Dynamic Principal-Multiple Agent Contracts

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Abstract

I explore the nature of optimal static and dynamic contracts in an environment with moral hazard, where individuals contracting with the same principal receive correlated productivity shocks. The environment resembles the one considered in relative compensation theory (i.e tournament theory), but extends this theory by solving for the optimal static and dynamic contracts in this setting. I compute and analyze *independent* (each worker's compensation depends only on her output) and *relative* contracts (each worker's compensation depends on the outputs of all workers contracting with the same principal). Results imply that the optimal static relative contract is not substantially different from the optimal static independent contracts. However, the dynamic relative contract displays a strong a tournament feature; the contract gives the highest compensation to the worker who produces more than her counterparts and the lowest compensation to the least productive worker. I also characterize the stochastic processes for consumption and effort implied by dynamic contracts, and study the age-earnings profiles of the workers.

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1 Introduction

This paper analyzes the nature of optimal static and dynamic contracts in an environment with moral hazard, where workers can take private actions that affect their productivity and receive a common productivity shock. The environment considered resembles the environments of relative compensation theory (tournament theory), initiated by Lazear and Rosen (1981). Lazear and Rosen compare three compensation schemes in a two-worker model: a linear piece rate, comparison against a fixed standard and a rank-order tournament. In the first, each worker's compensation is a linear function of his output; in the second contract, each worker receives one of the two fixed payments depending on output being above or below a specified standard. In the third contract, workers participate in a tournament with two fixed prizes, awarded on the basis of rank order of their output levels. They show that if workers are risk-averse, and the variance of the common shock is relatively large, workers would prefer tournaments to the other contracts. Lazear and Rosen provide two reasons for considering rank-order tournaments as compensation schemes. They argue that first, it may be difficult to observe absolute output and second, relative comparisons difference out common noise that risk-averse workers may not like.

Lazear and Rosen's work has prompted a substantial amount of research, from empirical analyses of the application of rank-order tournaments to executive pay[1], to theoretical analyses of the efficiency of rank-order tournaments[2]. However, two very important questions remain. First, what exactly is the nature of the optimal contract in this setting? Namely, what is the nature of the optimal contract in environments with moral hazard where multiple workers contracting with the same employer receive correlated productivity shocks? Although Lazear and Rosen com-

[1]See, for example, Bull, Schotter and Weigelt (1987), Ehrenberg and Bognanno (1990), Main, O'Reilly and Wade (1993).

[2]Green and Stokey (1983) extend Lazear and Rosen's work by examining the efficiency of rank-order tournaments in a multiple-worker environment where workers observe private signals correlated with the common shock. Their results imply that given any distribution for the common shock, for a sufficiently large number of workers, optimal tournament dominates optimal independent contracts.

pare compensation schemes that are widely observed, there is no theoretical reason to expect optimal contracts in this setting to resemble a piece rate or a rank-order tournament. The second question that remains to be addressed is the characterization of the optimal *dynamic* contract. So far the papers in relative compensation have looked only at static contracts. Dynamic contracts condition on the history of workers' performances and allow individual effort and consumption to vary over time. There is, however, a drawback to history dependence: it complicates the analysis considerably. In computing and analyzing the optimal dynamic contracts in this setting, I make use of some of the insights and techniques developed in the dynamic contracting literature. In particular, I formulate the dynamic problem as a recursive problem in which the expected lifetime utility of a worker is used as state variable.[3]

This paper extends relative compensation theory; it does so by applying and extending some of the methods developed in the dynamic contracting literature. The dynamic incentive models of Spear and Srivastava (1987) and Phelan and Townsend (1991), Green (1987), Phelan (1995) have explored the nature of the optimal dynamic contracts in environments with hidden information or hidden action, but these models have been primarily concerned with contracts between one principal and one agent. The models developed in this paper extend the dynamic contracting literature by characterizing the optimal dynamic contract between a principal and multiple agents, and by considering correlated productivity shocks. To compare the nature of the optimal contracts in this setting with tournament theory and dynamic contracting problems between a principal and a single agent, two types of contracts are characterized and computed in both the static and the dynamic settings. These are the optimal independent and the optimal relative contracts. Informally, an *independent contract* is a contract that conditions the compensation of a worker only on the output of that particular worker. In the presence of a common shock, in-

[3] In their seminal paper, Spear and Srivastava provide a representation of the dynamic contract that avoids the intractabilities associated with history dependence. Building on Spear and Srivastava's work, Phelan and Townsend develop a method for computing solutions to a dynamic contracting problem between a principal and a continuum of agents in a social planner environment.

dependent contracts between each worker and the principal ignore the information conveyed by the relative performance of the workers. A *relative contract*, however, conditions a worker's compensation on the outputs of all workers. The relative contracts considered are not constrained to be rank-order; the set of feasible contracts permits arbitrary dependence of a worker's compensation on the entire vector of performances.

I analyze the nature of these contracts in a two-worker, multiple-action, multiple-output environment. Each worker's output is a stochastic function of her effort level, an idiosyncratic shock and a common shock. The principal maximizes her surplus by offering lotteries over effort levels, output, consumption (in static and dynamic contracts) and future utility (in dynamic contracts). Section 2 characterizes the independent and relative static contracts and displays results for different parameterizations of the model. Section 3 develops the dynamic independent and relative contracts and displays and discusses results. Results suggest that the optimal static relative contract is not substantially different from the optimal static independent contract. However, the dynamic relative contract displays a strong a tournament feature; the contract gives the highest compensation to the worker who produces more than her counterpart and the lowest compensation to the less productive worker. Furthermore, the computed surplus functions of the principal display an interesting feature: at low utility promises, the surplus function is upward sloping. This feature of the surplus function has important implications for dynamic contracts. For workers who have, at any point in time, been offered the lower bound on utility as a future utility promise, there is an opportunity for renegotiating to receive a slightly higher utility, since this higher utility is Pareto-improving. Consequently, the principal cannot credibly commit to utility promises corresponding to the upward sloping portion of the value function. Section 3.3 proposes a method for constructing the set of dynamic independent "credible" contracts and reports some preliminary results. Section 4 characterizes the stochastic processes for effort and consumption implied by the optimal dynamic contracts and displays the age-earning profiles of workers. Section 5 concludes.

2 Static Contracts

2.1 Optimal Independent Contract

In this section, I consider the optimal independent contract between the principal and each worker. This section builds on the seminal work of Phelan and Townsend (1991). They examine the dynamic social planning problem for an economy with a continuum of agents and unobserved effort. I adapt their computational method to solve for the optimal independent contract in the presence of common shocks and extend it to solve for the optimal relative contract with two workers. The optimal independent contract is the solution to a programming problem subject to a worker receiving exactly a prespecified ex-ante expected utility, w. The principal chooses a contract that maximizes her surplus (consumption good produced, but not consumed by the worker), and the contract must deliver this required utility, w, to the worker. Total surplus of the principal is the sum of the surplus from each independent contract.

The workers have an identical stochastic production technology, described by an exogenous probability distribution $P(q|a,\eta)$, where η is an unobserved common shock, drawn independently over time with probability $G(\eta)$. $P(q|a,\eta)$ represents the probability of producing the output level q given the common shock η and the effort level a. It is assumed that higher effort levels imply higher expected output for any realization of the common shock η. For higher η, given an effort level, expected output is higher. At the beginning of each date, each worker takes an unobservable action a from a finite set of possible actions A. This action results in a publicly observable output $q \in Q$. After output is observed, the contract gives each worker some consumption c from the finite set C. It is assumes that the contractual rule for determining a worker's action and consumption may be probabilistic. The utility function of the workers, $U : A \times C \to \Re$, is additively separable in consumption and effort, $u(c) - v(a)$. The function $u(c)$ has the constant relative risk aversion specification, $\frac{c^{(1-\sigma)}}{1-\sigma}$. $v(a)$ is the disutility of effort and it is assumed to be strictly convex in a. For all else equal, the worker prefers lower action and higher consumption. The lowest ex-ante utility possible for a worker is that of receiving the lowest consumption in C and the lowest effort level in A with certainty. The highest ex-ante utility is that of receiving the highest consumption if C and the lowest effort level in A with certainty. These utility levels are denoted $\underline{w}$ and $\overline{w}$ respectively. Any utility level w between $\underline{w}$ and $\overline{w}$ can be achieved by randomizing between these two bounds, thus the set of possible ex-ante expected utilities is an interval $W = [\underline{w}, \overline{w}]$.

For each ex-ante expected utility $w \in W$, the principal's choice variable is written as $\Pi^w(a, q, c)$: the probability for a worker with utility requirement w of taking action a, producing q, and receiving consumption c today. For a given $w \in W$, a *contract* is a probability measure Π^w which satisfies the following constraints.

