

Multi-agent modeling and simulation of a sequential monetary production economy

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Abstract. This paper presents a heterogeneous agent model of a sequential monetary production economy. A deterministic dynamic flow model is employed. The model is characterized by three classes of agents: a single homogeneous representative consumer, heterogeneous firms and a banking sector. There are three asset classes (or debts): a single homogeneous physical good, money and debt securities. The homogeneous commodity is produced by firms and, if saved, increases their capital stock. Firms issue debts to finance growth. Firms are homogeneous as regarding production technology but are heterogeneous relative to expected inflation. Consumers provide labor force and make the decision of consumption and saving of their income. They own all the equities of firms and banks. The banking sector collects consumer savings and provides credit supply to firms. The main result of the model is that real economic variables are strongly affected by the level of credit supply in relation to the level of savings.

Keywords: Heterogeneous agents, financial markets and the macroeconomy, computer simulation

JEL codes: D92, E17, E44

1. Introduction and motivation

In the last few years, a multi-agent market model named Genoa Artificial Stock Market (GASM) has been developed (Raberto et al., 2001; Marchesi et al., 2003). GASM showed that financial resources of agents, i.e., the money and the number of stocks, are highly relevant when evaluating the profitability of an investment strategy (Cincotti et al., 2003; Raberto et al., 2003). In the GASM framework, financial resources were modeled as exogenous variables. In this respect, the model presented in this paper is conceived to provide a suitable framework for the dynamics of financial resources and to exhibit favorable characteristics in order to be coupled to the GASM.

Connection between macroeconomic variables and financial markets has been a goal of financial economics for a long time (Lettau and Ludvigson, 2001) and is supported by the evidence that returns of common stocks appear to vary with business cycles. In the last years, increasing attention has been focused on this topic, and many empirical studies showed that a range of financial and macroeconomic variables can predict stock market returns (Campbell, 1987; Fama and French, 1989; Ferson and Harvey, 1993; Pesaran and Timmermann, 1995; Chen, 1991). Standard stock evaluation models predict that stock prices are affected by the discounted value of expected cash flows. It

has been shown that real economic activity, interest rate and stock returns are correlated (Fama, 1990). Most of these earlier studies consider the short-run relationship between stock market and financial and macro-economic variables, which may remove important information contained in the permanent component of economic activity. Conversely, long-run relationship between stock market and the economic variables has received little attention of researchers (Nasseh and Strauss, 2000). Results support the existence of a long-run relationship between stock prices and interest rates, consumer prices, real domestic macroeconomic innovations and international activity. Thus, stock prices as well as their returns are a function of underlying economic activity in the medium and long run.

In this framework, a dynamic macroeconomic model is presented. It is designed in order to be bi-directionally coupled to the GASM, with the purpose to observe the evidence of the mutual influence between financial markets and macroeconomic variables. In particular, the level of financial resources affects real economic activity, not only in the short run but also in the long run. Special attention has been dedicated to the effects of the variation of credit supply in relation to marginal propensity to consume. Results show that real economic activity is strongly affected by the level of credit supply in relation to the level of savings in the long run, and that this influence is not trivial.

2. The model

The proposed model of economy is characterized by three classes of agents: firms, consumers, and banks and three classes of assets: physical capital, savings and debt securities. Physical capital is owned by firms and it is employed with labor to produce output. Output is a single homogeneous good which can be used both for consumption and investment. Firms do not retain profits and issue debt securities to finance investments. Consumers provide the labor force and own all the equities of firms and banks. Consumers income is composed by salary, profits of firms and banks, and interest on cumulated savings. A fraction of consumers income is used to buy goods and the rest is saved. Banks collect consumers savings and own debt securities issued by firms.

Firms are taken heterogeneous whereas consumers and banks are considered homogeneous. Therefore, a representative consumer and a banking sector are considered.

