Inequality, Growth, and Overtaking

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Abstract

This research develops a theory about the role of within-country income inequality leading to overtaking in economic performance among countries. The theory captures two opposing effects of inequality on factor accumulation and suggests that the qualitative change in their combined effect is a prime cause of overtaking. Due to the initial dominance of the positive effect of inequality, a less egalitarian economy follows a higher growth path in the short run, with a lower growth path in the long run. It is also shown that divergence or convergence may arise instead of overtaking, depending on the initial levels of development and inequality.

Keywords: Income Distribution, Economic Growth, Overtaking, Divergence, Convergence.

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1 Introduction

In history, the evolution of global income distribution has been characterized by shifts in the ranking of countries, as well as by divergence or convergence among them. As documented by Maddison (2001, Table B-21), the Netherlands, whose per capita GDP had been the highest in Europe since 1600, was overtaken by the United Kingdom by 1870, and then economic leadership shifted to the United States at the beginning of the twentieth century. Outside the Western world, Japan and the newly industrializing countries (Hong Kong, Singapore, South Korea, and Taiwan) overtook Argentina and Chile during the second half of the twentieth century.[1] Among the former European colonies, a reversal in relative incomes has occurred over the last 500 years (Acemoglu, Johnson and Robinson 2002).

Growth theorists have attempted to construct the theoretical foundations that account for these unpredictable phenomena. Among others, Brezis, Krugman and Tsiddon (1993) argue that overtaking reflects a leading country's failure to switch to a new technology that is initially less productive than the existing technology. Goodfriend and McDermott (1998) develop a model in which familiarity with a trading partner facilitates knowledge inflows and enhances learning productivity, human capital accumulation, and economic growth. Overtaking results from unilateral familiarization of a less developed country with the leading country. Galor, Moav and Vollrath (2005) suggest that while land abundance is beneficial for the process of development in the early stages, land inequality hinders the implementation of educational reforms.[2] On the empirical side, Acemoglu, Johnson and Robinson (2002) document that the above-mentioned reversal among the former European colonies resulted from the colonizers' tendency to establish investment-promoting institutions in relatively poor regions.

This research considers the role of income inequality within countries leading to one country overtaking another in terms of economic performance. By focusing on two opposing effects of inequality on factor accumulation, it examines the link between income distribution and the pattern

[1] In Maddison's (1982, Table C.10) productivity ranking among 16 countries between 1870 and 1979, Abramovitz (1986) finds that Australia fell by 8 places, Italy by $2\frac{1}{2}$, Switzerland by 8, and the United Kingdom by 10, while the United States rose by 4, Germany by $4\frac{1}{2}$, Norway by 5, Sweden by 7, and France by 8. See Jones (1997) and Pritchett (1997) for empirical discussions on convergence and divergence.

[2] Among other related theories, Fischer and Serra (1996) demonstrate that a highly equal country tends to overtake an unequal country in the presence of a human capital production function characterized by concavity and externality. Mountford (1998) finds overtaking by a country with a high saving rate in a dynamic version of the standard Heckscher-Ohlin model.

of development. The important premises here are that individual savings are convex with respect to income, whereas returns on education are subject to diminishing marginal returns. In these circumstances, inequality promotes the accumulation of aggregate physical capital by stimulating the savings of the rich.[3] On the other hand, inequality prevents the accumulation of aggregate human capital by placing borrowing constraints on the poor with regard to education.[4]

The relationship between income distribution and economic growth has been one of the most controversial topics in macroeconomics over the last decade. Despite the considerable number of empirical investigations, little is known about the relationship between these two elements within a single country. Most studies in the 1990s support the view that inequality is a hindrance to growth, while some recent articles find that their relationship turns positive in the short run.[5] Although these puzzling results would reflect, to some extent, differences in estimation methods and data qualities, it appears that this empirical ambiguity may reflect opposing forces that operate simultaneously.[6]

The proposed theory attributes the overtaking phenomena to a qualitative change in the combined effect of inequality on factor accumulation. The positive effect on physical capital formation is dominant at low levels of output. This is because, under low output and thus low wage rates, the saving-rate differential between the rich (capitalists) and the poor (workers) is significant, while investment in education provides few benefits.[7] However, the convexity of savings limits the capital-enhancing effect to the underdeveloped stages, whereas the negative effect on human capital accumulation increases with returns on education. This is why the overall effect of inequality on output reverses at high output levels.

Note that this reversal is not enough to generate a development trap, as high wages, associated

[3]See Keynes (1936), Kaldor (1978), Stiglitz (1969) and Moav (2002) for theoretical considerations, and Mayer (1966) and Dynan, Skinner and Zeldes (2004) for empirical evidence. The last paper supports bequest motives as in Becker and Tomes (1986), in explaining higher saving rates for higher-income groups.

[4]Galor and Zeira (1993) present a seminal theory in this field. Flug, Spilimbergo and Wachtenheim (1998) draw evidence from cross-country and panel regressions that credit market imperfections and unequal wealth distribution have negative impacts on average secondary enrollment. Perotti (1996) empirically supports the view that income equality encourages both male and female educational attainment.

[5]See Barro (2000) as well as Benabou's (1996) careful overview of the empirical studies in the early 1990s. A recent empirical work by Forbes (2000, p. 885) concludes that "the relationship between inequality and growth is far from resolved".

[6]For instance, Banerjee and Duflo (2003) argue that the differences in previous estimates can be explained by the linearity of the estimated models. Atkinson and Brandolini (1999) find it inappropriate to simply use "high quality" observations in Deininger and Squire's (1996) data set on income inequality.

[7]An empirical study by Perotti (1996) finds that income equality encourages investment in education more significantly in a group of high-income countries.

with high output, permit educational investment by wage earners. What is additionally necessary is highly unequal initial income distribution; this delays human capital accumulation significantly and thereby generates the reversal in the early stages of development. In this case, the economy converges to a steady state where wages are not sufficient for the poor to invest in education. A very egalitarian economy, on the other hand, is driven by universal investment in education and converges to a steady state characterized by higher output and persistent equality.[8] Comparing their output reveals that the former economy would follow a higher growth path in the short run, with a lower growth path in the long run.

These results indicate that initial income distribution plays a significant role in determining both long-run economic performance and the welfare of individuals.[9] The underdeveloped steady state acts as a development trap from which countries cannot escape without a substantial improvement in equality brought about by exogenous forces. Contrary to the macroeconomic viewpoint, however, it is shown that such a drastic redistribution is undesirable for the rich in both the long and short run, as they acquire the largest steady-state wealth in the aforementioned trap. This implies practical difficulties when actually pursuing a drastic redistribution.[10]

In addition to the two economies above, it is shown that an economy with moderate inequality may catch up with an egalitarian economy after being overtaken. Moderate inequality mitigates delays in the spread of education across individuals, and thus wages can reach a level that permits educational investment by the poor. Then credit constraints become less binding among the poor with the reduction in inequality, and the resulting universal investment in education leads the economy to a higher-level steady state. As a result, the evolution of inequality displays an inverted U-curve, as conjectured by Kuznets (1955).

The theory also reveals that convergence or divergence may occur instead of overtaking, depending on the initial degrees of inequality and the respective development stage of the countries concerned: Countries with similar degrees of inequality converge to similar growth paths, as long as their initial resources are sufficient to ensure the subsequent capital accumulation. Countries

[8] Due to the convexity of individual savings, more egalitarian economies need more initial resources to ensure capital accumulation. This paper does not go into the case of no (or negative) growth resulting from scarce resources.

[9] This is the notion emphasized by two seminal papers, Banerjee and Newman (1993) and Galor and Zeira (1993)

[10] While this is a meaningful implication, in this article, initial inequality is taken as exogenous and it is not political factors but (endogenously determined) low wages that generate persistent inequality. Galor and Moav (2000) alternatively propose a political economy view that capitalists would be willing to support the accumulation of human capital by workers in order to sustain their profit rates.

with differing degrees of inequality tend to diverge from each other if they are already at an intermediate stage of development. In this sense overtaking is perhaps less probable than divergence in the current world economy, which is more developed than ever.[11]

The general tendency towards divergence is supported by some empirical evidence. Benabou (1996) examines the role of inequality in the economic development of South Korea and the Philippines, which were similar with respect to all major macroeconomic variables such as GDP per capita, population, urbanization, and secondary school enrollment in the early 1960s. As a key factor to interpret South Korea's superior economic performance over the next 25 years, he points out significant differences in their initial distributions of income and land ownership: Inequality was much lower in South Korea as a result of its successful land reform following World War II. In fact, the combination of equality and rapid growth was also achieved by other East Asian economies (Hong Kong, Indonesia, Japan, Malaysia, Singapore, Taiwan, Thailand) during 1965-89 (Birdsall, Ross and Sabot 1995). Engerman and Sokoloff (2002) and Galor, Moav and Vollrath (2005) propose the relevance of different distributions of land ownership and human capital to the divergence in income levels between North and Latin America in the second half of the twentieth century.

The analytical framework is based on Galor and Moav's (2004) unified growth model that features capital market imperfections, altruistic linkage, capital-skill complementarity, and the above-mentioned contrasting properties of the saving and human capital functions.[12] There are three clear aspects that distinguish this research from theirs. First, Galor and Moav do not address the issue of overtaking and divergence. Their approach is to divide the process of industrialization into four stages, and examine the effect of inequality in one stage on subsequent growth within the same stage (i.e., short-term growth). This paper, by contrast, studies longer-term growth beyond the initial stage so as to observe diverse patterns of development.[13] Second, their analysis executes moderate redistributions of wealth so that the ex-ante state of the economy is maintained, whereas this paper considers drastic redistributions that can shift the initial economic regime. Third and finally, their research has positive as well as normative aspects. They trace a typical development

[11] The other central reason is that, as argued by Galor and Moav (2004), nowadays international capital markets encourage the flow of capital across borders, making domestic savings less important for physical capital accumulation.

[12] When physical capital is complementary with human capital in production, returns on and the demand for skills rise with capital intensity. See Goldin and Katz (1998) for empirical evidence.

[13] Galor and Weil (2002) and Galor and Moav (2004) theoretically analyze the long-run transition from stagnation to sustained growth. Yet unlike this research, they do not discuss the role of income inequality.

path of currently developed countries, showing that the role of inequality in economic growth has changed over time.[14] This article puts more emphasis on normative considerations.

The rest of the paper is organized as follows. Section 2 outlines the basic structure of the model, and Section 3 derives short-run equilibrium. Section 4 finds the multiplicity of steady-state equilibria, and then elucidates the global behavior of the dynamical system that governs the evolution of inequality. Utilizing these results, Section 5 analyzes the impact of income distribution on the behavior of output growth, by comparing the growth paths of hypothetical economies that differ only in their initial wealth distributions. Section 6 summarizes the discussion and proposes future research. Proofs of technical results are placed in the Appendix.

2 The Model

Consider a closed overlapping-generations economy operating over an infinite discrete time horizon, starting with period 0. Individuals with perfect foresight invest in assets and education in the presence of imperfect capital markets. In perfectly competitive environments, producers generate a single final good that can be consumed or passed on to the next generation. Population and technology are exogenously determined and stationary over time.