First the contract must implement the utility requirement of a worker. Namely, the expected utility delivered to a worker with required utility w is indeed w,

$$w = \sum_{A \times Q \times C} U(a, c)\ \Pi^w(a, q, c). \tag{1}$$

Since workers' efforts are not publicly observable, the contract cannot be conditioned on these effort levels. To induce workers to take the recommended actions, the contracts must provide incentives for workers not to deviate from these recommended actions. The next set of constraints ensure that the expected utility of obeying the recommendation is greater than that of each possible deviation. These constraints take the general form for all assigned and possible alternative action pairs $(a, \hat{a}) \in A \times A$,

$$\sum_{Q \times C} U(a, c)\{\Pi^w(c|q, a) \sum_{\eta} P(q|a, \eta) G(\eta)\}$$
$$\geq \sum_{Q \times C} U(\hat{a}, c)\{\Pi^w(c|q, a) \sum_{\eta} P(q|\hat{a}, \eta) G(\eta)\},$$

where $\Pi^w(c|q, a)$ is the conditional probability implied by $\Pi^w(a, q, c)$. The expression $\{\Pi^w(c|q, a) \sum_\eta P(q|a, \eta)\}$ is the probability of a given (q, c) combination given that action a is recommended and that this action is taken. Analogously, the expression $\{\Pi^w(c|q, a) \sum_\eta P(q|\hat{a}, \eta)\}$ is the probability of a given (q, c) combination given that action a is recommended and deviation action $\hat{a}$ is taken instead. By using the equality $\{\Pi^w(c|q,a)\sum_\eta P(q|a,\eta)G(\eta)\} = \Pi^w(q,c|a)$, one can solve for $\Pi^w(c|q,a)$ and substitute it into the previous equation. The incentive compatibility condition can now be expressed as,

$$\sum_{Q\times C} U(a,c)\ \Pi^w(q,c|a) \geq \sum_{Q\times C} U(\hat{a},c)\ \frac{\sum_\eta P(q \mid \hat{a},\eta)G(\eta)}{\sum_\eta P(q \mid a,\eta)G(\eta)}\ \Pi^w(q,c|a).$$

By multiplying both sides of the above equation by the marginal probability of action a (where $\Pi^w(a) = \sum_{Q\times C}\Pi^w(a,q,c)$), one can express the incentive compatibility condition as a linear function of the choice variable $\Pi^w(a,q,c)$. Consequently,

$$\sum_{Q\times C} U(a,c)\ \Pi^w(a,q,c) \geq \sum_{Q\times C} U(\hat{a},c)\ \frac{\sum_\eta P(q \mid \hat{a},\eta)G(\eta)}{\sum_\eta P(q \mid a,\eta)G(\eta)}\ \Pi^w(a,q,c) \tag{2}$$

The next set of constraints ensure that a given probability measure $\Pi^w(a,q,c)$ is consistent with the exogeneous probability distribution $P(q|a,\eta)$. Therefore for all $(\overline{a},\overline{q}) \in A\times Q$

$$\sum_C \Pi^w(\overline{a},\overline{q},c) = \sum_\eta P(\overline{q} \mid \overline{a},\eta)G(\eta)\sum_{Q\times C}\Pi^w(\overline{a},q,c). \tag{3}$$

Lastly, it is required that Π^w actually represent a probability measure, or,

$$\sum_{A\times Q\times C} \Pi^w(a,q,c) = 1 \text{ and for all } (a,q,c) \in A\times Q\times C\ \ \Pi^w(a,q,c) \geq 0 \tag{4}$$

For a given $w \in W$ a *static independent contract* is a function Π^w which satisfies constraints one through four. Let $\mathcal{F}_1$ denote the set of static independent contracts. The principal chooses a contract from the set $\mathcal{F}_1$ that maximizes her surplus, (amount of consumption goods produced, but not consumed by the workers). The *optimal static independent contracts* is the solution to the following programming problem:

$$S(w) \ = \max_{\Pi^w\in\mathcal{F}_1} \sum_{A\times Q\times C} (q-c)\ \Pi^w(a,q,c).$$

Given a utility requirement w, the incentive constrained problem defined above is a linear programming problem. A revised simplex algorithm can be used to solve for the optimal contract. The details of the computational method follows the discussion on relative contracts.

2.2 Optimal Relative Contract

The contract defined in the previous section delivers an expected consumption and utility to a worker independent of the contract offered to the other worker. The productivity shock η, however, affects both workers. If the contract delivers a high consumption for high output and a low consumption otherwise, it may be punishing the worker for receiving an adverse productivity shock beyond the control of the worker. On the other hand, if the contract gives high consumption for low productivity to insure workers against adverse common shocks, workers may not have enough incentives to exert high effort. Compensation schemes that base a worker's reward on the performance of her peers as well as her own performance, take advantage of the fact that the vector of output levels for the whole group is a source of information about the common shock and therefore a source of information about the effort levels.

An independent static contract is defined as a lottery over a worker's effort, output and consumption. A contract based on both workers' performance is a lottery over the pairs of efforts, outputs and consumptions. These contracts are not constrained to be rank-order; the set of feasible contracts permits arbitrary dependence of a worker's compensation on the entire vector of performances. The choice variable for the optimal relative contract is denoted by $\Pi^{w_1,w_2}(a_1, a_2, q_1, q_2, c_1, c_2)$: the probability for workers one and two with utility requirements (w_1, w_2) of taking actions a_1 and a_2, producing q_1 and q_2, and receiving consumptions c_1 and c_2 respectively. As in the independent contracts, the chosen probabilities must satisfy certain conditions.

First, the analogues of constraint 1, that a relative contract must deliver the utility requirements, w_1 and w_2, of each worker, or,

$$w_1 = \sum_{A^2 \times Q^2 \times C^2} U(a_1, c_1)\ \Pi^{(w_1,w_2)}(a_1, a_2, q_1, q_2, c_1, c_2), \tag{5}$$

$$w_2 = \sum_{A^2 \times Q^2 \times C^2} U(a_2, c_2)\ \Pi^{(w_1,w_2)}(a_1, a_2, q_1, q_2, c_1, c_2). \tag{6}$$

Second, the contract must be incentive compatible. The relative contract recommends an action for each worker, and the expected utility of obeying the recommendations must be greater than that of each possible deviation. The incentive compatibility constraints for worker 1 and 2 take the following form: for all assigned and alternative action pairs $(a_1, \hat{a_1}) \in A \times A$,

$$\sum_{A \times Q^2 \times C^2} U(a_1, c_1)\ \{\Pi^{(w_1,w_2)}(c_1, c_2 \mid q_1, q_2, a_1, a_2)\ \sum_{\eta} P(q_1 \mid a_1, \eta) P(q_2 \mid a_2, \eta) G(\eta)\}$$
$$\geq \sum_{A \times Q^2 \times C^2} U(\hat{a_1}, c_1) \{\Pi^{(w_1,w_2)}(c_1, c_2 \mid q_1, q_2, a_1, a_2)\ \sum_{\eta} P(q_1 \mid \hat{a_1}, \eta) P(q_2 \mid a_2, \eta) G(\eta)\}.$$

And for all assigned and alternative action pairs $(a_2, \hat{a_2}) \in A \times A$,

$$\sum_{A \times Q^2 \times C^2} U(a_2, c_2)\ \{\Pi^{(w_1,w_2)}(c_1, c_2 \mid q_1, q_2, a_1, a_2)\ \sum_{\eta} P(q_1 \mid a_1, \eta) P(q_2 \mid a_2, \eta) G(\eta)\}$$
$$\geq \sum_{A \times Q^2 \times C^2} U(\hat{a_2}, c_2) \{\Pi^{(w_1,w_2)}(c_1, c_2 \mid q_1, q_2, a_1, a_2)\ \sum_{\eta} P(q_1 \mid a_1, \eta) P(q_2 \mid \hat{a_2}, \eta) G(\eta)\}.$$

The expression $\{\Pi^{(w_1,w_2)}(c_1, c_2 \mid q_1, q_2, a_1, a_2)\ \sum_{\eta} P(q_1 \mid a_1, \eta) P(q_2 \mid a_2, \eta) G(\eta)\}$ is the probability of a given (q_1, q_2, c_1, c_2) combination given that actions a_1 and a_2 are recommended and that those actions are taken by workers 1 and 2 respectively. Analogously, the expression $\{\Pi^{(w_1,w_2)}(c_1, c_2 \mid q_1, q_2, a_1, a_2)\ \sum_{\eta} P(q_1 \mid \hat{a_1}, \eta) P(q_2 \mid a_2, \eta) G(\eta)\}$ is the probability of a given (q_1, q_2, c_1, c_2) combination given that actions a_1 and a_2 are recommended and deviation action $\hat{a_1}$ is taken by worker 1 instead.[4] By

[4] The expression $\{\Pi^{(w_1,w_2)}(c_1, c_2 \mid q_1, q_2, a_1, a_2)\ \sum_{\eta} P(q_1 \mid \hat{a_1}, \eta) P(q_2 \mid a_2, \eta) G(\eta)\}$ is the symmetric case for worker 2. Namely, it is the probability of a given (q_1, q_2, c_1, c_2) combination given that actions a_1 and a_2 are recommended and deviation action $\hat{a_2}$ is taken by worker 2.

