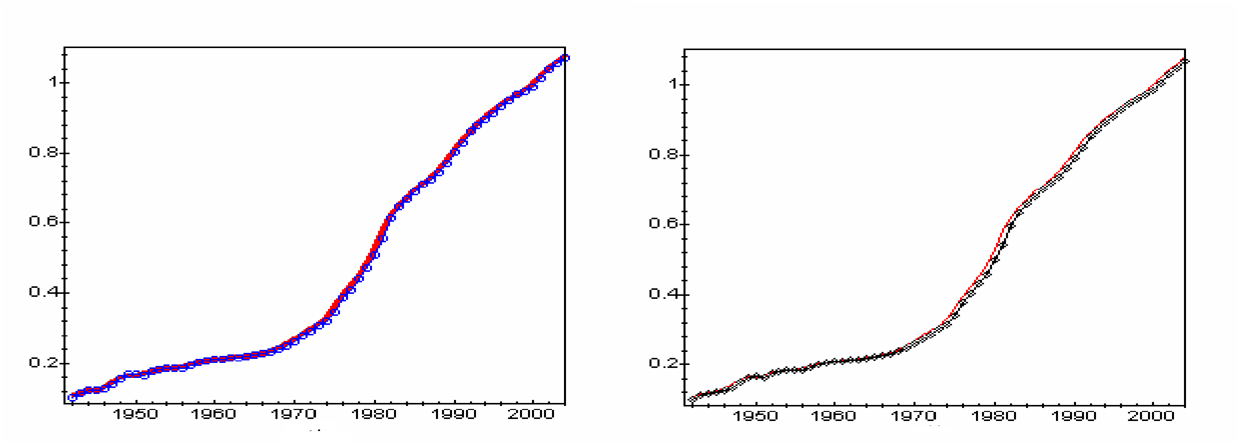


Figure 2 The curves of US economic levels $\mu_1(t)$, the developowers $\sigma_1(t)$ and developower rate $v_1(t)$ in the time period from 1941 to 2004. $\mu_1(t)$ is valued by GDP (chained) price index (Fiscal Year 2000=1.000). There is the comparison between the curves of real US economy and fitting on golden growth law in (a), and the comparison between the curves of real US economy and fitting on average growth law is in (b).

If we use separately the models of average error and relative error to analyze the effect of fitting as follow

The models of average error: $e_1^* = \frac{\sqrt{\sum_{t=1942}^{2004}[X_1^*(t)-\mu_1(t)]^2}}{2004-1942}$ and $e_1 = \frac{\sqrt{\sum_{t=1942}^{2004}[E_1(t)-\mu_1(t)]^2}}{2004-1942}$

and

The models of relative error: $e_2^* = \frac{\sqrt{\sum_{t=1942}^{2004}[X_1^*(t)-\mu_1(t)]^2}}{\sum_{i=1942}^{2004}\mu_1(t)}$ and $e_2 = \frac{\sqrt{\sum_{t=1942}^{2004}[E_1(t)-\mu_1(t)]^2}}{\sum_{i=1942}^{2004}\mu_1(t)}$

Then the results of error on US GDP (Chained) price index are

e_1^*=0.00847006861, e_2^*=0.002234007407, e_1=0.01420798692, e_2=0.003747401525

Because $e_1^* < e_1$ and $e_2^* < e_2$, the golden growth law matches US economic process rather than the average growth law. Again, the error e_2^*=0.002234007407 is so small that we can think the golden growth law to be effective for real economic process.

As for US GDP in billions of dollars, we can replace separately $X_1^*(t)$, $\mu_1(t)$ and $E_1(t)$ in the error models above by $X_2^*(t)$, $\mu_2(t)$ and $E_2(t)$. The results of error are

e_1^*=120.4741818, e_2^*=0.004904053749, e_1=189.0383114, e_2=0.007695043250.