2.1 Producers

The amount of aggregate output produced at time t, Y_t, is determined by the aggregate stocks of physical and human capital at time t, K_t and H_t respectively. The production function takes the Cobb-Douglas form:

$$Y_t = AK_t^{\alpha}H_t^{1-\alpha} = Ak_t^{\alpha}H_t \equiv f(k_t)H_t, \tag{1}$$

where $\alpha \in (0, 1)$, $k_t \equiv K_t/H_t$, and $A > 0$ stands for the level of technology. The market price of the final good is normalized to 1.

In contrast to individuals' loans taken out to cover the cost of education, producers freely rent the services of capital and labor from households through competitive factor markets.[15] Hence, they maximize their profits given the market wage per unit of human capital, w_t, and the rental price per unit of physical capital, r_t. This problem is to maximize $f(k_t)H_t - w_tH_t - r_tK_t$ with

[14] The development path proposed by Galor and Moav corresponds to the case of moderate inequality explained above.

[15] This assumption is supported by the fact that compared to human capital, physical capital is easily collateralized.

respect to K_t and H_t, and the associated first-order conditions are

$$\begin{aligned} r_t &= f'(k_t) = \alpha A k_t^{\alpha-1} \equiv r(k_t); \\ w_t &= f(k_t) - f'(k_t)k_t = (1-\alpha)A k_t^{\alpha} \equiv w(k_t). \end{aligned} \tag{2}$$

Note that the rate of return on human capital, w_t, increases with physical capital due to the complementarity between the two types of capital. Physical capital depreciates at a constant rate $\delta \in [0,1]$ in each period.

2.2 Households

2.2.1 Environment

A new generation of individuals is born in every period, living over the course of two periods. Namely, there are two generations in society at any point in time. Individuals may be different in their initial wealth, yet they are homogeneous in terms of all other aspects. The population size of each generation is normalized to one, and an individual born in period t is referred to as a member $i \in [0,1]$ of generation t.

In the first period of life, when young, a member i of generation t engages in skill acquisition. Human capital formation is augmented by physical investment, without which an individual will obtain only basic skills. In this circumstance, the individual allocates transfers from her single parent, b_t^i, between education, e_t^i, and savings, s_t^i. That is, $b_t^i = e_t^i + s_t^i$.

In the second period of life, when an adult, the individual acquires human capital $h_{t+1}^i = h(e_t^i)$, where $h(\cdot)$ is an increasing and strictly concave function defined on $\mathbb{R}_+$, satisfying $h(0) = 1$ and the Inada conditions.[16] Wage income is earned by supplying human capital inelastically in competitive labor markets. In addition, those who have savings rent out capital services to producers at the market price. Accordingly, the second-period wealth of a member i of generation t, I_{t+1}^i, is

$$\begin{aligned} I_{t+1}^i &= w_{t+1}h_{t+1}^i + s_t^i R_{t+1} \\ &= w_{t+1}h(e_t^i) + (b_t^i - e_t^i)R_{t+1}, \end{aligned} \tag{3}$$

where $R_{t+1} = 1 + r_{t+1} - \delta \equiv R(k_{t+1})$.

[16] Alternatively, $h(0)$ can be viewed as the level of human capital acquired by public primary and secondary education. In this case, inequality still generates differences in individual attainments in higher education.

The preferences of a member i of generation t are defined over c_{t+1}^i, consumption in period $t+1$, and b_{t+1}^i, transfers to her single child.[17] They are represented by the utility function

$$u(c_{t+1}^i, b_{t+1}^i) = (1-\beta)\ln c_{t+1}^i + \beta \ln(\bar{\theta} + b_{t+1}^i), \tag{4}$$

where $\beta \in (0,1)$ and $\bar{\theta} > 0$. The underlying premise of (4) is that intergenerational transfers are a luxury good and are motivated by "joy of giving". The budget constraint faced by the individual is

$$c_{t+1}^i + b_{t+1}^i \leq I_{t+1}^i. \tag{5}$$

2.2.2 Optimization

Each member of generation t maximizes her utility from (4) subject to (5). The optimal amount of transfers chosen by a member i of generation t is

$$b_{t+1}^i = \begin{cases} 0 & \text{if } I_{t+1}^i < \theta; \\ \beta(I_{t+1}^i - \theta) & \text{if } I_{t+1}^i \geq \theta, \end{cases} \tag{6}$$

where $\theta \equiv \bar{\theta}(1-\beta)/\beta > 0$. The convexity of this transfer function asserts that inequality in wealth I_{t+1}^i across individuals enhances aggregate transfers.

Noting that the indirect utility strictly monotonically increases with I_{t+1}^i, this member chooses educational expenditures so as to maximize I_{t+1}^i in (3). Hence, the optimal level of education where no credit constraints exist, denoted as e_t, is

$$e_t = \arg\max_e [w_{t+1}h(e) - eR_{t+1}], \tag{7}$$

where the factor prices are taken as given and predicted accurately. In light of (2) and the properties of $h(\cdot)$, the education level e_t is a unique maximum satisfying the first order condition

$$w(k_{t+1})h'(e_t) = R(k_{t+1}) \qquad \text{for } k_{t+1} > 0. \tag{8}$$

Without loss of generality, it is assumed that no physical resources are invested in education if the economy is expected to be inactive in the next period; i.e., $e_t = 0$ if $k_{t+1} = 0$. It then follows that there exists a continuous single-valued function

$$e_t = e(k_{t+1}) \qquad \text{for } k_{t+1} \geq 0, \tag{9}$$

[17] One may suppose that c_{t+1}^i includes the consumption of her child.

where $e(0) = 0$ and $e'(k_{t+1}) > 0$, implying that $e_t > 0$ as long as $k_{t+1} > 0$.[18] The intuition of the positive reaction of educational expenditures to a rise in the capital-labor ratio is straightforward: due to the capital-skill complementarity, a rise in k_{t+1} enhances the return on human capital, w_{t+1}, while reducing the return on savings, R_{t+1}.

Note that e_t is the amount that any member of generation t is willing to invest if she can. In this economy, however, imperfect capital markets completely limit individuals' access to credit and all of them cannot necessarily afford e_t. Bearing this in mind, the optimal level of education for a member i of generation t, e_t^i, is

$$e_t^i = \begin{cases} b_t^i & \text{if } b_t^i < e_t; \\ e_t & \text{if } b_t^i \geq e_t. \end{cases} \tag{10}$$

Hence, e_t^i may differ across individuals, depending on the transfers they receive. The resultant savings are

$$s_t^i = b_t^i - e_t^i = \begin{cases} 0 & \text{if } b_t^i < e_t; \\ b_t^i - e_t & \text{if } b_t^i \geq e_t. \end{cases} \tag{11}$$

It follows from (6) that individual savings s_t^i are convex with respect to wealth I_t^i.[19] Substituting (10) and (11) into (3), the second period's wealth is now modified to

$$I_{t+1}^i = \begin{cases} w_{t+1}h(b_t^i) & \text{if } b_t^i < e_t; \\ w_{t+1}h(e_t) + R_{t+1}(b_t^i - e_t) & \text{if } b_t^i \geq e_t. \end{cases} \tag{12}$$

This shows that members receiving more transfers will earn more income, due to the monotonicity of returns on investment (in both physical and human capital).

2.3 The Initial Distribution of Wealth

In period 0, society is divided into two income groups, R (Rich) and P (Poor), which respectively comprise fixed fractions $\lambda \in (0, 1)$ and $1 - \lambda$ of adults. There exists inequality in the initial wealth I_0^i between, but not within, these groups. Accordingly, members of each group behave identically in every period, and those of each generation may be indexed by $i = P, R$. Let k_0^i be the initial

[18] Unlike Galor and Moav (2004), this paper assumes that $h'(e) \to \infty$ as $e \to 0$, and thereby omits their Regime I, where $e_t = 0$ and $k_{t+1} > 0$. As will become apparent, however, this omission is not essential for one country to overtake another in economic performance.

[19] This convexity holds within each household, but not for each individual, in the sense that s_t^i is the savings by a member i of generation t, whereas I_t^i is the wealth owned by her parent. By contrast, Galor and Moav (2004) assume that adult individuals accumulate savings, so that individuals' savings are convex with respect to their own wealth. Such difference is not essential for the main results below, and to simplify the exposition this paper does not follow their assumption.

capital owned by an adult member of group i. Her wealth is then $I_0^i = w_0 h_0^i + R_0 k_0^i$, where it is assumed that $h_0^R = h(e(k_0)) > h_0^P = 1$ and $k_0^R > k_0^P = 0$.[20]

The government may execute a redistribution policy in period 0. For the sake of simplicity, it is assumed that redistribution is accomplished by using a lump sum transfer among adults, in such a way that $I_0^R - \varepsilon \equiv \tilde{I}_0^R \geq I_0^P + \varepsilon \equiv \tilde{I}_0^P$.[21] It then follows from (6) that initial transfers are

$$b_0^i = \beta \max(\tilde{I}_0^i - \theta, 0). \tag{13}$$

Hence, in light of (12),

$$I_t^R \geq I_t^P \geq 0 \quad \text{and} \quad b_t^R \geq b_t^P \geq 0, \quad \forall t \geq 0. \tag{14}$$

In other words, the initial ranking of wealth among dynasties never reverses in the future. This results reflect is generated by the unequal initial distribution of wealth, the monotonicity of returns on investment, and the monotonicity of the transfer function.

3 Short-Run Equilibrium

This section considers the determination of economic variables in each period.

3.1 The Capital-Labor Ratio

In this closed economy, aggregate savings are the only source for aggregate physical capital in the next period. Moreover, note that credit constraints are not binding for members of group R in equilibrium (i.e. $b_t^R \geq e_t$); otherwise no aggregate savings lead to $e(k_{t+1}) = e(0)$, a contradiction to the fact $b_t^R \geq 0$. It thus follows from (11) that

$$\begin{aligned} K_{t+1} &= \lambda s_t^R + (1-\lambda) s_t^P \\ &= B_t - \tau_t e_t - (1-\tau_t) b_t^P, \end{aligned} \tag{15}$$

where $B_t \equiv \lambda b_t^R + (1-\lambda) b_t^P$ is aggregate transfers, and τ_t represents the fraction of young members for whom credit constraints are not binding at time t. On the other hand, (10) yields the aggregate

[20] The assumption that $h_0^R = h(e(k_0))$ is made so that (31) below can be applied to period 0. However, this is not essential for the qualitative results and one may alternatively assume that members of group R are initially unskilled.

