# SEARCH

## VARIABLE SAMPLE SIZE

## A COMPUTATIONAL SOLUTION[1]

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ABSTRACT

Morgan (1983) guaranteed that VSS dominated both FSS and SSR. But it is difficult to calculate the optimal sample size and the optimal reservation price both without recall and with full recall. As VSS without recall is a simplification of VSS with full recall, we will present on appendix a VB30 program that calculates only the full recall case. As known, on VSS, the search is sequential and in each period the sample size is variable.

As normal, we will extract sellers prices from F(x) that is common knowledge, temporal horizon is T, goods are homogenous, no discount and consumer buys once just one unity of goods.

### 1. WITHOUT RECALL

#### 1.1. DETERMINATION OF THE SAMPLE SIZE, N(T)*.

Buyer must calculate how many prices to ask. In the beginning of each period, he will chose n that minimizes expected value of expenditure, Pi + K(n).

$$\left\{n(T)^* \ : E\left[V(n(T)^*,T)\right] = E\left[\ Min\left(Min(P1\ldots Pn), E\left[V\left(n(T-1)^*,T-1\right)\right]\right)\right] + K(n(T)^*)\right\} \quad (1)$$

[1] Nothing we present is new. We just try to expand discussion on search as a way to find the “invisible hand”.

We can see that there is a reservation price, P(T)*.

$$P(T)^* = E\left[V\left(n(T-1)^*, T-1\right)\right] \quad (2)$$

Using, on equation (1), M (x) instead of Min(P1…Pn) and P(T)* instead of $E\left[V\left(n(T-1)^*, T-1\right)\right]$, it gives the next equation.

$$E[V(n)] = E\left[Min\{M(n), P(T)^*\}\right] + K(n) \quad (3)$$

Using expectation we will have the next equation, that we will minimize using n as variable.

$$E[V(n)] = \int_0^{P(T)^*} x.\left(n.[1-F(x)]^{n-1}.f(x)\right).dx + \int_{P(T)^*}^{\infty} P(T)^*.n.[1-F(Z)]^{n-1}.f(x).dx + K(n) \quad (4)$$

We can see on Fig 1 the evolution on the optimal size with the diminution of the time horizon, Prices from [10,20], uniform, and K(n)= 0.15n.

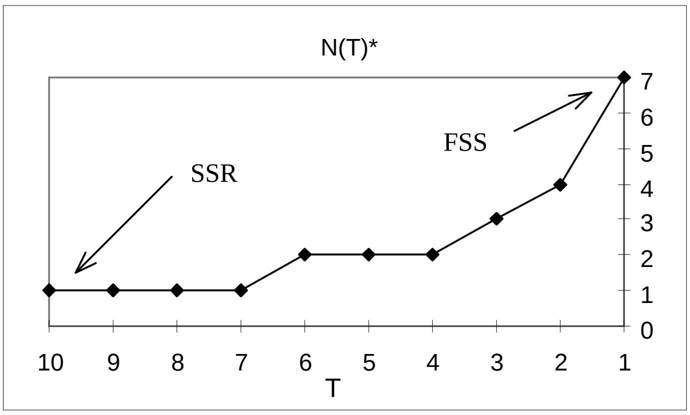


**Fig 1- Sample Size Without Recall**

## 1.2. DETERMINATION OF THE RESERVATION PRICE, P*

Now that we know the optimal sample size, n*, we have the reservation price, P(T+1)*, as the minimum of equation (4).

Equation (4), that we show with p(T+1)* on next equation, will permit calculate, by backward induction, the reservation prices and sample size at all periods. Being $H(x) = \int_0^{P(T)*} \left(P(T)*-x\right).n*.\left[1-F(x)\right]^{n*-1}.f(x).dx$, the marginal gain of search, P(T+1)*-P(T) will be the cost of losing one opportunity.

$$P(T+1)*-P(T)*+K(n*) = H\left(P(T)*\right) \tag{5}$$

We can see on Fig 2 the evolution on the reservation price with the diminution of the time horizon, prices from [10,20], uniform, and K(n)= 0.15n.

