Business Cycle Model on the Basis of Method of Systems Potential.

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A brief description of Method of Systems Potential (MSP) was given in the previous report [8]. Deduction of evolution equations for the normal evolution[1] of Complex Adaptive Systems (CAS) was considered. We assumed that the Economic System is some Complex Adaptive System[2] which evolve on the basis of universal laws of such systems. These laws were considered in paper [8].

The goal of this short message is to demonstrate how abstract ideas and conclusions of MSP-approach can be successfully applied in the economic theory. Interpretation of basic system variables and evolution parameters of MSP as applied to the economic CAS would be given below. Solution of this problem leads to reformulation of evolutionary equations in terms of the economic variables and economic parameters. Such reformulation of CAS-structure and CAS-dynamics in economic terms gives some new explanation of the Economic System dynamics. The new business cycle model follows from such reformulation. We show that this model can be interpreted as some quantitative version of with well-known N.Kaldor' [5] model.

Let us choose the following state-variables of CAS: "potential", Φ, "conditions of realization", U, "realizable part of potential", Φ_R. MSP evolutionary equations in these variables turn into the following equations:

$$\dot{\Phi}_R = a \cdot \Phi_R ; \tag{1}$$

$$\dot{\Phi} + d \cdot \Phi = (a + d) \cdot \Phi_R , \tag{2}$$

$$\dot{U} + \Lambda \cdot U = \nu \cdot \Phi_R , \tag{3}$$

To apply these equations to the Economic System abstract values, Φ, U, and Φ_R should be connected with some macro-economical variables characterizing the properties of economy as united system. Such properties could be for example output Y, fixed capital, K, human capital, and (or) some other characteristics of the Economic System.

We start from considerations of simplicity and naturalness. Our deductions and reasoning were built in the following way. Activity of the Economic System is a process of production[3]. Economical index reflecting value of production activity is the gross output. As the value of production activity depends directly upon realizable part of economic potential we came to the following conclusions:

Supposition 1: *Issue of final product (gross output) could be considered as economical index of realizable part of the Economic Potential.*

$$\Phi_R = Y . \tag{4}$$

The definite value of costs can be used more or less effectively i.e. these costs can ensure more or gross output in economics. We suppose that there is the only one

[1] The 'normal evolution' of Complex Adaptive System is such way of development when only two forces determinate the process: (1) activity of a System as only source of increments in 'potential' and 'conditions of realization' of this System and (2) entropy influence as destroying force [8].

[2] Determination of CAS and description of its structure and function is given in paper [8].

[3] We use this term in a very wide sense i.e. "production" here include any economically necessary activity of economic agents – trade, marketing and management in particularly.

highest possible gross output with certain given expense for each value of expenses. This highest possible (or 'potential') output characterizes maximal ability of given Economic System to produce the gross output[4]. It follows that:

Supposition 2: Potential output, $Y_{\max}$, is an economical index of Potential of the Economic System:

$$\Phi = Y_{\max} . \qquad (5)$$

Finally last supposition concerns the economical interpretation of term "conditions of realization".

Supposition 3: *Value of fixed capital, K, could be considered as index of "conditions of realization" in the Economic System.*

This statement requires explanations.

First, supplies of fixed capital in the country with the capitalistic economy are necessary conditions of productive application of available country's resources.

Thus, low developed countries that own supplies of resources could not use them because of absence of necessary supplies of capital. Involving of any resources to the production process requires some definite capital investments. In this sense the capital is condition of productive application of these resources, i.e. condition of realization of available economical potential.

Second, the equation (3) describes the process of accumulation of some value. Formally this equation looks the same as equation of accumulation of fixed capital if identify parameter Λ with depreciation rate, and parameter ν with norm of gross investments:

$$\dot{K} + \Lambda \cdot K = \nu \cdot Y . \qquad (6)$$

Third, statistics testifies that capital productivity $\frac{Y}{K}$ is stable enough value during long periods of time. For example in USA economy this ratio in fact did not changed from 1909 to 1929, and in average was equal approximately ≈ 0.3. Constance of this ratio lies in the base of acceleration principle of contemporary economic theory.