[21] The level of ε is determined at the beginning of period 0, so that all individuals take ε as given.

stock of human capital:

$$\begin{aligned} H_{t+1} &= \lambda h(e_t^R) + (1-\lambda)h(e_t^P) \\ &= \tau_t h(e_t) + (1-\tau_t)h(b_t^P). \end{aligned} \tag{16}$$

Accordingly, in view of (9), the capital-labor ratio in period $t+1$ is

$$k_{t+1} = \frac{B_t - \tau_t e_t - (1-\tau_t)b_t^P}{\tau_t h(e_t) + (1-\tau_t)h(b_t^P)} \equiv q(k_{t+1}, B_t, b_t^P, \tau_t). \tag{17}$$

Given the properties of $h(\cdot)$ and $e(\cdot)$, this equation implies a continuous single-valued function

$$k_{t+1} = k(B_t, b_t^P, \tau_t), \tag{18}$$

where $k(0, 0, \tau_t) = 0$, $\lim_{B_t \to \infty} k(\cdot) = \infty$, $k_B(\cdot) > 0$, $k_b(\cdot, \lambda) < 0$ for $b_t^P > 0$, and $k_b(\cdot, 1) = 0$. Noting that credit constraints are never binding for group R, τ_t is determined in a way that

$$\tau_t = \left\{ \begin{array}{lll} \lambda & \text{iff} & b_t^P < e(k(\cdot, \lambda)) \\ 1 & \text{iff} & b_t^P \geq e(k(\cdot, 1)) \end{array} \right\} \equiv \tau(b_t^R, b_t^P). \tag{19}$$

Figure 1 illustrates the determination of τ_t on the (b_t^R, b_t^P) space, where $b_t^R \geq b_t^P \geq 0$. The Credit Constraint Frontier, the CC locus, is defined as the set of all pairs (b_t^R, b_t^P) for which[22]

$$b_t^P = e(k(\cdot, 1)) = e(k(\cdot, \lambda)).$$

The frontier approaches the origin as b_t^R and thus k_{t+1} go to zero, and its slope is between zero and one. Observe that $e(k(\cdot, 1)) < b_t^P$ on the region above the frontier and $e(k(\cdot, \lambda)) > b_t^P$ on the region below the frontier—both cases are consistent with the definition of τ_t.[23] Noting that the effectiveness of credit constraints depends on between-group inequality, the (b_t^R, b_t^P) space in the diagram can be divided into three regimes:[24]

Regime 1 ($b_t^R > e_t > b_t^P = 0$)**:** This regime occurs on the vertical and the horizontal axes, where inequality in transfers is high. Credit constraints are binding for members P of generation t, who acquire only basic skills with no savings.

[22] These two equalities hold because $q(\cdot, 1) = q(\cdot, \lambda)$ if and only if $b_t^P = e(k_{t+1})$. Without loss of generality, one can choose $\tau_t = 1$ if $b_t^P = e_t$, so that τ_t can be defined in the above way.

[23] Likewise, $e(k(\cdot, \lambda)) < b_t^P$ on the region above the frontier and $e(k(\cdot, 1)) > b_t^P$ on the region below the frontier. However, both cases are inconsistent with the definition of τ_t.

[24] Regimes 1-3 in the present paper are the counterparts of Stages I-III (of Regime II) defined by Galor and Moav (2004). Since, unlike their economy, this paper's economy does not necessarily go through each stage in the process of development, Stages I-III are renamed Regimes 1-3. The counterpart of their Regime I does not exist in this paper as mentioned in Footnote 18.

Regime 2 ($b_t^R > e_t > b_t^P > 0$)**:** This regime occurs on the region between the b_t^R axis and the CC locus. While all members of generation t invest their endowments in education, credit constraints are binding for group P.

Regime 3 ($b_t^R \geq b_t^P \geq e_t$)**:** This regime occurs on the remaining region, where inequality is low. All members of generation t attain the educational level e_t, and credit constraints are not binding for any of them.

It is now clear that the equilibrium capital-labor ratio is expressed as a continuous single-valued function

$$k_{t+1} = \kappa(b_t^R, b_t^P). \tag{20}$$

3.2 Aggregate Output

In order to simplify the following analysis of the dynamical system, complete capital depreciation, $\delta = 1$, is assumed so that aggregate income (output) equals aggregate wealth in each period.[25] Then, substituting (15) and (16) into (1), aggregate output in period $t+1$ is expressed as

$$Y_{t+1} = \lambda I_{t+1}^R + (1-\lambda) I_{t+1}^P = Y(B_t, b_t^P, \tau_t), \tag{21}$$

where $Y(0,0,\tau_t) = 0$ and $Y_B(\cdot) = R_{t+1}$, as (2) and (7) imply that $e_t = \arg\max Y_{t+1}$. Thus due to the properties of $k(\cdot)$ in (18), the function $Y(\cdot)$ is increasing and strictly concave in B_t, and the slope $Y_B(\cdot)$ diminishes toward zero as B_t goes to infinity. These results reflect the neoclassical properties of the production function with respect to physical capital. Also, $Y_b(\cdot, \lambda) > 0$ for $b_t^P \in (0, e_t)$ and $Y_b(\cdot, 1) = 0$, implying that a rise in b_t^P enhances human capital h_{t+1}^P and thus output Y_{t+1} as long as credit constraints are binding for group P.

4 The Dynamical System

Equations (2) and (20) assert that the second period's income I_{t+1}^i in (12) is affected by b_t^j as well as b_t^i ($i, j = R, P$) through wage and interest rates (yet not through e_t because $e_t = \arg\max I_{t+1}^i$). One may thus write

$$I_{t+1}^i = I(b_t^i, k_{t+1}) = I^i(b_t^R, b_t^P), \tag{22}$$

[25] Assuming $\delta \in [0, 1)$ would not contaminate any qualitative properties of the dynamical system.

where $I(0,0) = I^i(0,0) = 0$, as no aggregate transfers lead to no physical capital and no output in the next period. In addition, the Appendix shows that for $b_t^R > b_t^P > 0$,

$$\partial I^i(\cdot)/\partial b_t^i > 0; \qquad \partial I^P(\cdot)/\partial b_t^R > 0; \qquad \partial I^R(\cdot)/\partial b_t^P \begin{cases} > 0 & \text{if } b_t^P < e_t \\ < 0 & \text{if } b_t^P \geq e_t. \end{cases} \tag{23}$$

The first two properties above, together with (21), yield that $\partial I^i(\cdot)/\partial b_t^R \to 0$ as $b_t^R \to \infty$. This Inada condition reflects the non-increasing returns to scale in physical and human capital investment, in addition to the neoclassical properties of the production function. Regarding the cross derivatives $\partial I^i(\cdot)/\partial b_t^j$, the difference in their signs is explained by the following three facts. First, as implied by (39) in the Appendix, capital income is more important than wage income for group R, while the opposite is true for group P. Second, a rise in b_t^R raises the capital-labor ratio and thus the return on efficient labor. Third, if credit constraints are binding, a rise in b_t^P will decrease the capital-labor ratio and thus increase the interest rate, otherwise the effect reverses.

Substituting (22) into (6), a trajectory $\{b_t^R, b_t^P\}_{t=0}^{\infty}$ is fully governed by a two-dimensional first-order autonomous system:

$$b_{t+1}^i = \phi(b_t^i, k_{t+1}) = \psi^i(b_t^R, b_t^P) \qquad \text{for } i = R, P, \tag{24}$$

with the initial condition (b_0^R, b_0^P) in (13).

4.1 Steady-State Equilibria

This subsection characterizes steady-state equilibria of the dynamical system (24).

4.1.1 Egalitarian Case

First, consider an egalitarian steady-state equilibrium where $b_{t+1}^R = b_t^R = b_t^P > 0 \; \forall t$. This symmetry implies that all individuals earn the same income and credit constraints are not binding in the steady state. It then follows from (18) and (21) that $I_t^R = Y_t$ and $k_b(\cdot, 1) = Y_b(\cdot, 1) = 0$. In these circumstances the system (24) yields

$$b_t^R = \phi(b_t^R, k(b_t^R, 0, 1)) = \beta[Y(b_t^R, 0, 1) - \theta]. \tag{25}$$

Due to the properties of $Y(\cdot)$, this condition is satisfied by at most two positive values of b_t^R. In order to assure their existence, it is assumed that the technological level A is sufficiently high:

$$A > \underline{A} \equiv \underline{A}(\alpha, \beta, \bar{\theta}), \tag{A1}$$

where $\underline{A}$ is the critical level which yields a unique solution in (25).

4.1.2 Inegalitarian Case

Next, consider an inegalitarian steady-state equilibrium where $b^R_{t+1} = b^R_t > b^P_t = 0 \ \forall t$. This asymmetry and (19) imply that credit constraints are binding for group P in the steady state (i.e. $e_t > b^P_t = 0$). It then follows from (24) that

$$b^R_t = \phi(b^R_t, k(\lambda b^R_t, 0, \lambda)). \tag{26}$$

In light of (17) and (39) in the Appendix, one finds that the income $I(b^R_t, k(\lambda b^R_t, 0, \lambda))$ strictly decreases with λ for a given $b^R_t > 0$. This result, together with the properties of $I^R(\cdot)$ and Assumption (A1), assures that at least two positive values of b^R_t satisfy (26). The inegalitarian steady-state equilibrium occurs if b^R_t additionally satisfies the condition $I^P(b^R_t, 0) = w_{t+1} \leq \theta$.

Now, note that (25) and (26) can be viewed as the steady-state equilibria of a one-dimensional system:

$$b^R_{t+1} = \phi(b^R_t, k(\tau b^R_t, 0, \tau)); \qquad \tau \in (0, 1]. \tag{27}$$

Let $\bar{b}$ and $\underline{b}$ denote the largest transfers in a locally *stable* and locally *unstable* steady-state equilibrium, respectively, of this system. Then it follows that $\bar{b} = \bar{b}(\tau) > \underline{b} = \underline{b}(\tau)$ for all τ, where $\bar{b}(\cdot)$ and $\underline{b}(\cdot)$ are single-valued functions.

Let $\hat{b}$ be the critical level of transfers such that

$$w(\hat{k}) = \theta, \quad \text{where } \hat{k} \equiv k(\tau\hat{b}, 0, \tau), \tag{28}$$

implying a single-valued function $\hat{b} = \hat{b}(\tau)$. It follows that if $\bar{b}(\lambda) \leq \hat{b}(\lambda)$, the pair $(b^R_t, b^P_t) = (\bar{b}(\lambda), 0)$ does not lead to a wage rate greater than θ, generating the inegalitarian steady-state equilibrium.[26] The existence of the steady-state equilibrium depends on the sign of the difference $\hat{b}(\tau) - \bar{b}(\tau)$.

Lemma 1 *Under (A1), the difference $\bar{b}(\tau) - \hat{b}(\tau)$ is increasing in $\tau \in (0, 1]$.*

Proof. See the Appendix. □

[26] In this case, the pair $(\underline{b}(\lambda), 0)$ also generates one of the steady-state equilibria because $\underline{b}(\lambda) < \bar{b}(\lambda)$.

In light of this result, the sign of $\bar{b}(\tau) - \hat{b}(\tau)$ depends on the level of τ if there exists a value $\tau^* \in (0, 1)$ such that[27]

$$\bar{b}(\tau^*) = \hat{b}(\tau^*). \tag{A2}$$

This condition assures that $\bar{b}(1) > \hat{b}(1)$ and $\bar{b}(\lambda) < \hat{b}(\lambda)$ for a sufficiently small λ (i.e. high inequality). It excludes the case that the wage rate remains either above or below θ for any τ, and allows us to examine the role of initial inequality in determining the long-run wealth distribution. The analysis below builds on Assumption (A2).

4.2 Global Dynamics

This subsection analyzes the long-run evolution of intergenerational transfers by utilizing a phase diagram. In order to consider various degrees of initial inequality, suppose that the fraction of group R, λ, can be either $\lambda^S \in (0, 1/2)$ or $\lambda^L \equiv 1 - \lambda^S$ in period 0. Group R is referred to as Group S (Small) if $\lambda = \lambda^S$, and as Group L (Large) if $\lambda = \lambda^L$.