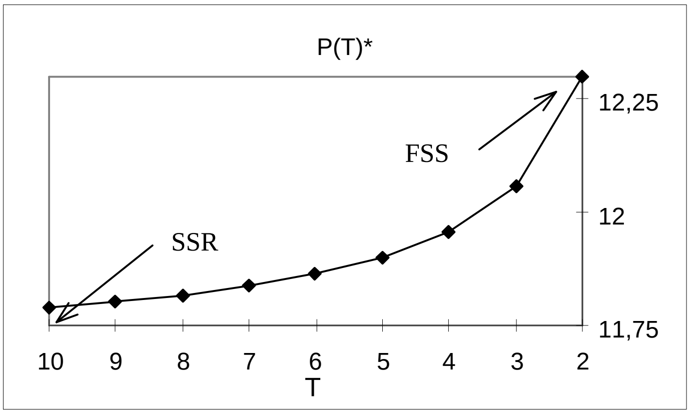


**Fig 2- Reservation Prices Without Recall**

We have SSR and FSS on Fig 1 and Fig 2 to show that VSS dominate them.

## 2. WITH PERFECT RECALL

Apparently, this is a very complex problem as the reservation price changes with the best price found till now, Z, but changes too with the present price, Pi, as we can see on next equation (we use n* = 1 on expressions to simplify them with no lost).

$$P(Z,T+1)* = Min\left(Pi, Z, P\left(Min(Pi,Z),T\right)*\right) + c \tag{6}$$

Using expectation, we will have a variable of integration as limit of the integration as owe can see on next equation (P(x, T)*<Z with no lost).

$$P(Z,T+1)^* = \int_0^{P(x,T)^*} x.f(x).dx + \int_{P(x,T)^*}^{Z} P(x,T)^*.f(x).dx + \int_Z^{\infty} P(Z,T)^*.f(x).dx + c \quad (7)$$

We will call P(Z,T) the Reservation Function because it changes with Z.

The reservation price will exists only under the next condition, Lippman and McCall (1976, eq. 24).

$$P(Z,T)^* - P(Z',T) \le Z - Z' \quad (8)$$

We see on Fig 3 that the reservation function is convex to origin, and P(0,T) is positive, so reservation price exist as the solution to the next equation.

$$\{\Pr(T)^* = x : P(x,T)^* = x\} \quad (9)$$

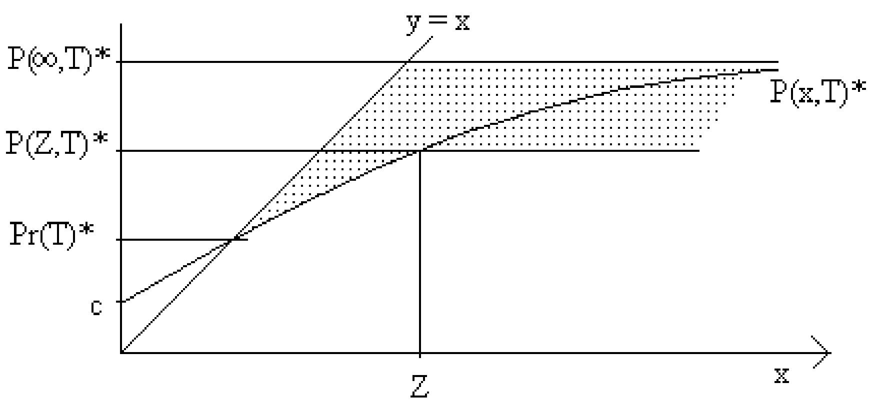


**Fig 3 - Reservation Function and Reservation Price with Perfect Recall**

The reservation price, Pr(T)*, don't depends on Z or on Pi, and is the limit of integration on equation (7)

## 2.1. DETERMINATION OF THE SAMPLE SIZE, N(T)*.

If Z ≤ Pr(T)*, the expected value of expenditure, known Z, is given by the next equation.

$$E[V(Z,n)] = \int_0^Z x.\left(n.[1-F(x)]^{n-1}.\, f(x)\right).dx + \int_Z^\infty Z.n.[1-F(x)]^{n-1}.\, f(x).dx + K(n) \quad (10)$$

If Z > Pr(T)*, we will have instead the next equation.

$$\begin{aligned} E[V(Z,n)] = & \int_0^{\Pr(T)^*} x.\left(n.[1-F(x)]^{n-1}.\, f(x)\right).dx + \\ & + \int_{\Pr(T)^*}^{Z} P(x,T).n.[1-F(x)]^{n-1}.\, f(x).dx + \\ & + \int_Z^\infty P(Z,T).n.[1-F(x)]^{n-1}.\, f(x).dx + K(n) \end{aligned} \quad (11)$$

The sample size minimizes equations (10). or (11). We can see on Fig 4 the relation between sample size, best price till now, and time horizon. Prices on [10,20], rectangular, and K(n)= 0.15n.