Two markets are open at each time period: the good market and the credit market. The two markets are cleared simultaneously. Clearing gives the price of goods and the interest rate (the price of credit) at which demand of goods (or credit) matches supply.

2.1. FIRMS

The economy consists of \mathcal{N} firms. Each firm produces the same single good subject to the same production function. The real output $Y_{i,t}$ of the firm i at any instant t is given by the following production function:

$$Y_{i,t} = K_{i,t}^\alpha N_{i,t}^\beta \quad i = 1, \dots, \mathcal{N}, \quad (1)$$

where $Y_{i,t}$ is the number of produced goods, both consumer and capital goods, and $K_{i,t}$ and $N_{i,t}$ are the factors of production, i.e. capital and labor, respectively, employed by firm i at time t . The production function is assumed to be characterized by positive though diminishing marginal products of capital and labor. Thus, $\alpha, \beta \in (0, 1)$.

At each time instant, firms decide the optimal levels of production $Y_{i,t}$ and investment $I_{i,t}$ according to a profit maximizing behavior with respect to factors of production.

2.1.1. Production

The production decision $Y_{i,t}$ of each firm at time t is taken according to the maximization of an objective function representing economic profit. The maximization is taken only with respect to the employment of labor N_i , while capital stock K_i is considered as fixed. Firms are able

to vary employment instantaneously while they are unable to change the capital stock simultaneously with the production decision.¹ Firms are supposed to operate in a competitive labor market in which, at any moment, they can hire all the labor they want at the going money wage w measured in euro per man per unit of time. The objective function of each firm is its economic profit Π_i^e defined as:

$$\Pi_{i,t}^e = p_t K_{i,t}^\alpha N_{i,t}^\beta - w_t N_{i,t} - p_t (r_t - \pi_{i,t}) K_{i,t} \quad i = 1, \dots, \mathcal{N}, \quad (2)$$

where p_t is the price of the produced good (both consumer and capital good), r_t is the instantaneous rate of interest on debt securities and $\pi_{i,t}$ is the rate of increase in the price of capital goods expected by the i th firm. The difference $r_t - \pi_{i,t}$ can be regarded as the real interest rate perceived by firm i or also as the cost of capital. The economic profit Π_i^e differentiates from the accounting profit Π_i^a defined as:

$$\Pi_{i,t}^a = p_t K_{i,t}^\alpha N_{i,t}^\beta - w_t N_{i,t} - r_t L_{i,t}, \quad i = 1, \dots, \mathcal{N}, \quad (3)$$

¹ Assuming that capital is fixed to each firm at each time step implies the absence of a perfect market for the existing stock of capital, in which individual firms can purchase or sell (or rent) capital, and so make a simultaneous change in their capital stock. The absence of this market might be rationalized by posting that, once in place, capital becomes completely specialized to each firm. Firms simply have no use for the existing capital of another firm, so that there is no opportunity for making a market in existing capital.

where $L_{i,t}$ is the debt accumulated by firm i th at time t . The accounting profit measures the real cash flow of firms but does not take into account the cost-opportunity of capital which should be considered in the decision making process of production and investment.

The price p_t of produced goods is the clearing price of the goods market at time t , i.e., the price at which real aggregate supply $Y_t = \sum_i Y_{i,t}$ is equal to real aggregate demand Z_t , composed by the real demand for consumption $Z_{C,t}$ of the representative consumer and the real demand for investment $Z_{I,t}$ made by firms themselves. Then, price is not exogenously given and, strictly speaking, firms are not price takers. However, the large number \mathcal{N} of firms and their heterogeneity allow us to make the simplifying assumption that the dependence of the value of p_t on the decision making of the single firm is negligible. Then, firms are assumed to be price takers.