Basic properties of global dynamics are illustrated in Figure 2, where $(b_t^S, b_t^L) \in \mathbb{R}_+^2$ since either groups S or L can be wealthier than the other. The space is divided by the two Credit Constraint Frontiers, the CC^S and CC^L loci, on which $b_t^S = e_t$ and $b_t^L = e_t$ respectively. Note that $b_t^S > e_t$ ($b_t^S < e_t$) on the region below (above) the CC^S locus, whereas $b_t^L > e_t$ ($b_t^L < e$) on the region above (below) the CC^L locus. Therefore, Regime 3 occurs on the space surrounded by the two frontiers.

Lemma 2 *Under (A1), the dynamical system (24) is characterized as follows.*

(a) $b_{t+1}^P > b_t^P$ *if* $b_t^R > \hat{b}(\lambda)$ *and* $b_t^P < \bar{b}^*$.

(b) $\underline{b}^S < \underline{b}^L < \underline{b}^* < \bar{b}^* < \bar{b}^L < \bar{b}^S$.

(c) *There are only two nontrivial steady-state equilibria,* $(\bar{b}^*, \bar{b}^*)$ *and* $(\underline{b}^*, \underline{b}^*)$, *in Regime 3.*

Proof. See the Appendix. □

Recalling (28), the first result means that members of group P can accumulate transfers up to a certain level as long as the wage rate is greater than θ. The upper bound $\bar{b}^*$ is generated by the

[27] In light of (2) and (28), Assumption (A2) holds if

$$[\theta/(1-\alpha)A]^{1/\alpha} = k(\tau\bar{b}(\tau), 0, \tau),$$

for $\tau = \tau^*$. This condition is feasible because $\bar{b}(\tau)$ and thus $k(\bar{b}(\tau), 0, \tau)$ increase with A.

neoclassical properties of the aggregate production function. The second result is owing to the fact that concentrating capital ownership in few hands will promote capital revenues, which are more important than wage income for capitalists.

In the diagram, $g^* \equiv g(1)$ and $g^i \equiv g(\lambda^i)$ for a function $g(x)$. The BB^i loci are defined as the set of all pairs (b_t^S, b_t^L) for which $b_{t+1}^i = b_t^i$, and includes the interval $[0, \hat{b}^j]$ on the b_t^j axis.[28] The system displays the multiplicity of steady-state equilibria, which occur at the intersections of the BB^S and BB^L loci. In view of Lemma 1, it is assumed that the difference between λ^S and λ^L is so large that $\hat{b}^S \geq \bar{b}^S$ and $\hat{b}^L < \bar{b}^L$. As discussed earlier, the first inequality generates an inegalitarian steady-state equilibrium, and the pair (b_t^S, b_t^L) converges to one of the points $(0,0)$, $(\bar{b}^S, 0)$ and $(\bar{b}^*, \bar{b}^*)$, depending on the initial amount and distribution of aggregate transfers. Observe that no steady-state equilibrium occurs at $(0, \bar{b}^L)$.

Now we are ready to examine how the allocation of resources between the two groups affects the subsequent evolution of transfers within dynasties of each group. As will become apparent, initial inequality may play a significant role in determining individual living standards in the long-run, depending on the level of B_0. The economic intuition behind this result will be explained in the next section.

First consider the case of $B_0 \in (\underline{b}^*, \lambda^S \bar{b}^S)$ in the diagram.[29] If the initial transfers are entirely in the hands of group S (i.e. high inequality), Lemma 2 yields that $b_0^S \in (\underline{b}^S, \bar{b}^S)$ and $b_0^L = 0$, meaning Regime 1. Hence b_t^S grows over time while b_t^L remains zero, and the pair (b_t^S, b_t^L) converges to the inegalitarian steady-state equilibrium $(\bar{b}^S, 0)$ in the same regime. Since Lemma 2 implies that members of group $S(= R)$ obtain the highest steady-state income in the steady state, this initial condition is ideal for group S in the long run as well as in the short run. On the other hand, inequality is not persistent if instead group L holds the entire amount of the initial transfers (i.e. lower inequality). In this situation Lemma 2 yields that $b_0^L \in (\underline{b}^L, \bar{b}^S)$ and $b_0^S = 0$, meaning Regime 1 as in the first case. Due to the property $\hat{b}^L < \bar{b}^L < \bar{b}^S$, however, there is a period when b_t^L is greater than the critical level $\hat{b}^L$, and consequently the pair (b_t^S, b_t^L) converges to the egalitarian

[28] The BB^i loci reflect the properties of $I^i(b_t^R, b_t^P)$ in (23). They are plotted so as to be gradual, and this way of plotting may rule out some steady states that otherwise would exist. As will become apparent, this simplification does not affect the qualitative nature of the dynamical system.

[29] The analysis here focuses only on the case $\underline{b}^* < \lambda^S \bar{b}^S$. In light of (36) and (A3) below, this case occurs if λ^S is sufficiently small and A is sufficiently high.

steady-state equilibrium $(\bar{b}^*, \bar{b}^*)$ in Regime 3.[30] It should be noted that the economy does not even go through Regime 1 if the initial inequality is even lower. For instance, the perfectly egalitarian case $b_0^S = b_0^L$ leads to $b_t^S = b_t^L$ and $\tau_t = 1$ in all subsequent periods. In this case, transfers within each dynasty start out in the interval $(\underline{b}^*, \bar{b}^*)$, and monotonically increase to the steady-state level $\bar{b}^*$.

Second, consider the case of scarce resources, $B_0 \in (\lambda^S \underline{b}^S, \underline{b}^*)$. Unlike in the first case, egalitarian policies may result in a gradual diminishment of resources, as all adults spend a large fraction of their income on consumption. Equality therefore yields a long-run outcome that is not desirable for anyone. In contrast, higher inequality such as $b_0^S > b_0^L = 0$ promotes intergenerational transfers within dynasties of group S. Since this allocation yields $b_0^S \in (\underline{b}^S, \bar{b}^S)$ as in the first case, the economy converges to the nontrivial steady-state equilibria $(\bar{b}^S, 0)$ in Regime 1.[31] Thus initial inequality is more desirable than equality at least for the richer group in any period.

Third and finally, the allocation of B_0 does not affect the long-run outcome if $B_0 < \lambda^S \underline{b}^S$ or $B_0 > \lambda^S \bar{b}^S$. Regardless of initial inequality, transfers within each dynasty decrease toward zero in the former case, while converging toward $(\underline{b}^*, \bar{b}^*)$ in the latter case.

It is worthwhile mentioning that the diagram illustrates the growth path presented by Galor and Moav (2004). According to their scenario, the economy starts out with the pair $b_0^L \in (\underline{b}^L, \hat{b}^L)$ and $b_0^S = 0$. This is the case in which initial inequality is *not* extremely high, as the number of richer members is relatively large.[32] Under this circumstance, the initial state is Regime 1, where b_t^L increases over time and b_t^S remains zero. Once b_t^L exceeds $\hat{b}^L$, the economy enters Regime 2 and the level of b_t^S begins to ascend. At this stage, members in group S invest all transfers in education, thus ending up with no savings. The economy reaches Regime 3 when the pair (b_t^S, b_t^L) crosses the CC^L locus. Then credit constraints are no longer binding and transfers converge to the egalitarian steady-state equilibrium $b_t^S = b_t^L = \bar{b}^*$. Wealth inequality therefore improves in the long run, and its evolution displays an inverted U-curve over the process of development.

[30] In fact, as asserted by Lemma 3 below, the steady-state aggregate transfers and output are maximized in this egalitarian steady-state equilibrium.

[31] In case where $b_0^L > b_0^S = 0$, on the other hand, the long-run result depends on the level of B_0: The pair (b_t^S, b_t^L) converges to $(0, 0)$ if $B_0 \in (\lambda^S \underline{b}^S, \lambda^L \underline{b}^L]$, and to $(\bar{b}^*, \bar{b}^*)$ if $B_0 \in (\lambda^L \underline{b}^L, \underline{b}^*)$.

[32] The population size of group R, λ, can be less than $1/2$ in their scenario. Since, as shown in Footnote 27, the critical value τ in (A2) depends on the structural parameters, it follows that $\hat{b}(\lambda) < \bar{b}(\lambda)$ $\exists \lambda < 1/2$ if the technological level A is sufficiently large.

5 Output Growth

The preceding section showed that the dynamic transition of the economic regime depends on initial inequality. This section considers the underlying evolution of output and thereby examines the impact of income distribution on economic growth.

For the sake of simplicity, the analysis below focuses on the development path on which a chain of intergenerational transfers does not break once it emerges. That is to say, $b_{t+1}^P \geq b_t^P$ for all t, implying that $b_{t-1}^P = 0$ if $b_t^P = 0$. It thus follows from (6) and (12) that

$$\tau_{t-1} = \lambda \quad \text{and} \quad I_t^P = w_t < \theta \qquad \text{if } \gamma_t = \lambda, \tag{29}$$

where γ_t denotes the fraction of members leaving transfers in period t. Note that γ_t is either λ or 1, and that if $\gamma_t = \lambda$, the economy has been in Regime 1 until period t. As this chapter focuses on regime-changing redistribution, it is assumed that in period 0 no redistribution policy is executed within Regime 1; namely, $\varepsilon = 0$ and $\tilde{I}_0^P = w_0 < \theta$ if $\gamma_0 = \lambda$.

5.1 The Evolution of Output

By noting (6), (21) and (29), aggregate transfers throughout the three regimes are expressed as

$$\begin{aligned} B_t &= \beta \max[Y_t - (1-\gamma_t)\omega(Y_t, \gamma_t) - \gamma_t \theta, 0] \\ &\equiv B(Y_t, \gamma_t), \end{aligned} \tag{30}$$

where $\omega(Y_t, \gamma_t) \equiv w(\pi(Y_t, \gamma_t))$ and $\pi(Y_t, \gamma_t)$ equals the capital-labor ratio k_t satisfying

$$Y_t = Ak_t^{\alpha}[\gamma_t h(e(k_t)) + 1 - \gamma_t]. \tag{31}$$

In view of the properties of $h(\cdot)$ and $e(\cdot)$,

$$\pi_Y(\cdot) > 0; \qquad \pi_\gamma(\cdot) < 0; \qquad \pi(0, \gamma_t) = 0; \qquad \lim_{Y_t \to \infty} \pi(\cdot) = \infty. \tag{32}$$

It then follows that $B_Y(\cdot) > 0$ for $Y_t \geq \breve{Y}_t$, where $\breve{Y}_t$ is defined as the critical output level below which there are no aggregate transfers; i.e. $B(\breve{Y}_t, \gamma_t) = 0$.[33] This condition implies a single-valued function $\breve{Y}_t = \breve{Y}(\gamma_t)$ such that $\breve{Y}'(\gamma_t) > 0$, $\lim_{\gamma_t \to 0} \breve{Y}(\gamma_t) = 0$, and $\breve{Y}(1) = \theta$.[34]

[33] As follows from (2) and (31), $Y_t - (1-\gamma_t)\omega(Y_t, \lambda) = f(k_t)[\gamma_t h(e(k_t)) + \alpha(1-\gamma_t)]$, where $k_t = \pi(Y_t, \gamma_t)$. In light of (32), this assures the monotonicity $B_Y(\cdot) > 0$ and the existence of $\breve{Y}_t$.