solving for $\{\Pi^{(w_1,w_2)}(c_1, c_2 \mid q_1, q_2, a_1, a_2)$ and multiplying each side of the incentive compatibility conditions for worker 1 and 2 with the marginal probability of a_1 and a_2 respectively, one can express the incentive compatibility conditions as follows. For each recommended and alternative action pair $(a_1, \hat{a_1}) \in A \times A$,

$$\sum_{A\times Q^2\times C^2} U(a_1, c_1)\ \Pi^{(w_1,w_2)}(a_1, a_2, q_1, q_2, c_1, c_2)$$
$$\geq \sum_{A\times Q^2\times C^2} U(\hat{a_1}, c_1)\ \frac{\sum_\eta P(q_1 \mid \hat{a_1}, \eta) P(q_2 \mid a_2, \eta) G(\eta)}{\sum_\eta P(q_1 \mid a_1, \eta) P(q_2 \mid a_2, \eta) G(\eta)}$$
$$\times \Pi^{(w_1,w_2)}(a_1, a_2, q_1, q_2, c_1, c_2), \quad (7)$$

and for each recommended and alternative action pair $(a_2, \hat{a_2}) \in A \times A$

$$\sum_{A\times Q^2\times C^2} U(a_2, c_2)\ \Pi^{(w_1,w_2)}(a_1, a_2, q_1, q_2, c_1, c_2)$$
$$\geq \sum_{A\times Q^2\times C^2} U(\hat{a_2}, c_2)\ \frac{\sum_\eta P(q_1 \mid a_1, \eta) P(q_2 \mid \hat{a_2}, \eta) G(\eta)}{\sum_\eta P(q_1 \mid a_1, \eta) P(q_2 \mid a_2, \eta) G(\eta)}$$
$$\times \Pi^{(w_1,w_2)}(a_1, a_2, q_1, q_2, c_1, c_2). \quad (8)$$

The ratio $\frac{\sum_\eta P(q_1|a_1,\eta)P(q_2|\hat{a}_2,\eta)G(\eta)}{\sum_\eta P(q_1|a_1,\eta)P(q_2|a_2,\eta)G(\eta)}$ measures the likelihood of the occurance of an output pair (q_1, q_2) given actions a_1 and a_2 have been recommended, but actions a_1 and $\hat{a_2}$ are taken.

The last set of constraints are analogues of 3 and 4. First they ensure that a given probability measure $\Pi^{(w_1,w_2)}(a_1, a_2, q_1, q_2, c_1, c_2)$ is consistent with $P(q|a, \eta)$. Therefore for all $(\overline{a_1}, \overline{a_2}, \overline{q_1}, \overline{q_2}) \in A^2 \times Q^2$

$$\sum_{C^2} \Pi^{(w_1,w_2)}(\overline{a_1}, \overline{a_2}, \overline{q_1}, \overline{q_2}, c_1, c_2) = \sum_\eta P(\overline{q_1} \mid \overline{a_1}, \eta) P(\overline{q_2} \mid \overline{a_2}, \eta) G(\eta)$$
$$\times \sum_{Q^2\times C^2} \Pi^{(w_1,w_2)}(\overline{a_1}, \overline{a_2}, q_1, q_2, c_1, c_2). \quad (9)$$

Second, it is required that $\Pi^{(w_1,w_2)}$ actually represent a a probability measure, or,

$$\sum_{A^2\times Q^2\times C^2} \Pi^{(w_1,w_2)}(a_1,a_2,q_1,q_2,c_1,c_2) = 1$$

$$\text{and for all } (a_1,a_2,q_1,q_2,c_1,c_2) \in A^2 \times Q^2 \times C^2$$
$$\Pi^{(w_1,w_2)}(a_1,a_2,q_1,q_2,c_1,c_2) \geq 0. \quad (10)$$

For a pair of utilities (w_1, w_2), a *static relative contract* is a function $\Pi^{(w_1,w_2)}$ that satisfies constraints 5 through 10. Let $\mathcal{F}_2$ denote the set of static relative contracts. The principal chooses a contract from the set $\mathcal{F}_2$ that maximizes the joint surplus. The *optimal static relative contract* is the solution to the following programming problem:

$$S(w_1,w_2) \ = \ \max_{\Pi^{(w_1,w_2)}\in\mathcal{F}_2} \sum_{A^2\times Q^2\times C^2} (q_1+q_2-c_1-c_2)\ \Pi^{(w_1,w_2)}(a_1,a_2,q_1,q_2,c_1,c_2)$$

2.3 Computations and Results

This section displays results of the optimal independent and relative contracts. Workers' utility function is assumed to be of the following form:

$$U(a,c) = \frac{c^{1-\sigma}}{1-\sigma} - \gamma a^2,$$

where γ is the disutility of effort parameter. The set of feasible actions A is $\{0.2, 0.8\}$ and the output set is $Q = \{1, 2\}$. Consumption takes values between 0.1 and 2.25. The common shock can take one of the two possible values $\{low, high\}$ with probabilities $(0.5, 0.5)$. The technologies relating actions to the probability of each output, namely $P(q|a,\eta)$, for low and high values of the common shocks are:

low η

a	P(q=1)	P(q=2)
0.2	0.8	0.2
0.8	0.5	0.5

and

high η

a	P(q=1)	P(q=2)
0.2	0.6	0.4
0.8	0.2	0.8

For the results reported in this section, the set of utility requirements are determined by the finite action, A, and consumption, C, sets chosen above. For each combination of the risk aversion parameter σ and the disutility parameter γ, there is a set of utilities, W, that the contract can deliver. The upper bound of this set W is obtained by prescribing the upper bound on consumption and lower bound on action with certainty ($\overline{w} = U(\underline{a}, \overline{c})$) and the lower bound by prescribing the lower bound on consumption and the lower bound on effort ($\underline{w} = U(\underline{a}, \underline{c})$). For any utility $w \in W = [\underline{w}, \overline{w}]$, there exists a contract which delivers at least that utility. Figures 1-7 display expected consumption, effort and the maximized surplus as functions of w.

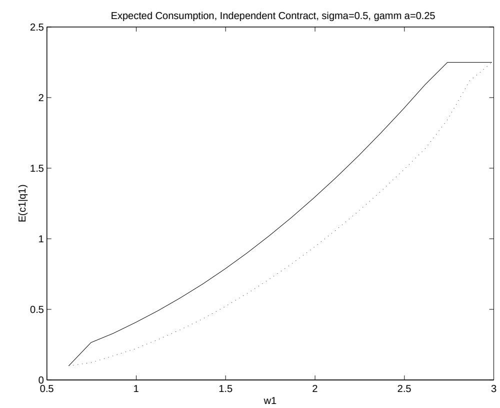

Figure 1: Independent contract

$\cdots$ q=1, — q=2

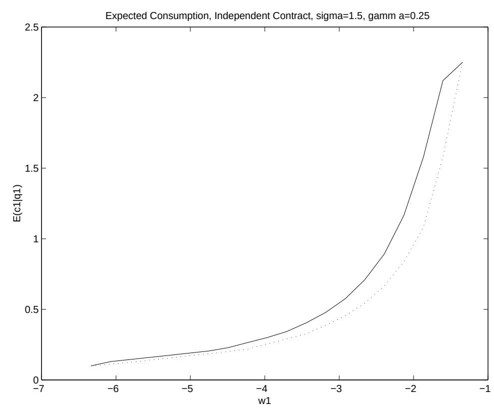

Figure 2: Independent contract

$\cdots$ q=1, — q=2

Figures 1 and 2 display the expected conditional consumption of worker 1, implied by the optimal independent contract, for $\gamma = 0.25$ and risk aversion parameters σ, 0.5 and 1.5 respectively. Figures 3 and 4 display expected conditional consumption of worker 1 for the relative contract, at symmetric (utility requirements of the workers are the same) utility requirements. The general feature is that holding output, q, constant, consumption is increasing in the required utility, w. For both types of contracts, higher consumption is associated with higher outputs to provide workers with an incentive to exert high effort. Consequently, workers bare some of the

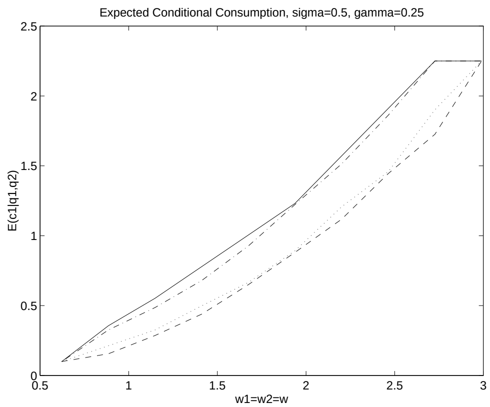

Figure 3: Relative contract

$\cdots$ $(q_1, q_2) = (1, 1)$, - - $(q_1, q_2) = (1, 2)$

-.-.- $(q_1, q_2) = (2, 2)$, — $(q_1, q_2 = (2, 1)$

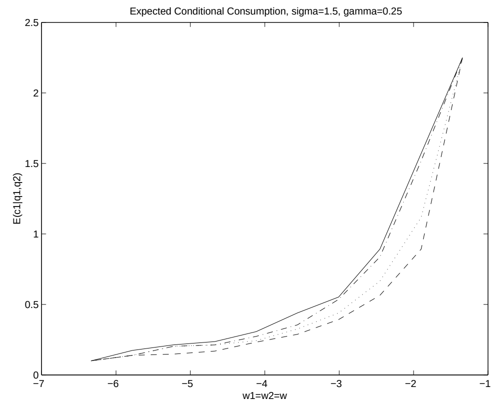

Figure 4: Relative contract

$\cdots$ $(q_1, q_2) = (1, 1)$, - - $(q_1, q_2) = (1, 2)$

-.-.- $(q_1, q_2) = (2, 2)$, — $(q_1, q_2 = (2, 1)$

risk. However, as workers become more risk averse or as effort becomes more costly, there is more insurance against shocks; consumption is smoother across states.