We also have $e_1^* < e_1$ and $e_2^* < e_2$. These interpret further that US economic process follows the golden growth law rather than the average growth law. In the period of 1950-2000, US economy develops in a higher efficiency, and its economic growth is durative and in higher speed in macro meaning. So we think that the golden growth law should be a basic law in economic process which is of high-efficiency. If like this, the golden growth law may be taken as a criterion by which we can judge the economic growth is high-effective or not.

6.3 Programming and managing the structure for economic growth

Suppose the economy includes three producing domains: primary industry (agriculture, etc.), secondary industry (manufacture, etc), tertiary industry (service, etc), and five functional realms: production, science& technology, education, management and policy. And $Z\in F_a(\mu, \sigma^2)$, $X_i\in F_a(\mu_i,\sigma_i^2)$, $Y_j\in F_a(\lambda_j, \kappa_j^2)$ and $X_{ij}\in F_a(\mu_{ij},\sigma_{ij}^2)$ are all known. The basic data and computed data of them are listed in table 2.

Table 2 The data of economic structure on golden growth law

Producing domains in economy	Functional realms in economy					Total productive value of domain on golden growth	Total developower of domain on golden growth
	Production Y_1	S&T Y_2	Education Y_3	Management Y_4	Policy Y_5		
Primary industry $X_1\in F_a(12, 6)$	$X_{11}\in F_a(5, 3)$	$X_{12}\in F_a(3, 1.2)$	$X_{13}\in F_a(2, 0.8)$	$X_{14}\in F_a(1, 0.4)$	$X_{15}\in F_a(1, 0.6)$	$X_1^*=$ 15.16988107	$\sqrt{D^*(X_1)}=$ 2.107452678
Secondary industry $X_2\in F_a(18, 10)$	$X_{21}\in F_a(7, 5)$	$X_{22}\in F_a(4, 2)$	$X_{23}\in F_a(3, 1.4)$	$X_{24}\in F_a(2, 0.9)$	$X_{25}\in F_a(2, 0.7)$	$X_2^*=$ 22.09066859	$\sqrt{D^*(X_2)}=$ 2.720709709
Tertiary industry $X_3\in F_a(24, 14)$	$X_{31}\in F_a(8, 5)$	$X_{32}\in F_a(7, 4.5)$	$X_{33}\in F_a(4, 2.2)$	$X_{34}\in F_a(3, 1.2)$	$X_{35}\in F_a(2, 1.1)$	$X_3^*=$ 28.93492846	$\sqrt{D^*(X_3)}=$ 3.219187141
Total productive value of realm on golden growth	$Y_1^*=23.83439829$; $Y_2^*=16.86210244$; $Y_3^*=11.20074663$; $Y_4^*=7.654219852$; $Y_5^*=6.644010919$					$Z^*=$ 66.19547812	
Total developower of realm on golden growth	$\sqrt{D^*(Y_1)}=3.102086350$; $\sqrt{D^*(Y_2)}=2.387413080$; $\sqrt{D^*(Y_3)}=1.804714653$; $\sqrt{D^*(Y_4)}=1.360354854$; $\sqrt{D^*(Y_5)}=1.332870105$						$\sqrt{D^*}=$ 4.712407448
Explanation	The real economic level $Z\in F_a(54, 30)$, $\mu=54$, $\sigma^2=30$						

According to table 2 and the conclusions in section 5.3, we know the real value of development rate at the end of this period is

$$v(Z^*)=\frac{\sqrt{D^*}}{\mu}=0.08726680459$$

i.e., the real growth rate on the golden law is $cv(Z^*)=0.05393385135$.

If we hope the final growth rate is e= 0.07, then the development rate in expectancy is $v_e=\frac{e}{c}=0.1132623792$ ($c=\frac{\sqrt{5}-1}{2}$). So, the increment of development rate is $\hat{v}^*=\sqrt{v_e^2-v^2(Z^*)}=0.07220021707$ in order to reach v_e=0.1132623792, or the increment of developower is $\hat{\sigma}^*=\sqrt{\mu^2 v_e^2-D^*}=3.898811722$.

If the data of industrial structure and three domains are known as in table 2, the developing planning for economic structure has been computed in table 3 according to the methods in section 5.4.1.