Being capital stock K_i fixed, the first-order condition for maximization of Eq. 2 with respect to N_i

$$\frac{\partial \Pi_i}{\partial N_i} = 0, \quad (4)$$

gives the expression for $N_{i,t}$:

$$N_{i,t} = \left(\beta \frac{p_t}{w_t} \right)^{1/(1-\beta)} K_{i,t}^{\alpha/(1-\beta)} \quad i = 1, \dots, \mathcal{N}. \quad (5)$$

Eq. 1 combined with Eq. 5 gives the expression for the output of each firm at time t :

$$Y_{i,t} = \left(\beta \frac{p_t}{w_t} \right)^{\beta/(1-\beta)} K_{i,t}^{\alpha/(1-\beta)} \quad i = 1, \dots, \mathcal{N}. \quad (6)$$

2.1.2. Investment

The investment decision of each firm $I_{i,t}$ is taken according to the maximization of the economic profit defined in Eq. 2 with respect to both employment N_i and capital stock K_i :

$$\frac{\partial \Pi}{\partial N_i} = 0, \quad (7)$$

$$\frac{\partial \Pi}{\partial K_i} = 0. \quad (8)$$

The joint resolution of Eq. 7 and Eq. 8 gives the optimal capital stock $K_{i,t}^*$ that firm would employ at time t :

$$K_{i,t}^* = \left(\beta \frac{p_t}{w_t} \right)^{-\beta/(\alpha+\beta-1)} \left(\frac{r_t - \pi_{i,t}}{\alpha} \right)^{(1-\beta)/(\alpha+\beta-1)} \quad i = 1, \dots, \mathcal{N}. \quad (9)$$

If $K_{i,t}^* > K_{i,t}$, firm i plans an investment $I_{i,t}^*$ in order to fill the gap between actual capital stock $K_{i,t}$ and desired one $K_{i,t}^*$. Because capital stock can not vary instantaneously, firms can employ the desired capital stock only at the successive instant $t + 1$. The dynamics of capital

accumulation is thus as:

$$I_{i,t} = \max(0, K_{i,t}^* - K_{i,t}) \quad i = 1, \dots, \mathcal{N}, \quad (10)$$

$$K_{i,t+1} = K_{i,t} + I_{i,t} \quad i = 1, \dots, \mathcal{N}. \quad (11)$$

It is worth noting that the values of α and β have to be restricted according to the relation $\alpha + \beta \neq 1$, due to the singular point of Eq. 9. Moreover, in order to take into account a realistic inverse dependence between desired real capital stock and real interest rates, the exponent $(1 - \beta)/(\alpha + \beta - 1)$ should be negative. These issues are addressed by restricting the bounds of α and β from $\alpha, \beta \in (0, 1)$ to $\alpha, \beta \in (0, 0.5)$.

The value of the interest rate r_t is determined in the credit market and is the value at which aggregate nominal demand for investment $p_t Z_{I,t} = p_t \sum_i I_{i,t}$ is equal to the credit M_t supplied by the banking sector.

The dynamics of firms debt is given by:

$$L_{i,t+1} = L_{i,t} + p_t I_{i,t} \quad i = 1, \dots, \mathcal{N},. \quad (12)$$

2.2. THE REPRESENTATIVE CONSUMER

The nominal income U_t of the representative consumer is composed by labor wages, interests on savings, accounting profits of firms and profits made by the banking sector, i.e.,

$$U_t = w_t \sum_i N_{i,t} + r_t D_t + \sum_i \Pi_{i,t}^a + \Pi_{B,t}, \quad (13)$$

where D_t represent cumulated savings. $\Pi_{B,t}$ are the profits made by the banking sector defined as the difference between interests received for loans L_t to firms and interests paid to the representative consumer for its deposits D_t

$$\Pi_{B,t} = r_t(L_t - D_t). \quad (14)$$

By substituting Eq. 14 and Eq. 3 into Eq. 13, the following identity holds:

$$U_t = p_t Y_t, \quad (15)$$

which means that the representative consumer collects the overall income of the economy. This income is partially saved and partially consumed. A one-period lag between the computation of incomes and the use of incomes for consumption and saving is assumed. If γ is the propensity to save, or saving rate, with $\gamma \in (0, 1]$, and C_t and S_t indicate