Most importantly, the static relative contract does not display a very strong tournament feature. Workers are awarded high consumption for producing high output and low consumption otherwise. The performance of one worker does not have a substantial effect on the compensation of the other worker.[5]

Figures 5 and 6 display expected effort for relative contracts, for different values of the cost of effort parameter, γ. Note that points not on the action grid are achieved through randomization. For low utility promises, the effort function slopes upward and this is due to the incentive compatibility constraints. The only incentive compatible way to give a worker ex-ante utility at the lower bound of consumption

[5] If, however, the static contract is repeated overtime, so that workers are offered the same contract every period, there are opportunities for collusion amongst workers. In Yeltekin (1997b), I show that if the optimal static relative contract is repeated infinitely many times, the subgame-perfect equilibrium value set includes payoffs to the workers that are higher than the Nash equilibrium payoffs of the optimal static contract. More specifically,in the infinitely repeated environment, there is an opportunity for workers to exert less effort and guarantee themselves a payoff higher than the payoff implied by the optimal static contract.

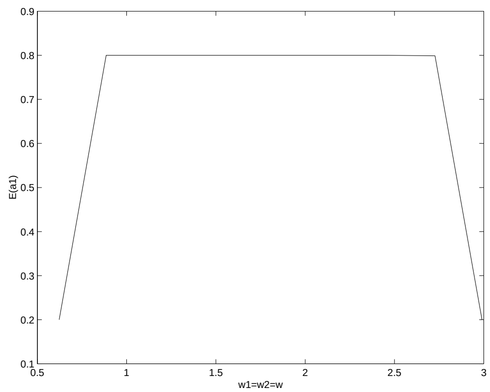

Figure 5: Relative contracts: Effort
$\sigma = 0.5$, $\gamma = 0.25$

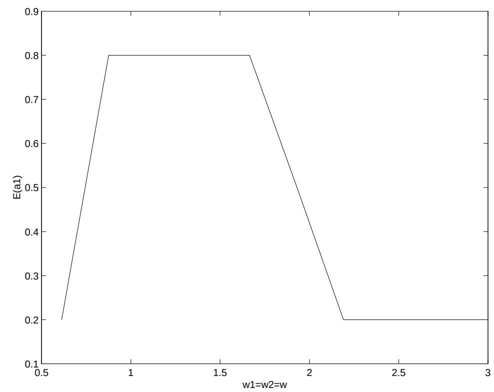

Figure 6: Relative contracts: Effort
$\sigma = 0.5$, $\gamma = 0.5$

and the lower bound on action is to actually give the lower bound on consumption and recommend the lower bound on action with certainty. If effort is more costly, as expected, effort is lower, specifically for high utility requirements. Lower expected effort implies lower expected output and consequently lower surplus for the principal. Figure 7 plots the diagonal cross-section (utility requirements for both workers are the same) of optimized surplus, for $\gamma = 0.25$ and $\gamma = 0.5$. If the cost of effort is low, the principal can recommend higher effort for workers and thus receive a higher surplus; the surplus function for $\gamma = 0.25$ lies above the surplus function for $\gamma = 0.5$. An interesting feature of these surplus functions is the upward sloping portions at low required utilities. This is a consequence of the incentive compatibility conditions. The only incentive compatible way to give a worker ex-ante utility at the lower bound of consumption and the lower bound on effort is to actually give the lower bound on consumption and recommend the lower bound on effort with certainty. For the left-end point on required utility, a higher required utility can deliver a higher surplus due to a higher expected level of action and thus a higher aggregate output from workers assigned that required utility. This higher output is more than enough to compensate the workers for the higher action they take and yields a higher surplus to the principal.

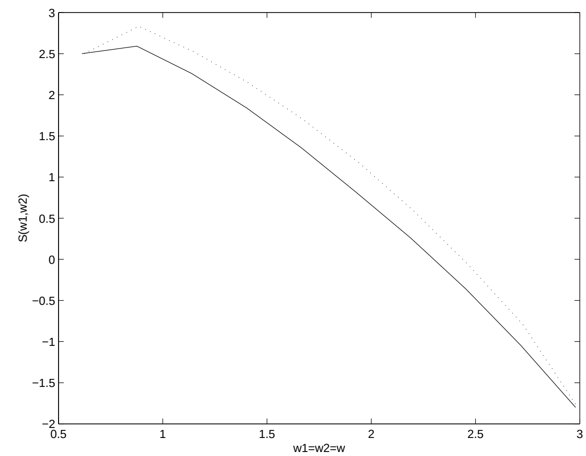

Figure 7: Relative contracts: Optimized Surplus
$\cdots$ $\sigma = 0.5, \gamma = 0.25$ and —- $\sigma = 0.5, \gamma = 0.5$

3 Dynamic Contracts

In sections 2.1 and 2.2 the optimal static independent and relative contracts have been analyzed. One question of particular concern is the evolution of the contract over time. Since most employment relationships are long-term, it is important to characterize the optimal dynamic contract and examine how the history of the workers' performances affect their current and future compensation. The dynamic contract allows a given individual's consumption and effort to be variable over time and conditions on this individual's history of performances. History dependence complicates the analysis substantially, but recent work in dynamic contracting literature has shown that, under certain conditions, there is a simpler representation of the dynamic contract that avoids these intractabilities.

In their seminal paper, Spear and Srivastava(1987) show that the optimal contract in a repeated agency model can be reduced to a two-period constrained optimum problem. In the first stage, risk is distributed between the principal and the agent in order to provide appropriate incentives to the agent. In the second stage, the contract specifies a promised utility payoff to the agent which is intertemporally optimal. It is also possible, under certain conditions, to summarize history dependence by including the agent's conditional expected utility as a state variable.[6]

Proceeding in an analogous way, the optimal independent contract is formulated recursively. The state variable for the dynamic independent contracts is the utility promise w and for the dynamic relative contract, it is the utility promises to both workers, w_1 and w_2. A dynamic contract maps a set of current utility promises to lotteries over action, output, consumption and future utility promise, w'. The subsequent sections characterize the recursive representation of the dynamic independent and relative contracts in detail.

3.1 Optimal Dynamic Independent Contract

The technology that maps effort levels to output, namely $P(q|a,\eta)$ is the same as in static contracts. It is assumed that time is discrete and workers discount at a common rate $\beta < 1$. The preferences of the workers and the principal are as follows[7]:

$$\text{Workers: } \; w = E_0 \sum_{t=0}^{\infty} \beta^t U(a_t, c_t)$$

$$\text{Principal: } \; S(w) = E_0 \sum_{t=0}^{\infty} \beta^t (q_t - c_t)$$

The principal's problem is to maximize her discounted surplus subject to each worker receiving an ex-ante expected discounted utility, w. The possible initial required discounted expected utilities can be derived along the same lines as for the static economies. The lower bound on expected discounted utility, $\underline{w}$, is that of receiving the lower bound on consumption and lower bound on effort with certainty at each date. The upper bound on utility, $\overline{w}$, is that of receiving the upper bound on

[6]For the justification of the application of these methods to this lottery characterization, see Phelan and Townsend (1991).

[7]In using discounted social surplus as the objective function it is implicitly assumed that the principal can borrow and lend at a constant rate of interest (equal to $(1-\beta)/\beta$). Alternatively, one can view this as the principal having the ability to invest in resources in a productive technology, accumulating reserves for subsequent consumption.

consumption and the lower bound on effort with certainty at each date. Any utility level w can be achieved by randomizing between these two bounds. Thus the set of expected utilities is an interval, $W_T = [\underline{w}, \overline{w}]$ for the T period contract. For the set of contracts examined in this section, T will be regarded as infinity. The choice variable for the optimal dynamic contract is denoted by $\Pi^w(a, q, c, w')$: the probability for a worker with a current utility promise w of taking action a, producing q, receiving consumption c today and being promised future utility w'. As in the static contracts, these probabilities are required to satisfy certain conditions.

First, the analogue of constraint 1, that ex-ante expected utilities are satisfied. This constraint is generally referred to as the "promise-keeping" constraint in the dynamic contracting literature. It guarantees that the sum of the utility from current consumption, $U(a, c)$, and the discounted future utility promise, $\beta w'$, is equal to the current utility promise, $w = E_0 \sum_{t=0}^{\infty} \beta^t U(a_t, c_t)$ of the worker. [8]

$$w = \sum_{A \times Q \times C \times W} \{U(a, c) + \beta w'\} \, \Pi^w(a, q, c, w') \tag{11}$$

Second, the dynamic contract must be incentive compatible. More specifically, for each recommended action, a, and possible deviation, $\hat{a}$, current utility plus the discounted future utility of taking the recommended action should be greater than that of the deviation action.[9] More specifically, for all action pairs $(a, \hat{a}) \in A \times A$,

$$\begin{aligned} &\sum_{Q \times C \times W} \{U(a, c) + \beta w'\} \, \Pi^w(a, q, c, w') \\ &\qquad \geq \sum_{Q \times C \times W} \{U(\hat{a}, c) + \beta w'\} \, \frac{\sum_\eta P(q \mid \hat{a}, \eta) G(\eta)}{\sum_\eta P(q \mid a, \eta) G(\eta)} \Pi^w(a, q, c, w') \end{aligned} \tag{12}$$

Third, the probability measure Π^w must be consistent with the exogenous proba-

[8] In the recursive formulation of the infinite-period problem, the set of continuation utilities, W', is the same as the set of discounted required utilities, W.