[34] It follows from (2), (31) and Footnote 33 that for $Y_t > 0$,

$$Y_t - (1-\gamma_t)\omega(Y_t, \gamma_t) > \gamma_t[w_t - (1-\gamma_t)\omega_\gamma(\cdot)],$$

where the term on the left-hand side goes to zero as Y_t goes to zero. Hence, $B_\gamma(\breve{Y}_t, \gamma_t) < 0$ and $\breve{Y}'(\gamma_t) > 0$.

Substituting (30) into (21), the evolution of output throughout the three regimes is given by

$$Y_{t+1} = Y(B(Y_t, \gamma_t), b_t^P, \tau_t) \equiv \Phi(Y_t, b_t^P, \gamma_t, \tau_t), \tag{33}$$

where

$$\begin{aligned} &\gamma_t = \lambda \text{ and } \tau_t = \lambda \quad \text{in Regime 1;} \\ &\gamma_t = 1 \text{ and } \tau_t = \lambda \quad \text{in Regime 2;} \\ &\gamma_t = 1 \text{ and } \tau_t = 1 \quad \text{in Regime 3.} \end{aligned}$$

As shown in Figure 2, the economic regime in period t is fully determined by b_t^R and b_t^P.

The analysis below compares and investigates the evolution of output in each regime.

5.1.1 Regimes 1 and 3

Noting that $b_t^P = 0$ in Regime 1 and $\Phi_b(\cdot) = 0$ in Regime 3, the evolution of output in these regimes is expressed as $Y_{t+1} = \Phi(Y_t, 0, \gamma_t, \tau_t)$. Recalling the definition of $\check{Y}_t$ above, one can find that

$$\Phi(Y_t, 0, \cdot) \begin{cases} = 0 & \text{for } Y_t \leq \check{Y}_t; \\ > 0 & \text{for } Y_t > \check{Y}_t, \end{cases}$$

where $\check{Y}_t$ is constant in each regime. Moreover, for $Y_t > \check{Y}_t$,

$$\begin{aligned} &\Phi_Y(Y_t, 0, \cdot) = R_{t+1} B_Y(\cdot) > 0; \\ &\Phi_\gamma(Y_t, 0, \cdot) = R_{t+1}[w_t - (1-\gamma_t)\omega_\gamma(\cdot) - \theta] \gtreqless 0; \\ &\Phi_\tau(Y_t, 0, \cdot) = [w_{t+1} h(e_t) - R_{t+1} e_t] - w_{t+1} > 0, \end{aligned} \tag{34}$$

using (7). Noting (32) and that $B_Y(\cdot) \leq \beta$, the first property shows that $\Phi_Y(Y_t, 0, \cdot) \to 0$ as $Y_t \to \infty$. The last two properties combined together reveal the difference in growth paths between the two regimes. Such difference represents the effect of drastic redistribution on economic growth.

The sign of $\Phi_\gamma(\cdot)$ above goes to negative infinity as Y_t decreases to $\check{Y}_t$, while it turns positive at $Y_t = \hat{Y}_t \equiv \hat{Y}(\gamma_t)$, where $\hat{Y}(\tau)$ is a single-valued function such that $\omega(\hat{Y}(\tau), \tau) = \theta$.[35] Namely, the above-mentioned egalitarian policies have negative impacts on aggregate transfers and thus on physical capital only at underdeveloped stages.[36] This growth enhancing effect of inequality is due to the convexity of the transfer function (6) with respect to income.

The property $\Phi_\tau(\cdot) > 0$ above reflects the concavity of $h(\cdot)$. Namely, in the presence of credit constraints, equality enhances aggregate human capital by raising the ratio of fully skilled workers.

[35] As shown in Footnote 34, $B_\gamma(\check{Y}_t, \gamma_t) < 0$. Since, in addition, $k(B(\check{Y}_t, \gamma_t), 0, \gamma_t) = 0$, equation (34) yields that $\Phi_\gamma(Y_t, 0, \cdot) \to -\infty$ as $Y_t \to \check{Y}_t + 0$. One can also find that $\check{Y}_t < \hat{Y}_t$ by noting that $\omega_\gamma(\cdot) < 0$ in the footnote.

[36] One can confirm from (17) that an increase in B_t leads to an increase in K_{t+1}.

Yet this positive effect of equality (i.e. the negative effect of inequality) is not dominant at immature stages of development; it approaches zero as Y_t decreases toward the zero-transfer level $\check{Y}_t$.[37] This is explained by the fact that these stages are characterized by scarce physical capital, which leads to low wage rates relative to interest rates (i.e. low returns on education relative to savings) due to the capital-labor complementarity in production. That is to say, the scarcity of physical capital, rather than income inequality with credit constraints, is the prime factor for low stocks of aggregate human capital in this situation. On the other hand, the opposite is true at higher levels of wages and output.

To summarize, while the positive effect of inequality outweighs the negative effect in initial development stages, the relative intensity between these opposing forces reverses in higher stages where sufficient physical capital boosts wage rates. More formally, for a sufficiently small value $\epsilon > 0$,

$$\Phi_\gamma(Y_t, 0, \cdot) + \Phi_\tau(Y_t, 0, \cdot) \begin{cases} < 0 & \text{for } Y_t \in (\check{Y}_t, \check{Y}_t + \epsilon); \\ > 0 & \text{for } Y_t \geq \hat{Y}^*, \end{cases} \tag{35}$$

noting that $\hat{Y}(\tau)$ is increasing in $\tau \in (0, 1]$. Note that $\hat{Y}^* \equiv \hat{Y}(1)$ since by definition $g^i \equiv g(\lambda^i)$ and $g^* \equiv g(1)$ for a function $g(\cdot)$.

Using (21) and (30), let

$$\underline{Y}(\tau) \equiv Y(\tau\underline{b}(\tau), 0, \tau); \qquad \bar{Y}(\tau) \equiv Y(\tau\bar{b}(\tau), 0, \tau); \qquad \bar{B}(\tau) \equiv B(\bar{Y}(\tau), \tau), \tag{36}$$

where $\underline{Y}(\tau) < \bar{Y}(\tau)$. As in the previous section, it is assumed that λ^S is sufficiently small and $\hat{b}^S \geq \bar{b}^S$, whereas λ^L is sufficiently large and $\hat{b}^L < \bar{b}^L$. Therefore, these functions yield aggregate output and transfers in the egalitarian and inegalitarian steady-state equilibria. Note that this is not the case for $\tau = \lambda^L$, as a steady-state equilibrium does not occur at $(b_t^R, b_t^P) = (0, \underline{b}^L)$.[38]

Lemma 3 *Under (A1)-(A2), $\bar{Y}^S < \bar{Y}^L < \bar{Y}^*$ and $\bar{B}^S < \bar{B}^L < \bar{B}^*$.*

Proof. Since $\pi(\hat{Y}(\tau), \tau) = \hat{k}$, (21), (28) and (31) yield

$$\begin{aligned} \hat{Y}(\tau) &= A\hat{k}^\alpha[\tau h(e(\hat{k})) + 1 - \tau] \\ &= Y(\tau\hat{b}(\tau), 0, \tau). \end{aligned}$$

[37] More formally, $e(k_{t+1})$ and hence $\Phi_\tau(Y_t, 0, \cdot)$ in (34) approaches zero as Y_t decreases to $\check{Y}_t$.

[38] One may interpret $\bar{Y}^L$ and $\bar{B}^L$ as the steady-state variables on the condition that the economy remains in Regime 1 (i.e. $b_t^P = 0$) in all periods.

It follows from (36) and Lemma 1 that $\bar{Y}^S \leq \hat{Y}^S$, $\hat{Y}^L < \bar{Y}^L$ and $\hat{Y}^* < \bar{Y}^*$. Hence, one can find that $\bar{Y}^S < \bar{Y}^L$ noting that $\hat{Y}(\tau)$ is increasing in $\tau \in (0, 1]$, and that $\bar{Y}^L < \bar{Y}^*$ by using (35). These results lead to $B(\bar{Y}^S, \lambda^S) < B(\bar{Y}^L, \lambda^L) < B(\bar{Y}^*, 1)$. □

This result and the analysis in Subsection 24 imply the proposition below.

Proposition 1 *Under (A1)-(A2), egalitarian policies are undesirable from the viewpoint of the rich in any period, even though they maximize the long-run aggregate output.*

Figures 3-4 depict the evolution of output in Regimes 1 and 3 for different values of λ: Figure 3 depicts the case where group R is small (i.e. high inequality), corresponding to the lower right part of Figure 2. Figure 4 depicts the case where group R is large (i.e. lower inequality), corresponding to the upper left part of Figure 2. In the diagrams,

$$\Phi^{1i}(Y_t) \equiv \Phi(Y_t, 0, \lambda^i, \lambda^i); \qquad \Phi^3(Y_t) \equiv \Phi(Y_t, 0, 1, 1),$$

where superscripts 1 and 3 respectively are used to denote functions for Regimes 1 and 3. Observe that $\Phi^3(Y_t)$ is strictly concave and identical between both diagrams. While $\Phi^{1i}(Y_t)$ is also strictly increasing in Y_t, it becomes lower than $\Phi^3(Y_t)$ at some output level below $\bar{Y}^*$.[39] Consistent with (36), the diagrams show that $Y_t = \Phi^{1i}(Y_t)$ for $Y_t = \underline{Y}^i, \bar{Y}^i$, and $Y_t = \Phi^3(Y_t)$ for $Y_t = \underline{Y}^*, \bar{Y}^*$.

As depicted in Figure 3, the analysis below concentrates on the case where $\underline{Y}^S < \underline{Y}^* < \bar{Y}^S$, so that there exists an initial output $Y_0 \in (\underline{Y}^*, \bar{Y}^S)$ that permits output growth in both egalitarian and inegalitarian economies. For this purpose, the technological level A is assumed to be sufficiently high to satisfy

$$\underline{Y}^* < \lim_{\tau \to 0} \bar{Y}(\tau) = [A(\alpha\beta)^\alpha]^{1/(1-\alpha)}, \tag{A3}$$

where the equality follows from the fact that $\lim_{\tau \to 0} \Phi(Y_t, 0, \tau, \tau) = A(\alpha\beta Y_t)^\alpha$.[40]

5.1.2 Regime 2

The evolution of output in Regime 2 is given by

$$Y_{t+1} = \Phi(Y_t, b_t^P, 1, \lambda) \equiv \Phi^2(Y_t, b_t^P),$$

[39] For simplicity in each diagram there are only two values of Y_t that satisfy $Y_t = \Phi^{1i}(Y_t)$, although, unlike $\Phi^3(Y_t)$, the concavity of $\Phi^{1i}(Y_t)$ is not guaranteed.

[40] This assumption is more restrictive than Assumption (A1), as the former requires the existence of $\underline{Y}^*$ and $\underline{b}^*$.

where $b_t^P \in (0, e_t)$ and $b_t^P \leq B(Y_t, 1) = \beta(Y_t - \theta)$. It follows that there is no impact of income inequality on aggregate transfers. Hence, for Y_t and b_t^P in Regime 2, $\Phi_\tau(\cdot) > 0$ and $\Phi_b^2(\cdot) > 0$, implying that

$$\Phi^2(Y_t, b_t^P) < \Phi^3(Y_t). \tag{37}$$

This result reflects that binding credit constraints cause production inefficiencies.