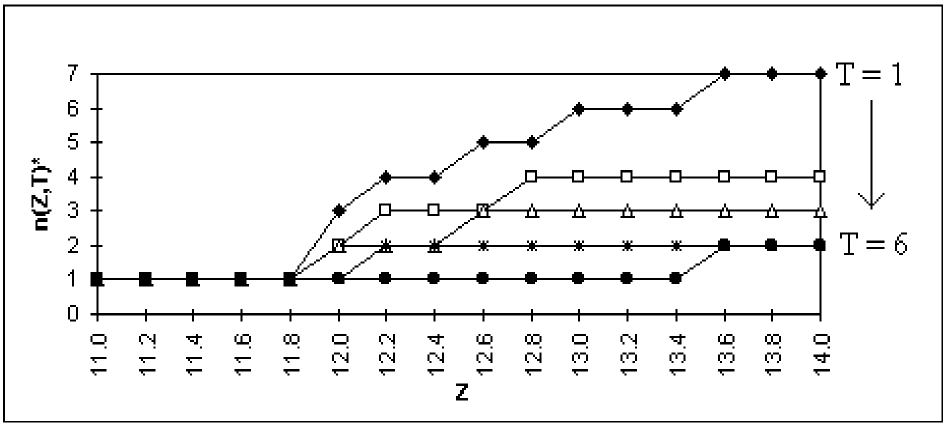


**Fig 4 - Sample Size function of Z and T, with Perfect Recall**

## 2.2. Determination of the Reservation Price, Pr(T)*

Has we have seen the reservation price is the solution to $\{\Pr(T)^* = x : P(x,T,n)^* = x\}$ being P(x, T, n)* the minim of equations (10). or (11).

We can see on Fig 5 the relation between the reservation function, the reservation price, the temporal horizon and Z.. Prices on [10,20], rectangular, and K(n)= 0.15n.

The reservation price is constant all periods but the last (Gal e al. 1981).

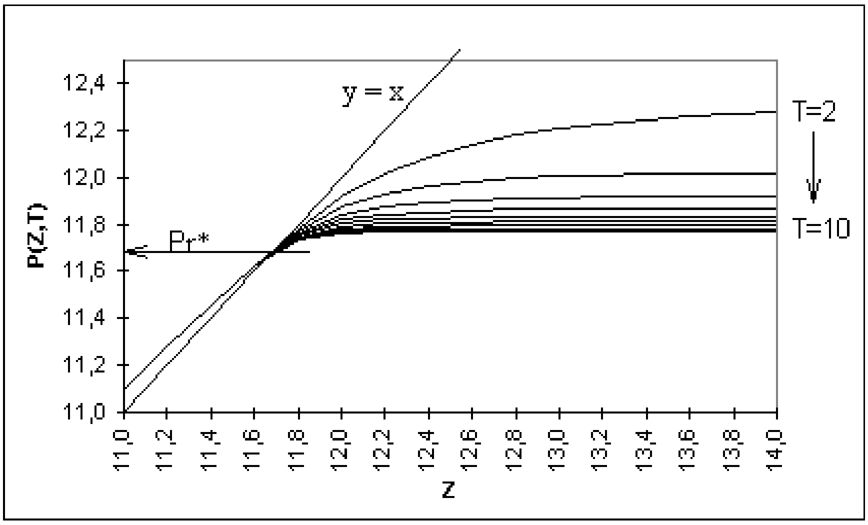


**Fig 5 - Reservation Function and Price, with Perfect Recall**

# APPENDIX

A program written in VB30 to calculate the VSS with recall.

We used a array with 101 points (0 to Resol)of the function P(Z,T) and, by backward induction, we calculate al periods. N_Optim is a variable global that is the optimal sample size.

Recursivity is computationally inefficient.