[9] The derivation of the incentive compatibility constraint is analogous to the derivation of the incentive compatibility constraints for the static economies. The above expression of incentive compatibility is obtained by following the steps discussed in Section 2.

bility distribution, $P(q|a,\eta)$. Therefore, for all $(\overline{a},\overline{q}) \in A \times Q$,

$$\sum_{C\times W} \Pi^w(\overline{a},\overline{q},c,w') = \sum_{\eta} P(\overline{q} \mid \overline{a},\eta)G(\eta) \sum_{Q\times C\times W} \Pi^w(\overline{a},q,c,w') \tag{13}$$

Lastly, the chosen probabilities must represent a probability measure, or,

$$\sum_{A\times Q\times C\times W} \Pi^w(a,q,c,w') = 1$$

$$\text{and for all } (a,q,c,w') \in A \times Q \times C \times W \quad \Pi^w(a,q,c,w') \geq 0 \tag{14}$$

For a utility promise, w, a *dynamic independent contract* is a function Π^w that satisfies constraints 11 through 14. Let $\mathcal{D}_1$ denote the set of dynamic independent contracts. The principal chooses a contract from the set $\mathcal{D}_1$ that maximizes her expected discounted surplus. The *optimal dynamic independent contract* is the solution to the following dynamic programming problem:

$$S(w) \;=\; \max_{\Pi^w\in\mathcal{D}_1} \sum_{A\times Q\times C\times W} \{(q-c)+\beta S(w')\}\Pi^w(a,q,c,w')$$

3.2 Optimal Dynamic Relative Contract

Analogous to the static case, the dynamic relative contract conditions a worker's compensation on both workers' performances. The relevant state variable for the dynamic relative contract is the utility promises to both workers. The choice variable for the optimal dynamic relative contract is denoted by $\Pi^{w_1,w_2}(a_1,a_2,q_1,q_2,c_1,c_2,w_1',w_2')$: the probability for workers one and two with utility promises (w_1,w_2) of taking actions a_1 and a_2, producing q_1 and q_2, receiving consumptions c_1 and c_2 today and being promised future utilities w_1' and w_2' respectively. For each pair of ex-ante discounted utilities (w_1,w_2), the principal's problem is to maximize total output less total consumption of the workers. As in the previous environments, the chosen probabilities must satisfy promise-keeping, incentive compatibility and consistency constraints.

First, the promise keeping constraints for each worker, which ensure that ex-ante expected utilities are satisfied.

$$w_1 = \sum_{A^2\times Q^2\times C^2\times W^2} \{U(a_1,c_1)+\beta w_1'\}\ \Pi^{(w_1,w_2)}(a_1,a_2,q_1,q_2,c_1,c_2,w_1',w_2') \tag{15}$$

$$w_2 = \sum_{A^2\times Q^2\times C^2\times W^2} \{U(a_2,c_2)+\beta w_2'\}\ \Pi^{(w_1,w_2)}(a_1,a_2,q_1,q_2,c_1,c_2,w_1',w_2') \tag{16}$$

The dynamic relative contract recommends a pair of actions, (a_1, a_2), for workers 1 and 2 respectively. To induce workers to follow these recommendations, the utility from taking the recommended action must be greater than that of each possible deviation. More specifically, for each recommended and alternative action pair $(a_1, \hat{a_1}) \in A \times A$

$$\begin{aligned}
&\sum_{A\times Q^2\times C^2\times W^2} \{U(a_1,c_1)+\beta w_1'\}\ \Pi^{(w_1,w_2)}(a_1,a_2,q_1,q_2,c_1,c_2,w_1',w_2') \\
&\quad\geq \sum_{A\times Q^2\times C^2\times W^2} \{U(\hat{a_1},c_1)+\beta w_1'\}\ \frac{\sum_\eta P(q_1\mid \hat{a_1},\eta)P(q_2\mid a_2,\eta)G(\eta)}{\sum_\eta P(q_1\mid a_1,\eta)P(q_2\mid a_2,\eta)G(\eta)} \\
&\qquad\qquad \times \Pi^{(w_1,w_2)}(a_1,a_2,q_1,q_2,c_1,c_2,w_1',w_2'),
\end{aligned} \tag{17}$$

and for each recommended and alternative action pair $(a_2, \hat{a_2}) \in A \times A$

$$\begin{aligned}
&\sum_{A\times Q^2\times C^2\times W^2} \{U(a_2,c_2)+\beta w_1'\}\ \Pi^{(w_1,w_2)}(a_1,a_2,q_1,q_2,c_1,c_2,w_1',w_2') \\
&\quad\geq \sum_{A\times Q^2\times C^2\times W^2} \{U(\hat{a_2},c_2)+\beta w_1'\}\ \frac{\sum_\eta P(q_1\mid a_1,\eta)P(q_2\mid \hat{a_2},\eta)G(\eta)}{\sum_\eta P(q_1\mid a_1,\eta)P(q_2\mid a_2,\eta)G(\eta)} \\
&\qquad\qquad \times \Pi^{(w_1,w_2)}(a_1,a_2,q_1,q_2,c_1,c_2,w_1',w_2').
\end{aligned} \tag{18}$$

Once again, the chosen probabilities must be consistent with the exogenous distribution $P(q|a,\eta)$, or, for all $(\overline{a_1}, \overline{a_2}, \overline{q_1}, \overline{q_2}) \in A^2 \times Q^2$

$$\begin{aligned}
\sum_{C^2\times W^2} \Pi^{(w_1,w_2)}(\overline{a_1},\overline{a_2},\overline{q_1},\overline{q_2},c_1,c_2,w_1',w_2') &= \sum_\eta P(\overline{q_1}\mid \overline{a_1},\eta)P(\overline{q_2}\mid \overline{a_2},\eta)G(\eta) \\
&\times \sum_{Q^2\times C^2\times W^2} \Pi^{(w_1,w_2)}(\overline{a_1},\overline{a_2},q_1,q_2,c_1,c_2,w_1',w_2')
\end{aligned} \tag{19}$$

Lastly, for each $(w_1, w_2) \in W \times W$, $\Pi^{(w_1,w_2)}$ must represent a valid probability measure. Therefore,

$$\sum_{A^2 \times Q^2 \times C^2 \times W^2} \Pi^{(w_1,w_2)}(a_1, a_2, q_1, q_2, c_1, c_2, w'_1, w'_2) = 1$$

$$\text{and for all } (a_1, a_2, q_1, q_2, c_1, c_2, w'_1, w'_2) \in A^2 \times Q^2 \times C^2 \times W^2$$

$$\Pi^{(w_1,w_2)}(a_1, a_2, q_1, q_2, c_1, c_2, w'_1, w'_2) \geq 0 \tag{20}$$

Given a pair (w_1, w_2), a *dynamic relative contract* is a function $\Pi^{(w_1,w_2)}$ that satisfies constraints 15 through 20. Let $\mathcal{D}_2$ denote the set of dynamic relative contracts. The *optimal dynamic relative contract* is a dynamic relative contract that maximizes the surplus $S(w_1, w_2)$ and it is the solution to the following dynamic programming problem.

$$\begin{aligned} S(w_1, w_2) &= \max_{\Pi^{(w_1,w_2)} \in \mathcal{D}_2} \sum_{A^2 \times Q^2 \times C^2 \times W^2} \{(q_1 + q_2 - c_1 - c_2) + \beta S(w'_1, w'_2)\} \\ &\qquad \times \Pi^{(w_1,w_2)}(a_1, a_2, q_1, q_2, c_1, c_2, w'_1, w'_2) \end{aligned}$$

3.3 Computations and Results

The infinite-period problems defined in sections 3.1 and 3.2 are solved by finding a surplus function $S(\cdot)$. This can be done by choosing an initial guess for $S(\cdot)$, and using it in the objective function of the dynamic programming problem for every $w \in W$ (for every (w_1, w_2) pair in $W \times W$ for relative contracts). The solution values of these programmes are then used as the next guess for $S(\cdot)$ and this procedure is repeated until the sequence of functions converges. As in the static model, the set W is discretized and the surplus function is solved for each $w \in W$ for the independent contract and for each pair $(w_1, w_2) \in W \times W$ for the relative contract.

A revised simplex algorithm, a policy function combined with a value function iteration are the main components of the computational method to solve for the optimal dynamic contract. Each iterationis composed of a linear programming problem for each $W \in W$ $((w_1, w_2) \in W \times W)$. However, there is a drawback: the large dimension of the linear programming problem. The number of variables for the relative contract is the product of the number of points in $A \times A \times Q \times Q \times C \times C \times W \times W$. Even coarse grids for each set result in a linear programming problem of enormous dimension. This is largely remedied by dividing the problem into subperiods. In the first subperiod, actions are taken and quantities are realized and a "middle-of-the-period continuation utility", w^m is promised. In the next subperiod, consumption and future utility are allocated, consistent with the w^m promise. For the relative contract, for example, this splits the lotteries into two smaller lotteries, $\Pi^{(w_1,w_1)}(a_1, a_2, q_1, q_2, w_1^m, w_2^m)$ and $\Pi^{(w_1^m,w_1^m)}(c_1, c_2, w_1', w_2')$. The second subperiod problem needs to satisfy only the promise-keeping and the constraint 20.