respectively the nominal consumption the nominal saving, we have:

$$C_t = (1 - \gamma)U_{t-1} = (1 - \gamma)p_{t-1}Y_{t-1}, \quad (16)$$

$$S_t = \gamma U_{t-1} = \gamma p_{t-1}Y_{t-1}. \quad (17)$$

Nominal saving S_t at time t increases the stock of cumulated savings D_t which are deposited in the bank:

$$D_{t+1} = D_t + S_t. \quad (18)$$

2.3. THE BANKING SECTOR

The banking sector collects savings of the representative consumer and provides credit to firms. The model assumes that the level of credit M_t supplied at time t is a fraction η of the nominal income of the economy at time $t - 1$, i.e.,

$$M_t = \eta p_{t-1}Y_{t-1}. \quad (19)$$

Both η and γ are intended to be two key control parameters of the model. Next section presents a study of different trajectories of the macroeconomic variables of the model for different values of the control parameters.

3. Simulation results

Different simulations have been performed with values of α and β both set to 0.3. Perception of expected inflation by firms $\pi_{i,t}$ has been assumed to depend on past inflation f_t , where f_t is given by:

$$f_t = \frac{p_t - p_{t-k}}{p_{t-k}}, \quad (20)$$

and $\pi_{i,t} = \xi_i f_t$. Values of ξ_i have been uniformly drawn in the range (0.75, 1.25) at the beginning of the simulation and the time window k for the computation of inflation has been set to 4. The nominal wage w has been assumed to grow at the same rate of nominal GDP. Initial conditions are the followings: $K_{i,0} = 10, L_{i,0} = 0, D_0 = 0 \forall i$.

Different values of the saving rate γ and of η have been investigated. The saving rate covers a realistic range of values from 7% to 14% of the nominal income, whereas η varies from 9% to 16%. The dynamics of the model depends on the simultaneous clearing of the good market and the credit market and this clearing determines price p and interest rate r . Trajectories representing the time evolution of the price level p and of the interest rate r for different values of η and γ are presented.

Figure 1 shows the evolution of the price level in a semi-logarithmic scale. Prices exhibit an exponential growth whose constant rate, i.e.,

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inflation, corresponds to the lines slopes. Fixed η to 13%, the rate of exponential growth increases monotonically with decreasing γ . This shows that the price level grows as the marginal propensity to consume rises. Moreover, fixing the saving rate γ , a higher credit supply also increments the price level growth, as shown in the bottom of Fig. 1. Fig. 2 indicates that inflation does not depend on the individual values of parameters η and γ but depends only on their difference.

The trajectories of nominal interest rate are reported in Fig. 3 with fixed η (top) and fixed γ (bottom). Results suggest that a decrease of the saving rate produces a rise of nominal interest rates. Besides, nominal interest rates are also increased by an increment of η .

Figures from 4 to 7 illustrates the growth of real and nominal GDP. In Fig. 4, where η is fixed, it can be observed that nominal GDP exhibits a constant exponential growth that increases with consumption. However, results are very different for real GDP whose rate of growth rises with savings in the long run. The effects of credit supply on GDP is presented in Fig. 5, where γ is constant. It can be noticed that the exponential growth of nominal GDP is a monotone increasing function of the amount of credit supply. The time evolution of real GDP deserves special attention. The short run confirms the results of nominal GDP, i.e., GDP growth rises with credit supply. Conversely, in the long run this growth effect is not assured. The simulation clearly shows that the

trajectory for $\eta = 0.14$ and $\eta = 0.16$ crosses at about time 100. It is worth noting that the growth rate of real GDP is sub-exponential. This is probably due the firms production functions, characterized by diminishing marginal products of capital and labor.