5.2 Inequality and the Patterns of Growth

This subsection investigates the impact of income distribution on the output behavior over the entire process of development. First, consider a case of low inequality such that

$$\underline{b}^* < b_0^i < \bar{b}^* \quad \text{for } i = P, R, \tag{A4}$$

where $\bar{b}^* = \beta(\bar{Y}^* - \theta)$ and $\underline{b}^* = \beta(\underline{Y}^* - \theta)$, noting (25) and (36). Figure 2 shows that given (A4), the pair (b_t^R, b_t^P) evolves in either Regimes 2 or 3 in all periods, converging to the egalitarian steady-state equilibrium $(\bar{b}^*, \bar{b}^*)$ in Stage 3.

Proposition 2 *Under (A2)-(A4), Y_t increases monotonically in either Regimes 2 or 3 for all periods, converging to the steady-state level $\bar{Y}^*$ in Regime 3.*

Proof. The results, except the monotonic growth, follow from (36) and the evolution of transfers described above. Since (6), (21) and ($A4$) yield $Y_0 \in (\underline{Y}^*, \bar{Y}^*)$, the properties of $\Phi^3(\cdot)$ and (37) show that $Y_t \in (\underline{Y}^*, \bar{Y}^*)$ $\forall t \geq 0$ and that $Y_{t+1} > Y_t$ if the economy is in Regime 3 in period t. If the economy is in Regime 2 in period t, it can be seen from Figure 2 that $(b_{t+1}^R, b_{t+1}^P) \gg (b_t^R, b_t^P)$. This implies that $Y_{t+1} > Y_t$, as $B_t = \beta(Y_t - \theta)$ $\forall t \geq 0$. □

Next, consider a case of higher inequality such that

$$\underline{b}(\lambda) < b_0^R < \bar{b}(\lambda) \quad \text{and} \quad b_0^P = 0, \tag{A4'}$$

meaning that the initial state is Regime 1. Recall that the fraction of group R, λ, determines the sign of $\hat{b}(\lambda) - \bar{b}(\lambda)$ and thus the existence of a nontrivial, locally stable, steady-state equilibrium in Regime 1. For the case $\lambda = \lambda^L$ in Figure 2, the economy goes through Regimes 1-3 sequentially over time and converges to the steady-state equilibrium where $b_t^R = b_t^P = \bar{b}^*$. Aggregate transfers monotonically increase over the first two regimes. On the other hand, for the case $\lambda = \lambda^S$ in the diagram, $b_t^S (= b_t^R)$ grows over Regime 1 in all periods, converging to the steady-state level $\bar{b}^S$.

Proposition 3 *Under (A2), (A3) and (A4′),*

(*a*) *If* $\lambda = \lambda^L$, *the economy goes through Regimes 1-3 sequentially.* Y_t *increases monotonically and converges to the steady-state level* $\bar{Y}^*$.

(*b*) *If* $\lambda = \lambda^S$, *the economy remains in Regime 1 for all periods.* Y_t *increases monotonically and converges to the steady-state level* $\bar{Y}^S$.

Proof. The results, except the monotonic growth in Regime 3, follow from (21), (36) and the evolution of transfers described above. Note that the output level is below $\bar{Y}(\lambda)$ during Regime 1, and that $B_t > 0$ and thus $Y_t > 0$ in all periods. It thus follows from the properties of $\Phi(\cdot)$, Lemma 3 and (37) that $Y_t \in (\underline{Y}^*, \bar{Y}^*)$ after Regime 1, implying monotonic output growth in Regime 3. □

Figure 4 depicts the evolution of output described in Proposition 3(*a*). Starting out at $Y_0 \in (\underline{Y}^L, \bar{Y}^L)$, output monotonically increases over Regime 1 toward the take-off level $\hat{Y}^L$, which is lower than the steady-state level $\bar{Y}^L$. Hence Y_t eventually exceeds $\hat{Y}^L$ and then the economy enters Regime 2. Since the above proof shows that $Y_t \in (\underline{Y}^*, \bar{Y}^*)$ afterwards, $\hat{Y}^L$ lies between $\underline{Y}^*$ and $\bar{Y}^*$ in the diagram. Figure 3 depicts the evolution of output described in Proposition 3(*b*). Starting out at $Y_0 \in (\underline{Y}^S, \bar{Y}^S)$, output monotonically increases over Regime 1 toward $\bar{Y}^S$, which is lower than $\bar{Y}^*$. Now that $\bar{Y}^S < \hat{Y}^S$, wage rates do not exceed θ and the economy remains trapped in the regime.

It is plausible to state that among the above three cases, Assumption (A4) yields the lowest degree of inequality, whereas Assumption (A4') with $\lambda = \lambda^S$ yields the highest. Hence, Propositions 2-3(*a*) reveal that highly or relatively egalitarian economies grow towards the steady-state equilibrium where $Y_t = \bar{Y}^*$ and $(b_t^R, b_t^P) = (\bar{b}^*, \bar{b}^*)$, despite the possibility of initially experiencing Regime 1.[41] On the other hand, inegalitarian economies are unable to converge to this steady state according to Proposition 3(*b*) and Lemma 3. Nevertheless, the theorem below shows that they grow faster than the others in early stages of development.

Theorem 1 (Overtaking) *Under (A2)-(A3), consider a group of countries that differ only in their initial income distributions. Less egalitarian countries may achieve faster output growth in the short run, depending on the initial output. However, they tend to end up with lower growth paths.*

[41] Since the economy in Proposition 3(*a*) has a relatively large λ and $\varepsilon = 0$, it would be plausible to say that it is more egalitarian than the economy in Proposition 3(*b*).

Proof. Suppose that λ is as small as λ^S. Then $\underline{Y}^S < \underline{Y}^* < \bar{Y}^S < \hat{Y}^S$ noting that $\bar{b}^S < \hat{b}^S$ and (A3). Thus given $Y_0 \in (\underline{Y}^*, \bar{Y}^S)$ and $\varepsilon = 0$, it is found that $\underline{Y}^S < \Phi^{1S}(Y_0) = Y(B_0, 0, \lambda^S) < \bar{Y}^S$, implying (A4'). On the other hand, given this level of Y_0 and a sufficiently large ε, (A4) is obtained (for any λ) by noting Lemma 3. The theorem therefore follows from Propositions 2-3 and Figures 3-4. □

The theorem reflects a reversal of the qualitative effects of inequality on factor accumulation in the process of development. As explained earlier, inequality has two opposing effects on factor accumulation: it promotes aggregate savings and thus physical capital accumulation, while constraining the spread of educational investment.[42] While the capital-enhancing force is initially dominant, it dissipates at high levels of wages (and thus output). Since, by contrast, the negative effect of inequality increases with output, their combined effect is negative at $\hat{Y}^*(< \bar{Y}^*)$ as shown in (35). This is why the potential steady-state output level in Regime 1, $\bar{Y}(\lambda)$, is lower than the steady-state output level in Regime 3, $\bar{Y}^*$.

Note that the reversal of the combined effect of inequality does not assure that $\bar{Y}(\lambda)$ is the actual steady-state output in Regime 1. It is the case if the initial conditions are characterized by (A4') and $\lambda = \lambda^S$; such high inequality significantly delays the accumulation of aggregate human capital and thereby generates the steady-state level $\bar{Y}^S$ below the take-off level $\hat{Y}^S$, as shown in Figure 3. Highly inegalitarian economies therefore converge to the underdeveloped steady state in Regime 1 while being overtaken by egalitarian economies converging to the developed steady state in Regime 3.

This phenomenon—overtaking followed by divergence—is likely to occur among countries characterized by a high marginal productivity of physical capital. In those countries a large value of α (i.e. disparity in returns between capital and labor) intensifies the adverse effect of credit constraints by delaying the growth in wages relative to output. As a result, the steady-state level $\bar{Y}(\lambda)$ tends to be lower than the take-off level $\hat{Y}(\lambda)$.[43] It should be noted that the share of labor income is less important for egalitarian economies where many individuals obtain asset earnings as well as wages.

In contrast to the above case, an economy satisfying (A4') and $\lambda = \lambda^L$ may catch up with the leaders after being overtaken. Such moderate inequality mitigates the adverse effect of credit

[42] The adverse effect of credit constraints is reflected in the expanding difference between e_t and $e_t^P(= 0)$ in the growth process.

[43] This is confirmed by the fact that if α is sufficiently large, $\hat{b}(\lambda)$, but not $\bar{b}(\lambda)$, goes to infinity.

constraints, and thus $\bar{Y}^L$ becomes higher than $\hat{Y}^L$ as depicted by Figure 4. Hence, although the initial state is Regime 1, output exceeds $\hat{Y}^L$ at some time and grows toward the highest steady-state level $\bar{Y}^*$ in Regime 3.

Lastly, the developed theory implies that convergence and divergence (without overtaking) are attributable in part to the initial income distributions of the countries concerned: Similarity in this respect leads them to similar growth paths in the long run, whereas dissimilarity propels semi-developed countries to diverge from each other. Such divergence is explained by the fact that in intermediate development stages, sufficiently high wages diminish the saving-rate differential between the rich and the poor, nullifying the capital-enhancing force of inequality.[44]

6 Concluding Remarks

This research has developed a theory about the role of income inequality leading to one country overtaking another in terms of economic performance. The theory highlights two opposing effects of inequality on factor accumulation. On the one hand, concentrating wealth in the hands of a small group promotes physical capital accumulation, due to the convex behavior of household savings with respect to income. On the other hand, such inequality, together with borrowing constraints, acts as a barrier to widespread investment in human capital—a prerequisite for sustained growth. The former effect works only in early development stages where wages are low, while the latter increases with returns on education and becomes more significant under wider inequality. The resultant qualitative change in their combined effect permits egalitarian countries to overtake highly inegalitarian countries, which remain underdeveloped.

The essential assumptions for the overtaking phenomenon above are the convexity of the saving function, the concavity of the human capital function, and borrowing constraints on individuals' education decisions. Since the convexity limits the saving-enhancing effect to underdeveloped stages with low wages, semi-developed countries will diverge if their degrees of initial inequality are diverse. By contrast, countries with sufficient initial resources converge to similar growth paths if they have similar degrees of inequality. Hence, the initial levels of inequality as well as of output play a significant role in determining a country's growth pattern.

[44] Income divergence is not predicted by Galor and Moav (2004), as they deal with only marginal changes in initial inequality.

The established theory confirms the political difficulty of implementing a drastic redistribution in reality, by showing that egalitarian policies are undesirable from the viewpoint of the rich even though they maximize the long-run aggregate output. This paper does not go into how to reach a compromise between them. This topic is left for future research.