By running Reserv_Function we calculate P(Z,T+1). For example, to calculate P(Z,10) we will do the next routine.

```
Initialise
For I=1 to 10
    Reserv_Function
Next i
```

And then we may print Pact(i). To print the optimal sample size we need to do it inside routine Reserv_Function.

```
´ Declarations
Const c = .15, Resol=100
Dim N_Optim As Integer, Pant(0 To Resol), Pact(0 To Resol)

Sub initialise ()
'P(Z,T) for T = 1
Dim i As Integer
```

```
For i = 0 To Resol
   Pant(i) = 20
Next i
End Sub
```

**Function g (N As Integer, x)**

```
'Distribution of minimum prices on a sample size n
Dim fp, Fg

'fp(x) - uniform on [10 e 20]
If (x >= 10) And (x <= 20) Then
   If (x = 10) Or (x = 20) Then fp = .05 Else fp = .1
Else
   fp = 0
End If

'Fg(x)
If (x >= 10) Then
   If x < 20 Then Fg = -1 + x / 10 Else Fg = 1
Else
   Fg = 0
End If

'g(n,x)
g = N * (1 - Fg) ^ (N - 1) * fp
End Function
```

**Sub Reserv_function ()**

```
'Calcul Resol+1 points from the reservation function
Dim Z, LimInt, i As Integer

   LimInt = Pr()
   For i = 0 To Resol
      Z = 10 + i *10/ Resol
      Pact(i) = Point_Resev_Function(Z, LimInt)
      'Here print variable N_Optim
   Next i
   For i = 0 To Resol ' Pant is the P(Z,T) and Pact is the P(Z,T+1)
      Pant(i) = Pact(i)
   Next i
End Sub
```

**Function Pr ()**

```
Dim L1 As Integer, L2 As Integer, L3 As Integer
Dim V2, X1, X3

L1 = 0: L3 = Resol
If (Pant(L3) - (10 + L3*10 / Resol) >= 0) Then
   Pr = 20
```

```
    Exit Function
End If
While L3 - L1 > 1
  L2 = Int((L1 + L3) / 2 + .01)
  V2 = Pant(L2) - (10 + L2 *10/ Resol)
  If V2 >= 0 Then L1 = L2 Else L3 = L2
Wend
X1 = 10 + L1*10 / Resol
X3 = 10 + L3 *10 / Resol
Pr = X1 - (X3 - X1) / (Pant(L3) - X3 - Pant(L1) + X1) * (Pant(L1) - X1)
End Function
```

**Function Point_Reserv_Function (ByVal Z, LimInt)**

```
Const L3 = 20
Dim L1, L2, ConstZ, N  As Integer
Dim V1, V2
'Case Z<=Pr* or otherwise
If Z <= LimInt Then
    L1 = Z: L2 = Z
    ConstZ = Z
Else
    L1 = LimInt: L2 = Z
    ConstZ = Pvss_inter(Z)
End If

'Minimization
N = 1
V1 = Integ(N, L1, L2, ConstZ) + N * c
V2 = Integ(N + 1, L1, L2, ConstZ) + (N + 1) * c
While V1 >= V2
    N = N + 1
    V1 = V2
    V2 = Integ(N + 1, L1, L2, ConstZ) + (N + 1) * c
Wend
N_Optim = N
Point_Reserv_Function = V1
End Function
```

**Function Integ (ByVal N As Integer, L1, L2, ConstZ)**

```
    Const Delta = .01, L3 = 20
    Dim Integral, x

Integral = 0
For x = 0 + Delta / 2 To L1 Step Delta
    Integral = Integral + x * g(N, x) * Delta
```

```
Next x
For x = L1 + Delta / 2 To L2 Step Delta
    Integral = Integral + P_inter(x) * g(N, x) * Delta
Next x
For x = L2 + Delta / 2 To L3 Step Delta
    Integral = Integral + ConstZ * g(N, x) * Delta
Next x
Integ = Integral
End Function
```

**Function P_inter (Z)**

```
Dim V1, V2, L1, L2
Dim i As Integer

i = Fix((Z - 10) * Resol / 10)
If i = Resol Then
    P_inter = Pant(Resol)
Else
    L1 = 10 + i *10 / Resol
    V1 = Pant(i)
    L2 = 10 + (i + 1) *10 / Resol
    V2 = Pant(i + 1)
    P_inter = V1 + (V2 - V1) * (Z - L1) / (L2 - L1)
End If
End Function
```

## REFERENCES


Gal, S., M. Landsberger and B. Levykson (1981). “A Compound Strategy for Search in the Labour Market.” *International Economic Review*, Vol. 22, No. 3, pp. 597-608.

Lippman, Steven A. and John J. McCall (1976). “The Economics of Job Search: A Survey.” *Economic Inquiry*, Vol. 14, pp.155 - 89.

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