Table 3 The data of programming on domains

The increments of developower of each domains	The increment coefficients of developower for elementary components				
$\hat{\sigma}_1^*=$ 2.517870230	$\gamma_{11}=$ 1.208588886	$\gamma_{12}=$ 1.146568091	$\gamma_{13}=$ 0.9361689254	$\gamma_{14}=$ 0.6619713956	$\gamma_{15}=$ 0.5404973812
$\hat{\sigma}_2^*=$ 2.925500842	$\gamma_{21}=$ 0.9424140590	$\gamma_{22}=$ 0.8514785504	$\gamma_{23}=$ 0.7632836423	$\gamma_{24}=$ 0.6346546395	$\gamma_{25}=$ 0.7196307193
$\hat{\sigma}_3^*=$ 3.296665978	$\gamma_{31}=$ 0.8825974083	$\gamma_{32}=$ 0.8140469364	$\gamma_{33}=$ 0.6652828287	$\gamma_{34}=$ 0.6755979061	$\gamma_{35}=$ 0.4704259994
Explanation	The final growth rate : e= 0.07 the real growth rate : $cv(Z^*)=0.05393385135$ Difference for growth rates : $e\text{-}cv(Z^*)$=0.016066149		Development rate in expectancy: v_e=0.1132623792 Real development rate: $v(Z^*)$=0.087266805 Developower increment: $\hat{\sigma}^*$ =3.898811722		
	Developower increment of elementary components: $\sqrt{\gamma_{ij}}\sigma_{ij}$, $\gamma_\sigma=\sqrt{\sum_{i=1}^{3}\sum_{j=1}^{5}\gamma_{ij}\sigma_{ij}^2}$ =5.076044940				

If the data of functional structure and five realms are known as in table 2, the developing planning for economic structure has been computed in table 4 according to the methods in section 5.4.2.

Table 4 The data of programming on realms

The increments of developower of each realms	$\hat{\kappa}_1^*=$ 3.144428434	$\hat{\kappa}_2^*=$ 2.860000232	$\hat{\kappa}_3^*=$ 2.432201582	$\hat{\kappa}_4^*=$ 2.151120016	$\hat{\kappa}_5^*=$ 1.829564727
The increment coefficients of developower for elementary components	$\varphi_{11}=$ 0.6763433864	$\varphi_{12}=$ 0.9415078202	$\varphi_{13}=$ 1.173556278	$\varphi_{14}=$ 1.257949279	$\varphi_{15}=$ 0.9506813665
	$\varphi_{21}=$ 0.7334506679	$\varphi_{22}=$ 0.9723850958	$\varphi_{23}=$ 1.330687741	$\varphi_{24}=$ 1.677265704	$\varphi_{25}=$ 1.760319835
	$\varphi_{31}=$ 0.8382293348	$\varphi_{32}=$ 1.134449278	$\varphi_{33}=$ 1.415362126	$\varphi_{34}=$ 2.178832062	$\varphi_{35}=$ 1.404249442
Explanation	The final growth rate : e= 0.07 the real growth rate : $cv(Z^*)=0.05393385135$ Difference for growth rates : $e\text{-}cv(Z^*)$=0.016066149		Development rate in expectancy: v_e=0.1132623792 Real development rate: $v(Z^*)$=0.087266805 Developower Increment: $\hat{\sigma}^*$ =3.898811722		
	Developower increment of elementary components: $\sqrt{\varphi_{ij}}\sigma_{ij}$, $\varphi_\sigma=\sqrt{\sum_{i=1}^{3}\sum_{j=1}^{5}\varphi_{ij}\sigma_{ij}^2}$ =5.65307531				

Comparing the results in table 3 and table 4, we know, the integrated indexes of elementary components on domains and realms are separately

$$\gamma_\sigma=\sqrt{\sum_{i=1}^{3}\sum_{j=1}^{5}\gamma_{ij}\sigma_{ij}^2}=5.076044940 \text{ and } \varphi_\sigma=\sqrt{\sum_{i=1}^{3}\sum_{j=1}^{5}\varphi_{ij}\sigma_{ij}^2}=5.653075310$$

The increment of developower caused by way of domains (in section 5.4.1) is equal to that of realms (in section 5.4.2), but $\gamma_\sigma<\varphi_\sigma$, this means more investment is need to increase the same developower by the way of realms than that of domains, so the way in domains should be better because it is more sparing in the use of investment. By the discussion above on comparing the way in domains with that in realms in their integrated index of elementary components, we could know which one is better at any time and should be applied to carry.