Appendix

Proof of Equation (23). Noting that $R_{t+1} = r_{t+1}$, the first order condition (8) becomes

$$k_{t+1}h'(e_t) = \frac{\alpha}{1-\alpha} \qquad \text{for } k_{t+1} > 0.$$

In light of this result and (17), the implicit function theorem yields

$$\begin{aligned} k_B(B_t, b_t^P, \tau_t) &= [H_{t+1} + \tau_t e'(k_{t+1})/(1-\alpha)]^{-1} > 0; \\ k_b(B_t, b_t^P, \tau_t) &= -(1-\tau_t)[h'(b_t^P)k_{t+1} + 1]k_B(B_t, b_t^P, \tau_t) \leq 0. \end{aligned}$$

It follows that

$$\begin{aligned} \partial k_{t+1}/\partial b_t^R &= \lambda k_B(B_t, b_t^P, \tau_t); \\ \partial k_{t+1}/\partial b_t^P &= (1-\lambda)k_B(B_t, b_t^P, \tau_t) + k_b(B_t, b_t^P, \tau_t). \end{aligned}$$

Now, (7) and (22) yield that for a given $k > 0$,

$$\begin{aligned} I_b(b_t^i, k) &= \begin{cases} w(k)h'(b_t^i) & \text{for } b_t^i < e(k) \\ R(k) & \text{for } b_t^i \geq e(k); \end{cases} \\ I_k(b_t^i, k) &= \begin{cases} w'(k)h(b_t^i) & \text{for } b_t^i < e(k) \\ (1-\alpha)R(k)[h(e(k)) - (b_t^i - e(k))/k] & \text{for } b_t^i \geq e(k). \end{cases} \end{aligned} \tag{38}$$

This, together with (17) and (19), yields that

$$I_k(b_t^R, k_{t+1}) \leq 0 \qquad \text{and} \qquad I_k(b_t^P, k_{t+1}) \geq 0 \qquad \forall k_{t+1} > 0, \tag{39}$$

where equalities hold only if $b_t^R = b_t^P$. These properties establish (23), noting that $\lambda(b_t^R - e_t) \leq K_{t+1}$ in (15). □

Proof of Lemma 1. When $k_{t+1} = k(\tau b_t^R, 0, \tau)$, where $b_t^P > 0$ and $\tau \in (0, 1]$, k_{t+1} is increasing in both b_t^R and τ. Thus by noting (27), (38) and (39), one finds that $\chi'(\tau) < 0$, where $\chi(\tau) \equiv k(\tau\bar{b}(\tau), 0, \tau)$, if and only if $\beta R(\chi(\tau))$ is slightly greater than 1. Since (25) implies that $\beta R(\chi(\tau)) < 1$ if τ is sufficiently close to 1, it is infeasible to have $\chi'(\tau) < 0$ for any $\tau \in (0, 1]$. On the other hand, $k(\tau\hat{b}(\tau), 0, \tau)$ is constant for all τ, and hence the result follows. □

Proof of Lemma 2. (a) Consider the region for Regime 2 in Figure 2. Due to the properties of $I^i(b_t^R, b_t^P)$, the BB^S and BB^L loci starting at $(\hat{b}^S, 0)$ and $(0, \hat{b}^L)$, respectively, are non-positively sloped. Hence, the result follows from Lemma 4 below, noting that the isoquants of the capital-labor ratio are non-negatively sloped.

(b) As follows from (17) and (39), $I(b_t^R, k(\lambda b_t^R, 0, \lambda))$ strictly increases with λ. In addition, the proof of Lemma 1 above implies that $\bar{b}(\tau)$ is continuous in $\tau \in (0, 1]$. Hence (27) and the definition of $\bar{b}$ yield the result.

(c) Let $B > 0$ be the aggregate (or equivalently, average) transfer in a steady-state equilibrium in Regime 3. Then the associated capital-labor ratio is $k \equiv k(B, 0, 1)$. Noting that $\phi_b(b_t^i, k) = \beta R(k)$ for $b_t^i \geq e(k)$, one can find a pair (b^R, b^P) such that $b^i = B = \phi(b^i, \dot{k}) > 0$. In this case B equals $\bar{b}^*$ or $\underline{b}^*$, as (25) and ($A1$) show that there are only two nontrivial egalitarian steady-state equilibria. Noting the fact $\beta Y_B(B, 0, 1) = \beta R(k) \neq 1$ reveals that there is no other nontrivial steady state in Regime 3. □

Lemma 4 *$b_{t+1}^P > b_t^P$ if $b_t^P \leq \bar{b}^*$ and $k_{t+1} > \hat{k}$.*

Proof. Recall that $\varphi(0, \hat{k}) = w(\hat{k}) - \theta = 0$ and let $\tilde{k}$ be the value such that $\beta R(\tilde{k}) = 1$. It then follows from (38) that for a given $k > \max(\hat{k}, \tilde{k})$, there exists a unique value of b_t^P for which $\phi(b_t^P, k) = b_t^P$ and below which $\phi(b_t^P, k) > b_t^P$. Also, if $k \in (\hat{k}, \tilde{k})$ then $\phi(b_t^P, k) > b_t^P\ \forall b_t^P \geq 0$. Now note that (25) and (38) yield $\phi_b(\bar{b}^*, \bar{k}^*) = \beta R(\bar{k}^*) < 1$ and $\phi_k(\bar{b}^*, \bar{k}^*) = 0$, where $\bar{k}^* = [\bar{b}^* - e(\bar{k}^*)]/h(e(\bar{k}^*))$. This result, together with (38), assures the existence of a continuous single-valued function $b_t^P = \mu(k)$ for $k > \tilde{k}$, such that $\phi(b_t^P, k) = b_t^P \geq \bar{b}^*$. The lemma therefore follows. □

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Notes of Figures

Figure 1. The Credit-Constraint Frontier (The CC Locus). On the frontier, $b_t^P = e_t$, meaning unbinding credit constraints on and no savings made by group P.

Figure 2. The Evolution of Transfers. The regions below and above the 45° line, respectively, depict the evolution of the transfers for $\lambda = \lambda^S$ and $\lambda = \lambda^L$. The pair (b_t^S, b_t^L) converges to one of the points $(0, 0)$, $(\bar{b}^S, 0)$ or $(\bar{b}^*, \bar{b}^*)$, depending on the initial amount and allocation of aggregate transfers.

Figure 3. The Evolution of Output for $\lambda = \lambda^S$. There exists a locally stable steady-state equilibrium in both Regimes 1 and 3. In the early stages of development, an inegalitarian economy operates in Regime 1 and produces higher output than a more egalitarian economy in Regime 3. However, the former's output is unable to reach the take-off level $\hat{Y}^S$, thus converging to the lower steady-state level $\bar{Y}^S$ in Regime 1.

Figure 4. The Evolution of Output for $\lambda = \lambda^L$. Unlike $\bar{Y}^S$ in Figure 3, $\bar{Y}^L$ is not a steady-state level of output. Hence, output of an inegalitarian economy exceeds the take-off level $\hat{Y}^L$ (i.e. departure from Regime 1), converging to the steady-state level $\bar{Y}^*$ in Regime 3.