This allows write down the following approximate equality:

$$\frac{\dot{K}}{K} \approx \frac{\dot{Y}}{Y} = a . \qquad (7)$$

From the equation (6) follows that capital productivity could not change much indeed, and it is approximately equal:

[4] Let's indicate also that potential output depends on the technological base of economy. Non-optimal resources allocation and other factors that prevent to full capacity utilization must be taken into account also. We do not discuss here these nuances in determination of potential output. Index of capacity utilization (widespread in the modern statistics as the index of business activity) and realization ration of economic CAS are almost identical terms. Logistic growth of realization ratio in MSP-model of Economic System dynamics indicates that degree of utilization of technical and organizational basic innovations must be taken in account. Our definition of potential output means the full utilization and optimal coordination of all production factors (including the technological factor also).

$$\frac{Y}{K} \approx \frac{a+\Lambda}{\nu}. \qquad (8)$$

Forth, the equation for the fixed capital accumulation is essential for any economical model. But we have only three evolution equations. As we identified Φ and Φ_R with $Y_{\max}$ and Y respectively, we could follow one of two ways: to identify the equation of accumulation of fixed capital with equation for value U or consider the equation of capital accumulation as an additional fourth evolutionary equation. But that should mean unjustified expanding of system of evolutionary equations.

Finally we did not found any other economical interpretation of value U. This value, for example could not be identified with human resources of the country, L, because productivity of labor as distinct from capital productivity is permanently increasing value while we should obtain the stable productivity of labor by substitution $K \to L$ in formula (6) if rate of growth of labor is constant.

According to the listed-above suppositions evolutionary **equations of MSP lead to the following economic model of long-run dynamics of the Economic System.**

1. Equation of the growth of gross output with the constant rate of growth:

$$\dot{Y} = a \cdot Y; \qquad (9)$$

2. Equation for fixed capital accumulation:

$$\dot{K} + \Lambda \cdot K = \nu \cdot Y \qquad (10)$$

3. Equation for adjustment of actual to potential output:

$$\frac{d[Y_{\max} - Y]}{dt} = -d \cdot [Y_{\max} - Y]. \qquad (11)$$

The last equation is only the other form of record of equation (2) taking into account the equation (9). Equations (9)-(11) describe the definite economical model. The basic properties of this system of equations were considered in paper [8]. These equations are completely equivalent to evolution equations of MSP.

Evolution parameters of the Economic CAS have very simple interpretation:

ν - Rate of gross investment;

a - Rate of growth of output;

Λ - Depreciation rate of fixed capital;

d - Adjustment coefficient in actual-potential output interaction.

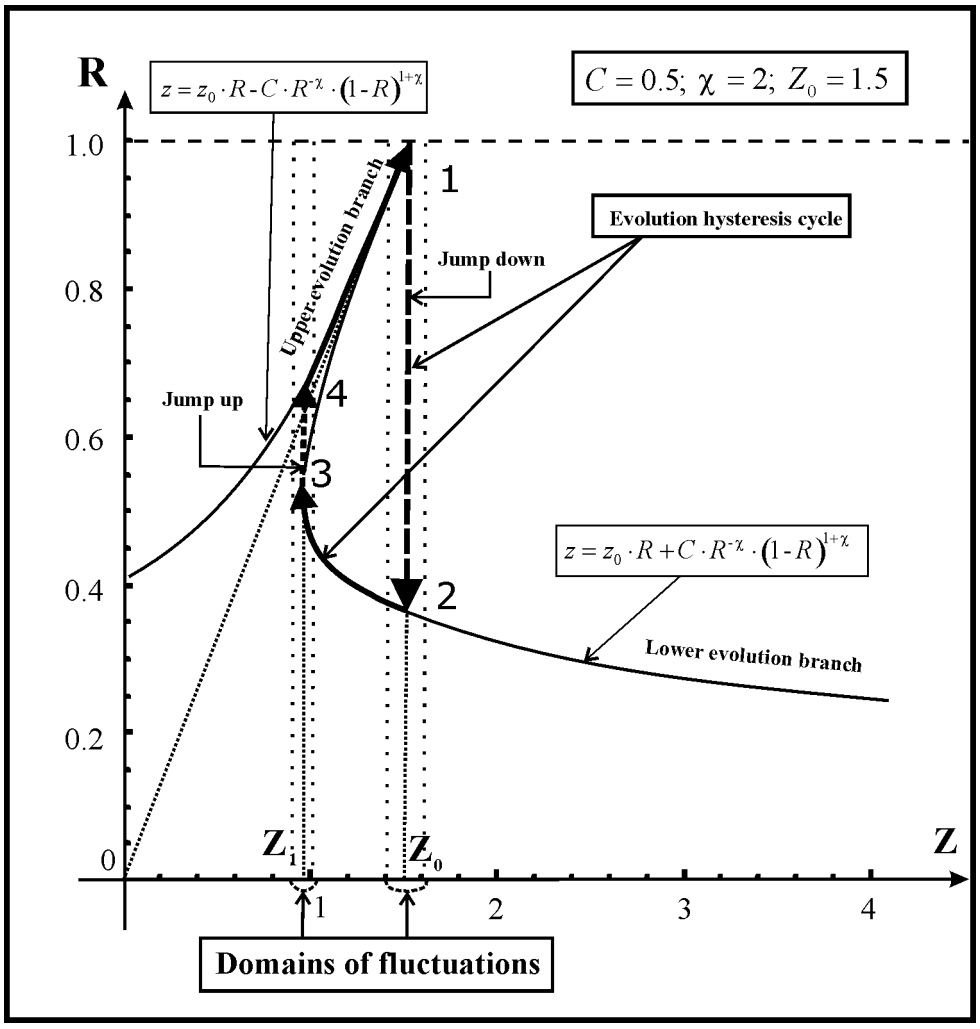

Figure 1. The business cycle of the Economic System according to MSP-approach.