Now we have introduced the application process about the method of programming and managing for economic growth on golden law in empirical way. It is similar to the method on average law. The difference between the methods on golden law and on average law is that the proper developower rate in expression (17) and the increment of developower in expression (18) are all different.

6.4 Analysis for developower to increase the productive value

In table 2, the total of basic levels on production realm is

$$\mu_1 = \sum_{i=1}^{3} \mu_{i1} = 20$$

at the end of this period, the total productive value (i.e., real level) on golden growth law is

$$Y_1^* = \sum_{i=1}^{3} (\mu_{i1} + c\sigma_{i1}) = 23.83439829$$

and the corresponding increment of total productive value is

$$Y_1^* - \mu_1 = 3.83439829$$

In another hand, the total productive value on the integrated developower (with all of realms) is

$$Z_1^* = \sum_{i=1}^{3} \left(\mu_{i1} + c\sqrt{\sum_{j=1}^{5} \sigma_{ij}^2} \right) = 25.78073443,$$

and the corresponding increment of total productive value is

$$Z_1^* - \mu_1 = 5.78073443$$

We see, the increment of productive value brought by the developower from the functional realms of nonproduction is 5.78073443-3.83439829=1.94633614, and the percentage of total increment is 33.6693575%. So the functional realms of nonproduction can make greater contributions to increment in productive value, and this contribution is in sustained existence even it will be smaller and smaller.

In general, if the real level of production realm is $Y_1 \in P(\lambda_1, \kappa_1^2)$, the total real level of all the nonproduction realm is $Y \in P(\lambda, \kappa^2)$, and the real level of industrial economy $Z_1 \in P(\lambda_1, \sigma^2)$, where $\sigma^2 = \kappa_1^2 + \kappa^2$, then we have the following inequation according to formula (2)

$$E(Z_1) > E(Y_1)$$

Namely the real average output of industrial economy is larger than that of production, this means the factors of nonproduction is redound to raise the real level of economy and its contribution for average value can be valuated as

$$\Delta = E(Z_1) - E(Y_1)$$

7 Conclusion

This paper has mainly done the works as follow :

● The developower is perfected by explaining that it is the measurement of uncertainty in economic process and giving the concept of developower rate. And the key problem of economic development is the interrelation and interaction between productivity and developower.

● By means of the partial distribution (the univariate and multivariate partial distribution), the models are established on the relation between developower and basic economic level and real economic level, including the models in average relation and in optimal relation, where the “optimal” means the product of the economic level and its occurred probability is maximum. The basic character of these models is that the real economic level is formed of basic economic level and economic increment, and the economic increment is caused from developower. In the process of developower pushing economy to grow, developower is released in the nonlinear not the linear (see also section 4.4).

● It is important that, we get a kind of the economic growth mode which is called the golden growth law here. We analyze the contribution of developower to economic increment, and compute the coefficient of total quantity of contribution in average law and in the golden law. And they are separately as I=1.394044438 and I^*=4.426034704.

● Based on the partial distribution, we build the economic structure model, a kind of programming and controlling model for economic system in golden law (see in table 1). According to the economy in division of producing domains and functional realms, the computing methods for the economic structure model are given.

● The empirical researches are put up on the US GDP data (chained price index and in billions of dollars) from 1940 to 2004. It is illustrated that the golden growth law is better than the average growth law in the fitting with real US economic process. So the golden growth law should be existed in the process of economic growth, and could be taken as a criterion by which we can judge the economic growth is high-effective or not.

Besides the coefficients of total quantity of contribution I and I^*, we also get the some of important constants as follow

1) The average growth coefficient b=0.2869947990 in formula (8).