For the results reported in this section, the selected action, output and consumption sets are the same as in static contracts.

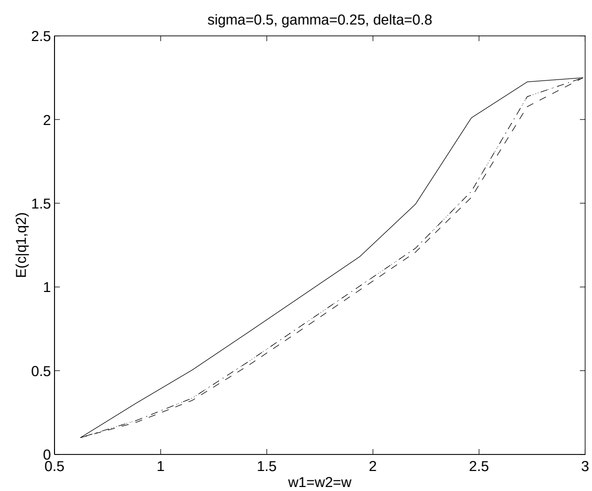

Figure 8: Relative contracts: Expected Conditional Consumption of Worker 1

$\cdots$ $(q_1, q_2) = (1, 1)$, - - $(q_1, q_2) = (1, 2)$

-.-.- $(q_1, q_2) = (2, 2)$, — $(q_1, q_2 = (2, 1)$

Figures 8 and 9 display expected conditional consumption for the relative and independent contracts, for $\sigma = 0.5$, $\gamma = 0.25$, and discount rate, $\beta = 0.8$. In both optimal plans, holding output, q, constant, expected consumption increases as utility

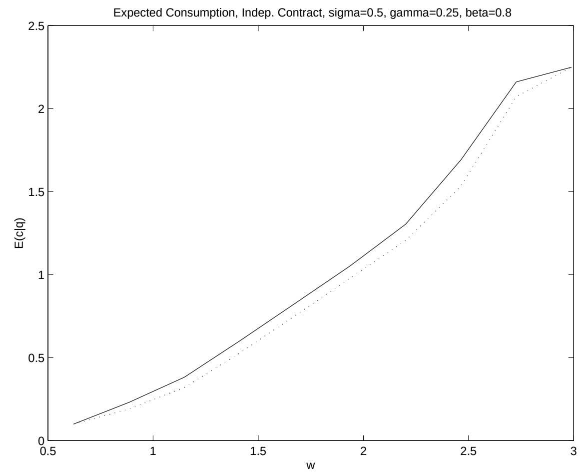

Figure 9: Independent contract: Expected Conditional Consumption

$\cdots$ q=1, — q=2

promises of workers increase. For the independent contract, workers are awarded high consumption for high output, and low consumption for low output. Compared to static independent contracts, current consumption depends less on current output and therefore allows better contemporaneous smoothing.

The optimal plan for the dynamic relative contract is significantly more complicated, and it takes advantage of the information on the relative performance of the workers. Workers are given high consumption for producing more than their counterpart and low consumption otherwise. This feature of the optimal dynamic relative plan displays the striking difference between the optimal static and dynamic relative contracts. While the static relative contract is not significantly different from the static independent contract, the dynamic relative contract displays a strong tournament feature and is substantially different from the dynamic independent contract. If both workers produce the same amount, such as $(1,1)$ or $(2,2)$, their expected consumption does not depend on their absolute performance. This is displayed by the fact that the dotted and dashed lines ($(q_1,q_2) = (1,1)$, $(q_1,q_2) = (2,2)$ respectively) overlap in the consumption figure. The lowest expected consumption delivered is when one of the workers produces less than the other. By giving the highest consumption to the "winning" worker, the principal provides incentives for workers to exert high effort.

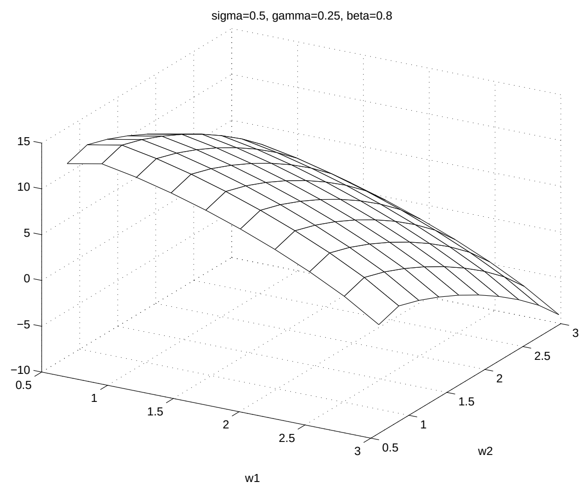

Figure 10: Relative Contract: Optimized Surplus

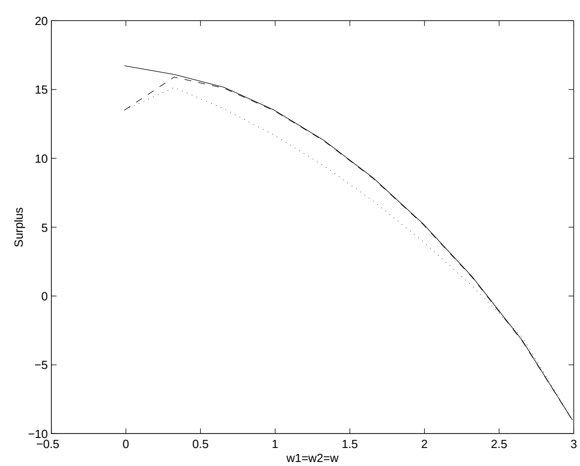

Figure 11: Relative contracts: Maximized Surplus

Figure 10 displays the optimized value function for the dynamic relative contract for all (w_1, w_2) pairs. The diagonal cross-section of this surplus function (so that workers' utility promises are the same) is displayed in figure 11, along with the full information value function and the surplus function associated with infinitely repeating the solution to the one-period relative contract.[10]

The full information contract maximizes the surplus $S(w_1, w_2)$ subject to constraints 15-16 and 19-20. Because workers' actions are observable in the full information environment, the full information contract is not subject to incentive compatibility conditions.[11]

The surplus from the dynamic relative contract lies everywhere below the surplus from the full-information problem, due to the added incentive constraints. However, it lies everywhere above the surplus from the static relative contract. Much of the surplus lost from the agency problem is recovered when history dependence is allowed.

Furthermore, the value function for the dynamic unobserved action case, is concave in the utility promises to the workers and as in static contracts, the value function peaks not at the lowest w, but at a slightly higher w. This is again a consequence of the incentive compatibility constraints. The only incentive compatible way to give workers ex-ante utility at the lower bound of consumption and the lower bound on action is to actually give the lower bound on consumption and recommend the lower bound on action with certainty every period. For the left-end point in required utilities, higher required utilities can deliver a higher surplus due to higher expected levels of action and thus higher aggregate output from workers assigned these required utilities. This higher output is more than enough to compensate the workers for the higher action they take and yields a higher surplus to the principal.

[10]Note that a static plan with required utility w for the worker and surplus $S(w)$ for the principal gives expected discounted utility of $(1/(1-\beta)) \cdot w$ to the worker and $(1/(1-\beta)) \cdot S(w)$ to the principal if infinitely repeated.

[11]Note that solutions to the full information problem in the static case are also solutions to the dynamic full-information problems.

3.4 Credible Contracts

The presence of an upward sloping portion of the value function has important implications. If the principal is better off offering slightly higher utility to workers with the lowest ex-ante expected utility, then issues of renegotiation would arise. For workers who have, at any point in time, been offered the lower bound on utility as a future utility promise, there is an opportunity for renegotiating to receive a slightly higher utility, since this higher utility is Pareto-improving. Consequently, the principal cannot credibly commit to utility promises corresponding to the upward sloping portion of the value function. Furthermore, the principal cannot credibly use the lower bound on utility promises as punishments for deviations from recommended actions. A question of particular interest is the set of "credible" contracts in this setting. This section proposes a method for constructing the set of dynamic independent credible contracts [12] and reports some preliminary results.

The set of credible contracts is constructed by sequentially raising the lower bound on utility promises until the value function is non-increasing everywhere. As mentioned before, the lower bound on utility grid is the utility from receiving the lowest consumption and exerting the lowest amount of effort, with certainty at each date and the highest utility is of receiving the highest consumption and exerting the lowest amount of effort with certainty each date. This defines a set of feasible utilities the contract can promise, $W = [\underline{w}, \overline{w}]$. Given W, one can solve for the surplus function $S(w)$ following the computational method outlined in section 3.3. If the $S(w)$ is not non-increasing everywhere, then the algorithm searches for the lowest $\hat{w}$ in W such that $S(w)$ is non-increasing for all $w \geq \hat{w}$. Then the set of required utilities is updated by setting $\hat{w}$ to be the new lower bound. This procedure defines a fixed point problem in the set W, and the algorithm searches for the fixed point, by updating the set W in each iteration. The steps of the algorithm that solves for the set of credible contracts is as follows:

1. Set initial W^0 such that $W^0 = [\underline{w}^0, \overline{w}]$, where $\underline{w}^0 = \sum_{t=0}^{\infty} \beta^t U(\underline{a}, \underline{c})$ and $\overline{w} =$

[12] The set of credible relative contracts can be constructed in an analogous way, but has not been examined here to avoid repetition and complicated notation.