Figure 1

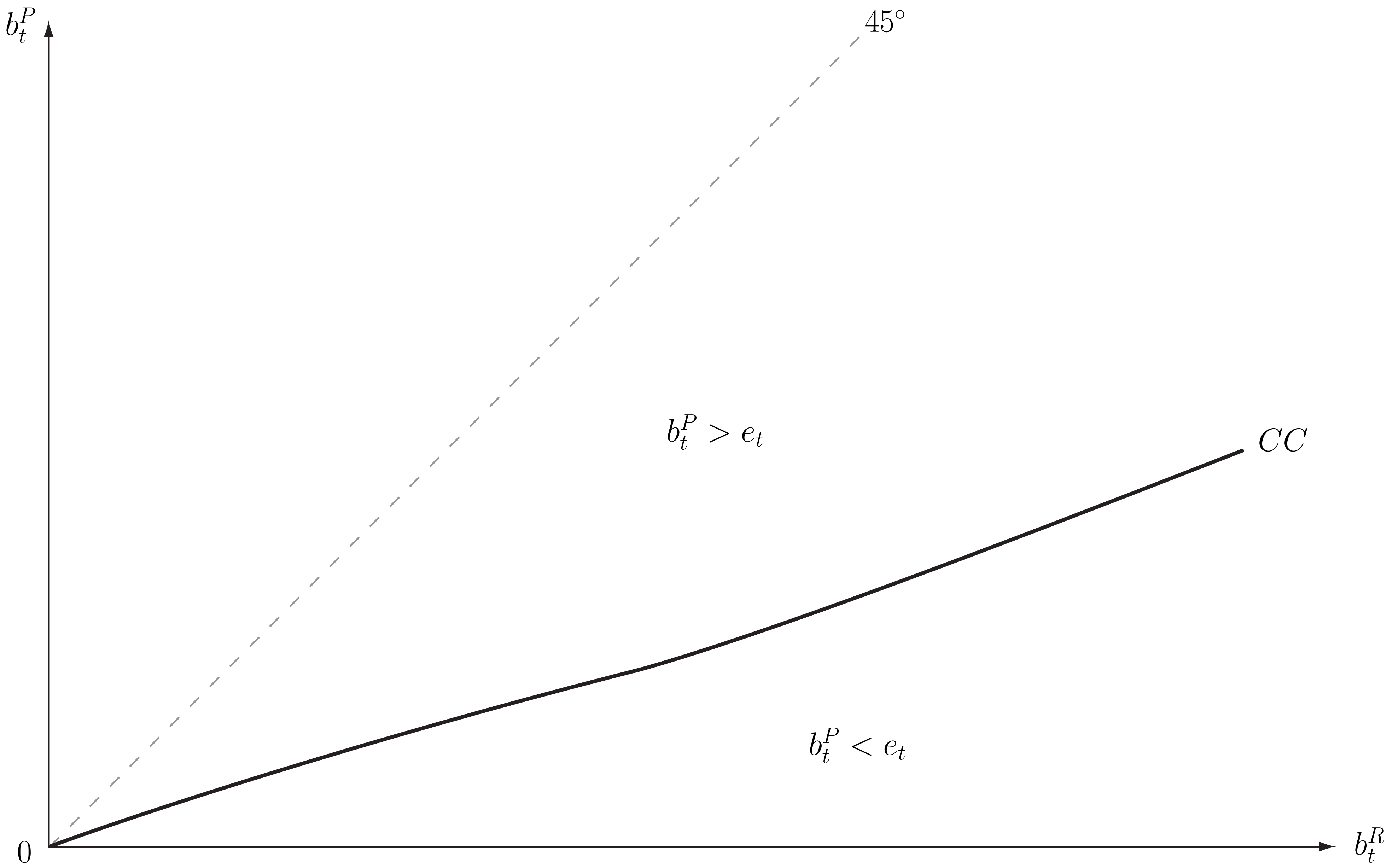
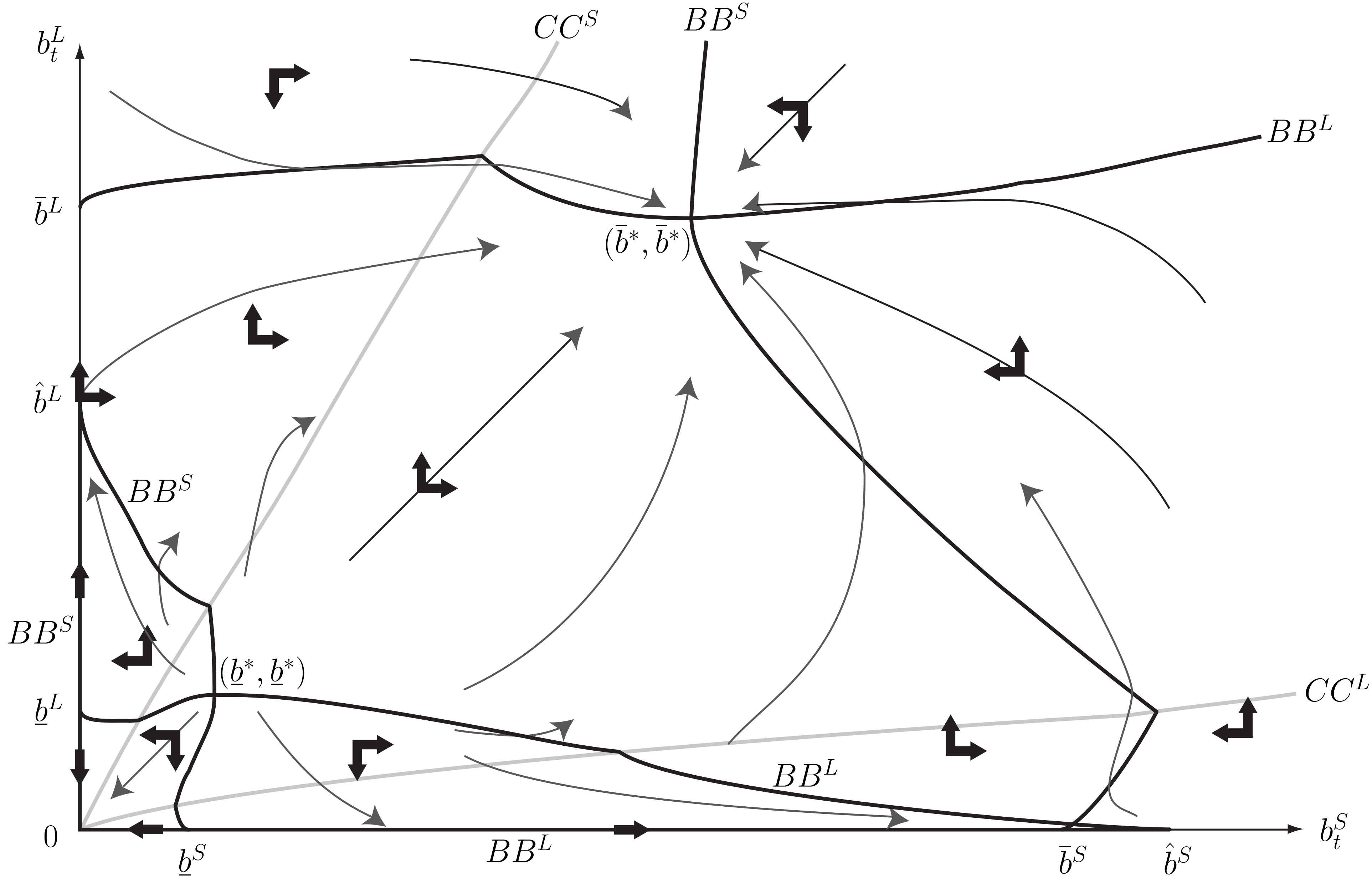
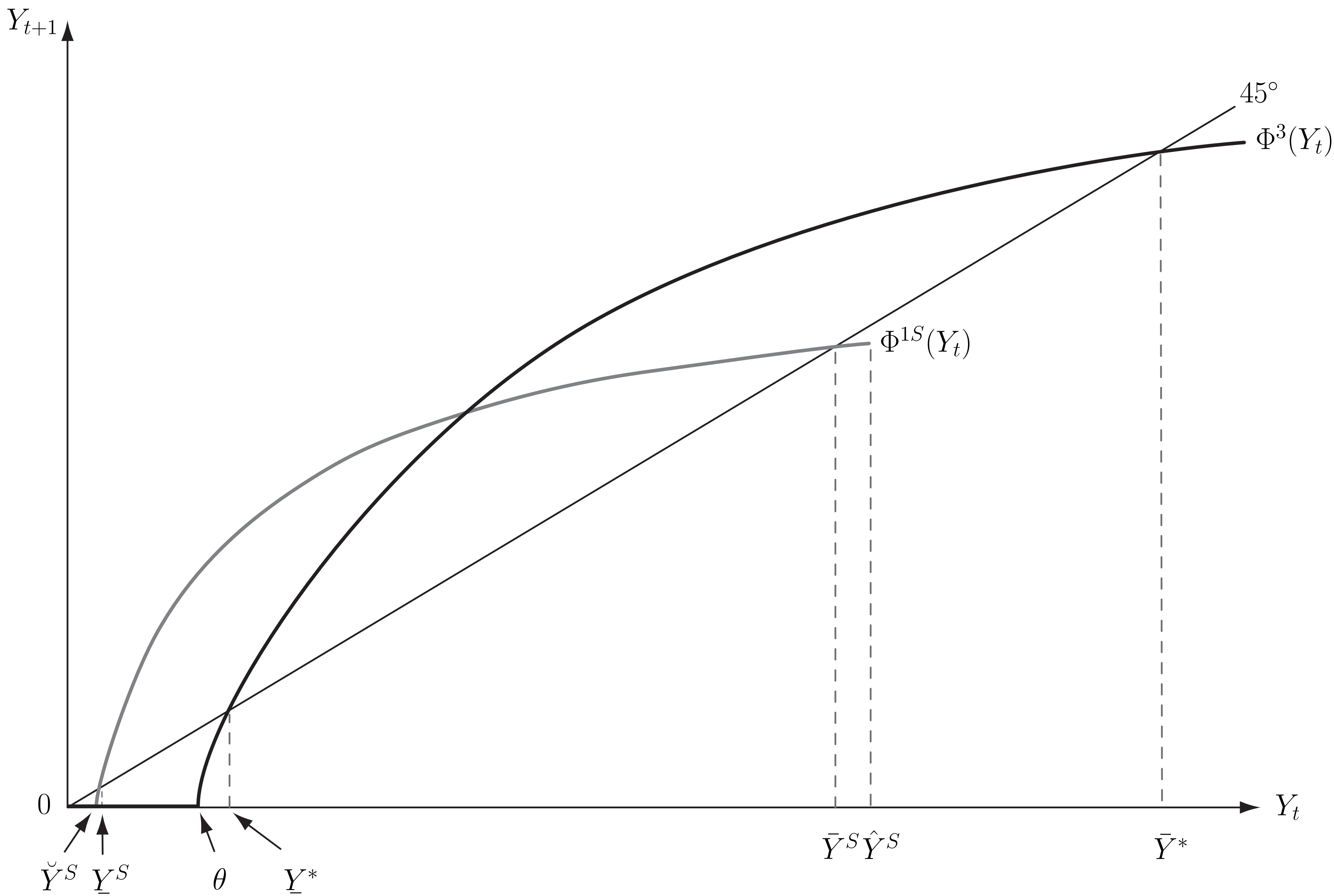


Figure 2



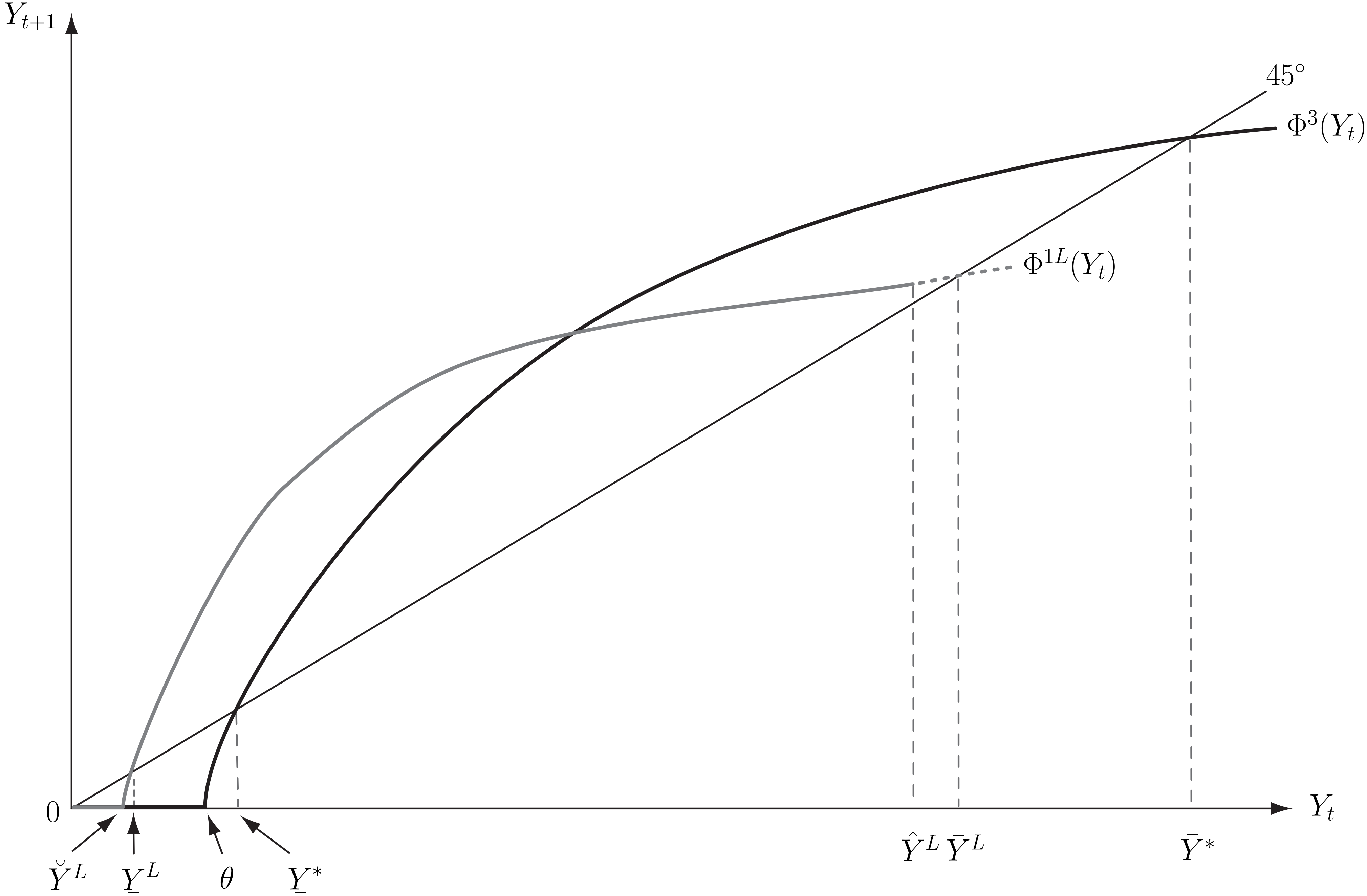
b_t^L
CC^S
BB^S
BB^L
$\bar{b}^L$
$(\bar{b}^*, \bar{b}^*)$
$\hat{b}^L$
BB^S
BB^S
$(\underline{b}^*, \underline{b}^*)$
$\underline{b}^L$
CC^L
BB^L
0
$\underline{b}^S$
BB^L
$\bar{b}^S$
$\hat{b}^S$
b_t^S

Figure 3



Y_{t+1}
Y_t
0
45°
$\Phi^3(Y_t)$
$\Phi^{1S}(Y_t)$
$\breve{Y}^S$
$\underline{Y}^S$
θ
$\underline{Y}^*$
$\bar{Y}^S$
$\hat{Y}^S$
$\bar{Y}^*$

Figure 4



Y_{t+1}
$45°$
$\Phi^3(Y_t)$
$\Phi^{1L}(Y_t)$
0
Y_t
$\breve{Y}^L$
$\underline{Y}^L$
θ
$\underline{Y}^*$
$\hat{Y}^L$
$\bar{Y}^L$
$\bar{Y}^*$