2) The preserved coefficient of developower in average growth $\bar{b}$ =0.7941279409 in formula (9).

3) The golden growth coefficient c=0.618033989 in formula (12).

4) The preserved coefficient of developower in golden growth $\bar{c}^*$ =0.8603639532 in formula (13).

These coefficients reflect some of important numeric relations in economic growth.

All the assumptions in this paper are according with practice or not, all the basic conclusions are right or not, and many of the details in models are perfect or not, all of these need be practiced and tested. The authors hope the works in this paper to be useful to policymaking for economic planning, also hope the economists to put forward their opinions.

Appendix

If $\left(\hat{\sigma}^*\right)^2=\sum_{i=1}^{n}\beta_i\left(\hat{\sigma}_i^*\right)^2$, then let $\beta_i=\frac{\sigma_i}{\sigma}$ (i.e., weights of branch of developower should correspond to its current proportion in total developower). When $\hat{\sigma}^*$ is given, the normal vector of hypersphere $\left(\hat{\sigma}^*\right)^2=\sum_{i=1}^{n}\beta_i\left(\hat{\sigma}_i^*\right)^2$ is $\left(2\beta_i\hat{\sigma}_i^*,\cdots,2\beta_n\hat{\sigma}_n^*\right)^{\mathrm{T}}$, let the normal vector be in the same direction as the vector $\left(\mu_1,\cdots,\mu_n\right)^{\mathrm{T}}$ (this means the increment structure of developower is consistent to the distribution structure of basic economic level, like this, the investment for developower reaches its maximum efficiency in increasing economic level), then

$$\frac{2\beta_i\hat{\sigma}_i^*}{\mu_i}=t \quad (i=1,\cdots,n)$$

i.e. $\hat{\sigma}_i^*=\frac{\mu_i}{2\beta_i}t$, and put it into $\left(\hat{\sigma}^*\right)^2=\sum_{i=1}^{n}\beta_i\left(\hat{\sigma}_i^*\right)^2$, we get $t=\frac{2\hat{\sigma}^*}{\sqrt{\sum_{i=1}^{n}\frac{\mu_i^2}{\beta_i}}}$, thus

$$\hat{\sigma}_i^* = \frac{\mu_i}{\beta_i} \frac{\hat{\sigma}^*}{\sqrt{\sum_{i=1}^{n}\left(\frac{\mu_i^2}{\beta_i}\right)}} \quad , i=1, \cdots, n.$$

Further, When $\hat{\sigma}_i^*$ and $X_{ij} \in P(\mu_{ij}, \sigma_{ij}^2)$ are given, the normal vector of hypersphere $\left(\hat{\sigma}_i^*\right)^2 = \sum_{j=1}^{m} \gamma_{ij}\sigma_{ij}^2$ is $\left(2\gamma_{i1}\sigma_{i1}, \cdots, 2\gamma_{im}\sigma_{im}\right)^{\mathrm{T}}$, let the normal vector be in the same direction as the vector $\left(\mu_{i1}, \cdots, \mu_{im}\right)^{\mathrm{T}}$, then

$$\frac{2\gamma_{ij}\sigma_{ij}}{\mu_{ij}} = t \quad (i=1, \cdots, n; j=1, \cdots, m)$$

i.e. $\gamma_{ij} = \frac{\mu_{ij}}{2\sigma_{ij}} t$, and put it into $\left(\hat{\sigma}_i^*\right)^2 = \sum_{j=1}^{m} \gamma_{ij}\sigma_{ij}^2$, we get $t = \frac{2\left(\hat{\sigma}_i^*\right)^2}{\sum_{j=1}^{m} \sigma_{ij}\mu_{ij}}$, thus

$$\gamma_{ij} = \frac{\mu_{ij}\left(\hat{\sigma}_i^*\right)^2}{\sigma_{ij}\sum_{j=1}^{m} \sigma_{ij}\mu_{ij}} \quad (i=1, \cdots, n; j=1, \cdots, m).$$

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