$\sum_{t=0}^{\infty} \beta^t U(\underline{a}, \overline{c})$.

2. Given W, solve for $S(w)$. Find $\hat{w} \in W$ such that for all $w \geq \hat{w}$, $w \in W$, $S(w) \leq S(\hat{w})$.

3. If $\hat{w} = \underline{w}$ STOP. Otherwise set the new $\underline{w} = \hat{w}$ and return to STEP 2.

Figures 12-15 display the value and policy functions from some of the intermediate iterations for computing the set of credible contracts.

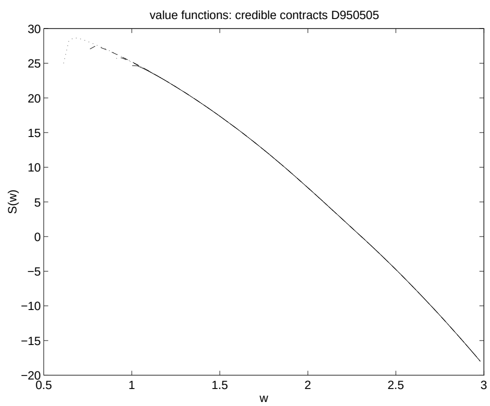

Figure 12: Value function

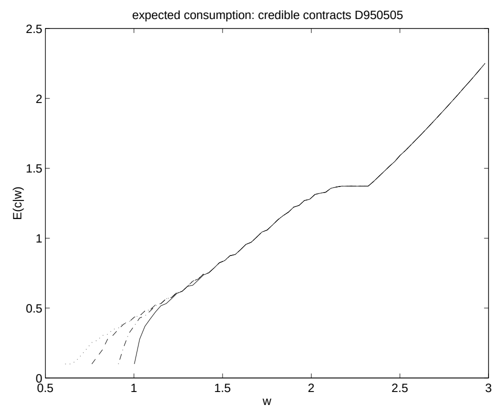

Figure 13: Consumption

An important feature of the value functions plotted in figure 12 is that as the lower bound on utility promises is increased, the frontier remains the same. This feature of the value function suggests that in the unconstrained case workers almost always receive utility promises that correspond to the non-increasing portion of the value function. Figure 12 supports this interpretation. The utility promises are monotonically increasing and thus workers are moving along the downward sloping portion of the value function.[13]

In the original set of contracts (as displayed in section 3.3), for the lower bound on utility promises, the principal can only give the lower bound on consumption and

[13]Further evidence of the positive drift in utility promises can be found in Section 4, which displays the transition functions and the time series for consumption and utilities for the optimal dynamic contracts.

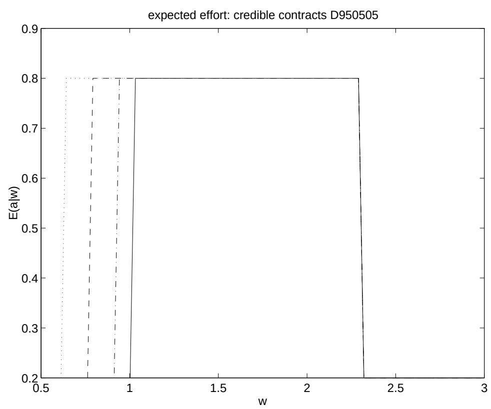

Figure 14: Effort

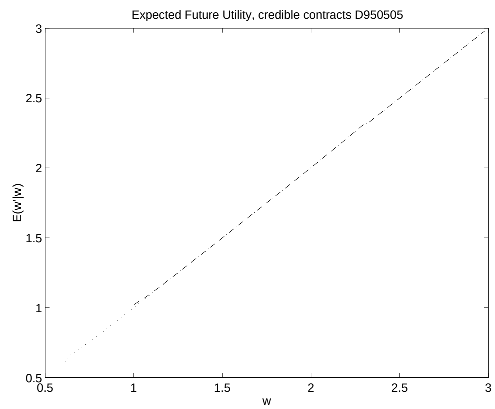

Figure 15: Utility Promise

recommend the lowest effort level with certainty at each date. For the principal to induce a worker to exert more effort, she must be able to provide a spread in future utility promises that induces that level of effort, but because the utility promises are bounded below, she is unable to do so. In set of credible contracts examined here, the principal is able to provide such a spread by using current consumption. [See Figures 16 and 17]

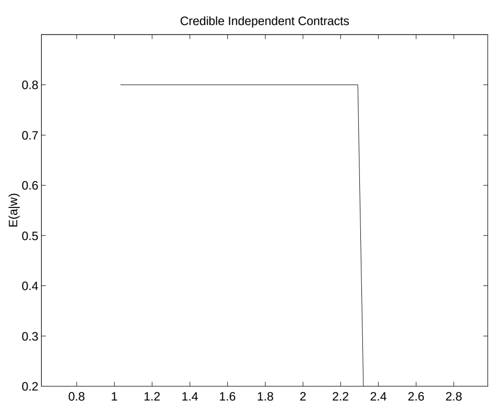

Figure 16: Effort

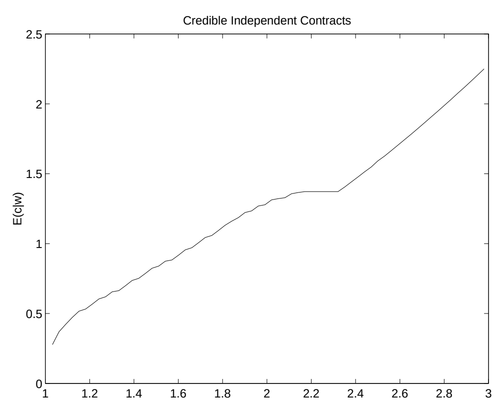

Figure 17: Consumption

4 Age-Earning Profiles

The optimal dynamic contracts computed in section 3 can be used to characterize the stochastic processes for consumption and effort. Given an initial w $((w_1, w_2))$ for the independent (relative) contracts and a random number generator, one can use the probability distributions Π^w $(\Pi^{(w_1,w_2)})$ to generate time series of effort and consumption. One simply keeps track of the w' $((w_1', w'2_))$ generated in each period and uses it as the initial condition for the next. Figures 18 and 19 display 2 such time series of consumption and effort for the relative contracts, for a discount rate of 0.96. The solid line is the individual time series for worker 1 and the dotted line is the time series for worker 2.

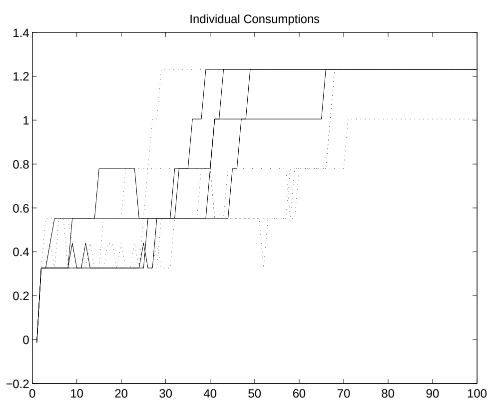

Figure 18: Consumption Series
Dynamic Relative Contracts
— Worker 1, ··· Worker 2

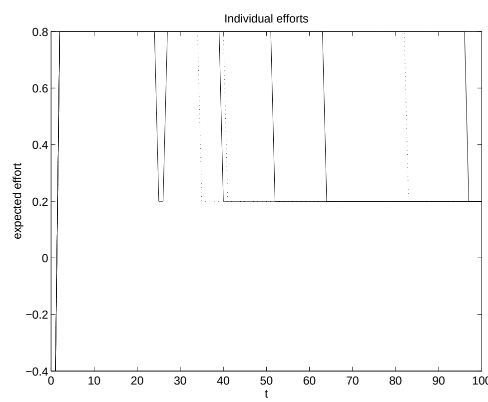

Figure 19: Effort Series
Dynamic Relative Contracts
— Worker 1, ··· Worker 2

Over time, consumptions of workers increase while their effort levels fall. A consequence of the optimal dynamic contracts is that workers who have been in the contract for a long time receive high compensation and exert little effort. The time series of workers' utility promises has a positive drift; with probability almost one, workers' utilities converge to the upper (or close to the upper bound, depending on the transition matrix for utilities) bound on the utility set, W.