All the properties of dynamics of CAS evolutionary equations solutions transfers to the properties of dynamics of solutions of economic model (9)-(11). Particularly when $\Lambda > d$ the cyclical dynamics of economic variables taking place.

The index of efficiency, R, and wealth-density z, of the Economic System satisfy to the following relations:

$$R = \frac{Y}{Y_{\max}}; \tag{12}$$

$$z = \frac{K}{Y_{\max}}. \tag{13}$$

Efficiency grows as logistic function[5]:

$$\dot{R} = (a + d) \cdot R \cdot (1 - R). \tag{14}$$

[5] This formula follows from (9) and (11). Logistic law means that potential output of our model includes the full capacity of technological factor of growth. Just such form of dependence corresponds to process of diffusion of radical innovations in economy. Many authors [1; 4; 7] proposed to approximate the long-term efficiency growth by logistic function. MSP explains why just such law of growth acts.

Efficiency, R, is the function of wealth-density of a system. This function satisfy to the following ordinary differential equation[6]:

$$R'_z \cdot \left[\left(\nu - (a+d) \cdot z \right) \cdot R + (d - \Lambda) \cdot z \right] - (a+d) \cdot R \cdot (1-R) = 0 . \qquad (15)$$

The graph of efficiency index has the shape of sharp lopsided ridge, and the top of it is the point of long-run equilibrium of a system. But this is the unstable point. That is why the system when reaches the ridge, jumps to the lower evolution branch and after that moves along the lower branch. Point, z_1, is the second unstable point of a system. In this point the system jumps to the upper evolutionary branch and moves along the upper branch to the unstable point, z_0 [7]. This cycle recurs.

Cycles composed from phases of smooth changes and catastrophic jumps are described in H. Varian' [9; 10] model of business cycles. But in his modernizing version of N. Kaldor's [5] model cycle does not recur because the long-term equilibrium point corresponds to the *stable* state. Exogenous shock is the necessary condition for generation of the cyclical process in H. Varian' model. Later George, D. [3] suggested to change the H. Varian's model in such a way that the equilibrium point becomes unstable. Cycles with unstable equilibrium point *formally* could be completely identified with the cycles generating in a system described by equations (9)-(11).

But in all other respects with the exception of the formal similarity our model strongly differs from these models.

Hysteresis loop in the plane $\left(\frac{K}{Y_{\max}}; \frac{Y}{Y_{\max}} \right)$ after the scaling transformation transforms into the classical N. Kaldor' loop of the business cycle in the plane $(K;Y)$[8].

The business cycle in MSP-model has the same properties as the typical business cycle according to W. Mitchell' [6] investigations. These properties were listed and discussed in article [8]. Besides that it is asymmetric cycle that is also the property of typical business cycle according to G. Gabish' and H. Lorentz' examination [2].

Note that duration of different phases of cycle in our model depends on fluctuations of evolutionary parameters and other disturbances of the Economic System (exogenous shocks). Duration of prosperity phase depends on perturbations

[6] Details of derivation and solution of this equation are contained in paper [8].

[7] Instability of these points is the direct consequence of the existence of mechanisms that stabilize the steady state of a system. Deviation of a system from point of evolution branch (which correspond to its steady state) activates the forces which tend to come back the system. In neighborhood of points, z_1 and z_0 this forces generate catastrophe jumps in efficiency of a system (revival and crisis phases of the cycle).

[8] Moreover MSP-model like to N.Kaldor' in some other aspects. Three equilibrium points (investment = saving) of N.Kaldor' model correspond to three possible values of efficiency for any $z_1 < z < z_0$. Regulating force in N.Kaldor' model (excess investment over saving) is equivalent of MSP stabilizing mechanisms that come back the system into its steady state. Cycle in both models consists of two catastrophe jumps and two phases of gradual change. Thus MSP-model can be interpreted as some quantitative version of N.Kaldor' model.

of a system very strongly since the duration of this phase tend to infinity as perturbations of a system tend to zero. Thus this cycle unites the properties of determined and stochastic processes that make the dynamics of MSP-model very complicated and multiform.

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