Figure 6 evidences that the growth rate of nominal GDP as a function of both η and γ is linear. Moreover, as in the inflation case, nominal GDP does not depend on the individual values of parameters η and γ but depends only on their difference.

Figure 7 reports the average values of growth rates for real GDP for different values of η and γ . This figure highlights several interesting features. First, the growth rate of real GDP with respect to the difference $\eta - \gamma$ is not monotonic. Second, growth does not depend only on $\eta - \gamma$ but also on their individual values, unlike in the case of nominal GDP. Third, the growth functions are non-linear and present maximum points, which strongly depend on the individual values of η and γ . These findings suggest the existence of several couples of parameters (η, γ) as optimal choices for the long run economic growth.

4. Discussion and conclusions

In this paper a dynamic macroeconomic model in a multi-agent context has been presented. The distinctive feature of the model is that heterogeneous firms make production and investment decisions which are constrained by the consumers choices and credit supplied by the banking sector. The simultaneous clearing of the goods and the credit markets determines the price of goods and the price of credit (interest rate) at which transactions occur. Credit supply and saving rate, controlled respectively by parameters η and γ are the fundamental drivers of the system.

Simulations have been performed for different values of η and γ . Results show that behavior of nominal variables depends only on the difference between η and γ , whereas real variables are also influenced by individual values of the two parameters. Furthermore, the study has shown that behaviors of real variables that are valid in the short-run are often no more valid in the long-run. For instance, a rise of credit supply to firms determines a faster growth of real GDP that does not necessarily last in the long-run.

To give an intuition behind these results, it is worth noting that the difference $\eta - \gamma$ drives the net inflow of credit-money in the system. The value $1 - \gamma$ sets the amount of money which is used for consumption

and thus returns to firms. On the other hand, the fraction γ of the nominal income, that is saved, leaves the production-consumption process. Parameter η rules the quantity of credit-money lent to firms, and therefore determines the money injected into the production-consumption process by means of purchasing goods for investment. This explains the monotonic dependence of nominal variables on the difference $\eta - \gamma$. However, concerning GDP, the dependence is far less trivial and the main result of the model is that a positive net inflow of credit-money in the system may not correspond to faster long-run real growth. This finding may find a justification in the fact that a positive net inflow of credit-money produces higher inflation which can determine a slower capital accumulation, due to more expensive investment goods.

The model presented in this paper is autonomous and uncoupled to the GASM, but it presents proper characteristics to be coupled to the GASM in the future, with the purpose to observe the evidence of the mutual influence between financial markets and macroeconomic variables.

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Fruitful discussions with Gian Carlo Cainarca and Sergio Focardi are acknowledged. This work has been supported by the University of Genoa and the Italian Ministry of Education, University and Research (MIUR) under grant FIRB 2001.