The set of graphs in figure 20 displays consumption variation across periods for

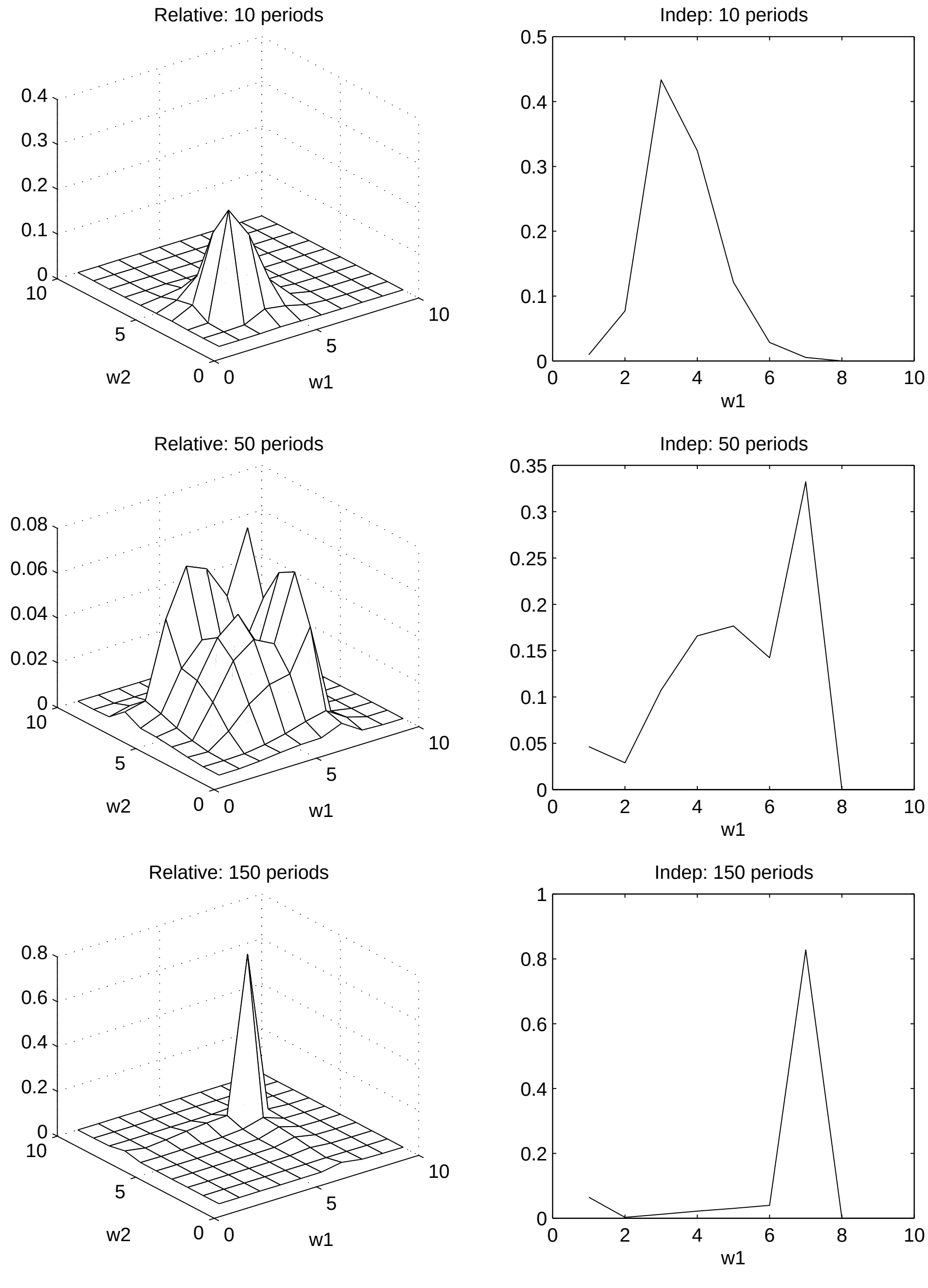

Figure 20: Consumption Variation

the relative and independent contracts. The economy is started at w $((w_1, w_2))$ such that w $((w_1, w_2))$ maximizes the surplus of the principal. Variance of consumption is low at first, is substantially larger after 50 periods, and low again after 150 periods. For the computed examples, the ergodic distribution of utilities and consumption is degenerate. There are, typically two sets of absorbing states, one at the lowest utility level and a few near the highest utility levels. The number of absorbing states at the high utility levels is sensitive to the risk aversion and disutility of effort parameters. With very high probability, both workers end up in one of the high absorbing states, unless one or both of them start at the lowest utility level, however the transition to these states is extremely slow. Figures 21-24 plot the marginal probabilities of a worker's utility state after 2000 periods for different discount factors. Almost all of the probability mass is placed on the absorbing states, with a significantly large proportion of this mass on the higher absorbing states.

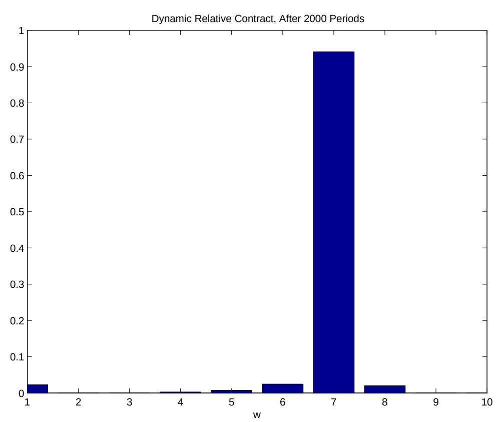

Figure 21: $\beta = 0.96$

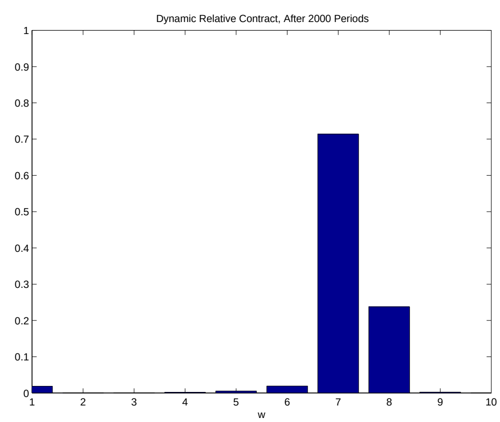

Figure 22: $\beta = 0.98$

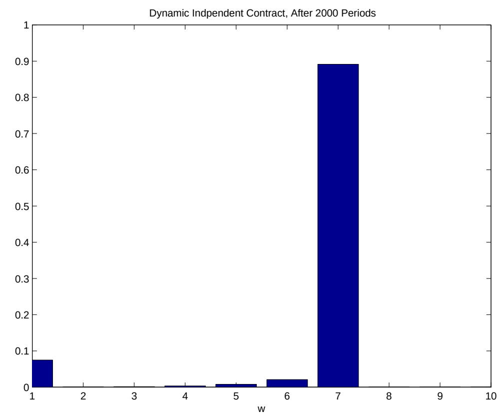

Figure 23: $\beta = 0.96$

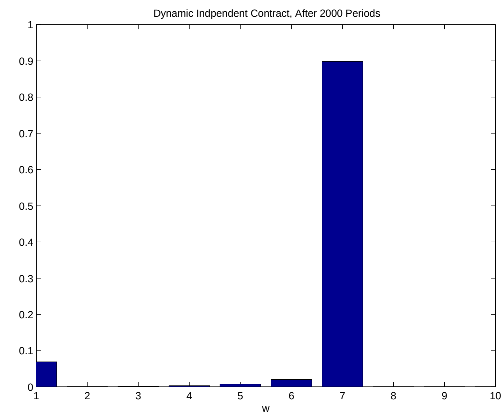

Figure 24: $\beta = 0.98$

5 Conclusion

The contracts developed and computed in the previous sections shed light on the nature of optimal static and relative contracts in an environment with moral hazard, where workers contracting with the same principal receive correlated productivity shocks. The static contract does not display a strong tournament feature. The dynamic relative contract, on the other hand, is substantially different from the dynamic independent contract. An important feature of the dynamic relative contract is the difference in expected consumption across output pairs; highest consumption award is given to the worker with the highest output. If both workers produce the same output, they receive the same amount of consumption, regardless of the absolute level of this output.

Certain aspects of the dynamic contracts examined provide additional issues for future research. Amongst these issues are collusion, variation in the information and shock structure, commitment and the investigation of the positive drift in workers' utilities over time. In the contracts constructed above, workers take their actions before the realization of the common shock. One possible way to modify the above model would be allow workers to observe the common shock before exerting effort. Another interesting extention would be to examine the nature of the dynamic contracts with common shocks that follow a Markov process. In such an environment, the history of workers' performances would reveal more information to the principal than in the case with i.i.d. common shocks. If one worker repeatedly produces low output while the other worker produces high output, the principal can deduce that the former worker is shirking. One obstacle in incorporating some of these features is the curse of dimensionality in the relative contracts. The dynamic and linear programming methods used need to be more efficient to be able to handle additional state variables.

The optimal contracts of section 2 and 3 are not collusion-proof. The opportunity of collusion among workers exists; for example, workers can repeatedly exert little effort and lead the principal to believe that they are getting hit by the adverse shock at all times or they can vary their effort levels to alternate "winning" the tournament. One would expect the collusion-proof contract to resemble the independent contract; if a contract relies less on relative performance for compensation, the opportunities for collusion may be reduced. Furthermore, one can investigate how different commitment assumptions affect the nature of the contracts and the age-earnings profiles. One possibility is to give both the principal and the workers the ability to terminate or quit the contract with outside options. Certain aspects of these commitment issues have been examined by Phelan(1995), but not in a multiple-agent setting.

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