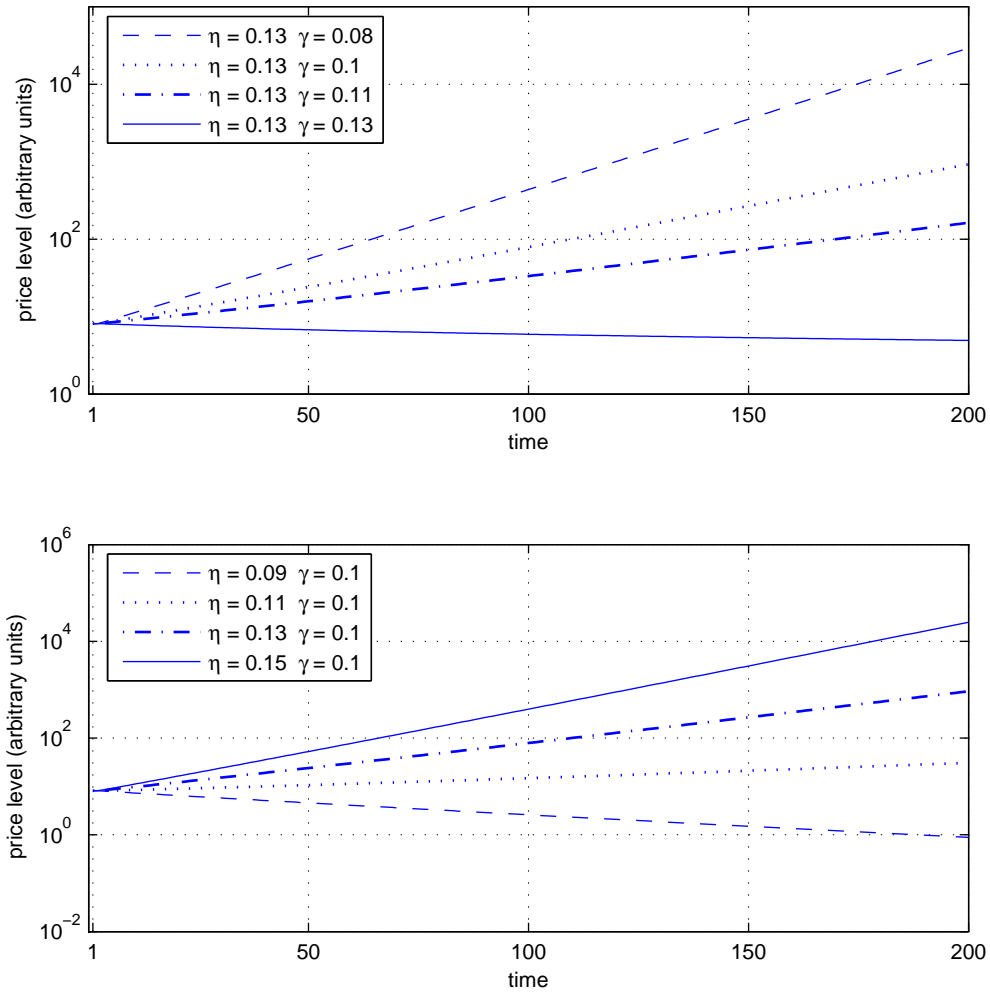


Figure 1. Time evolution of the price level with fixed η and variable γ (top) and fixed γ and variable η (bottom).

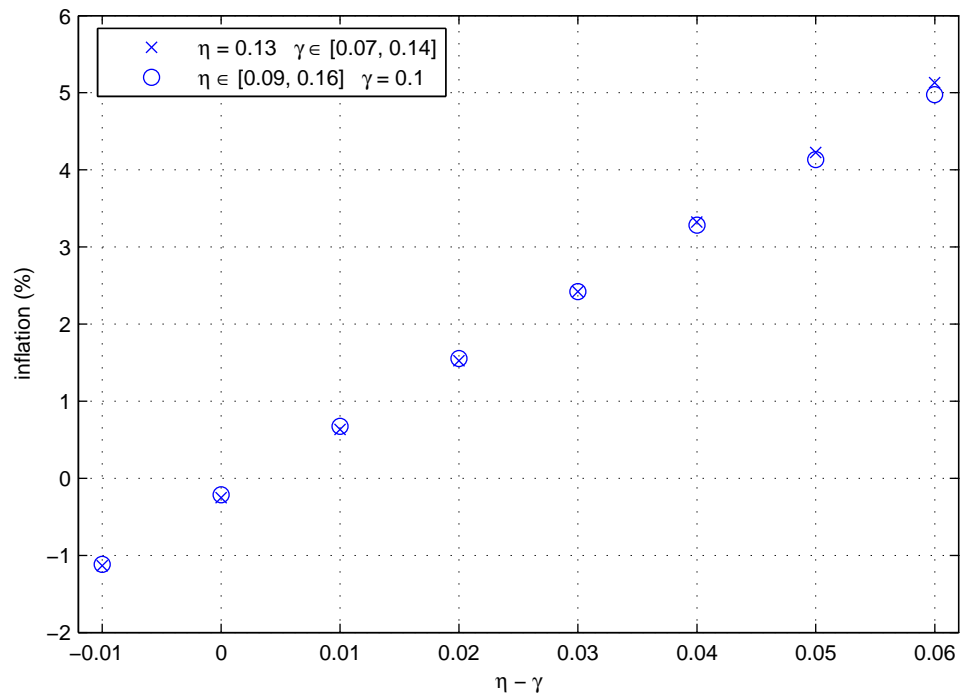


Figure 2. Inflation rate as a function of $\eta - \gamma$.

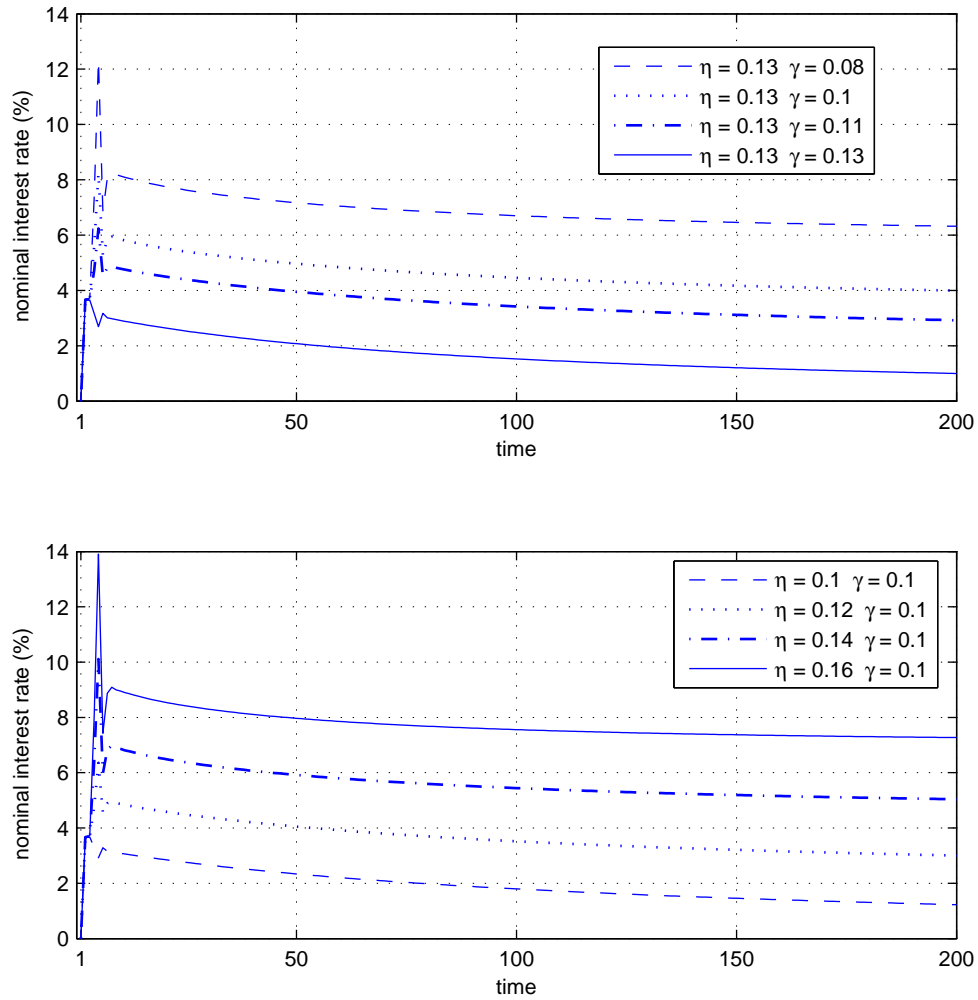


Figure 3. Time evolution of nominal interest rate with fixed η and variable γ (top) and fixed γ and variable η (bottom).

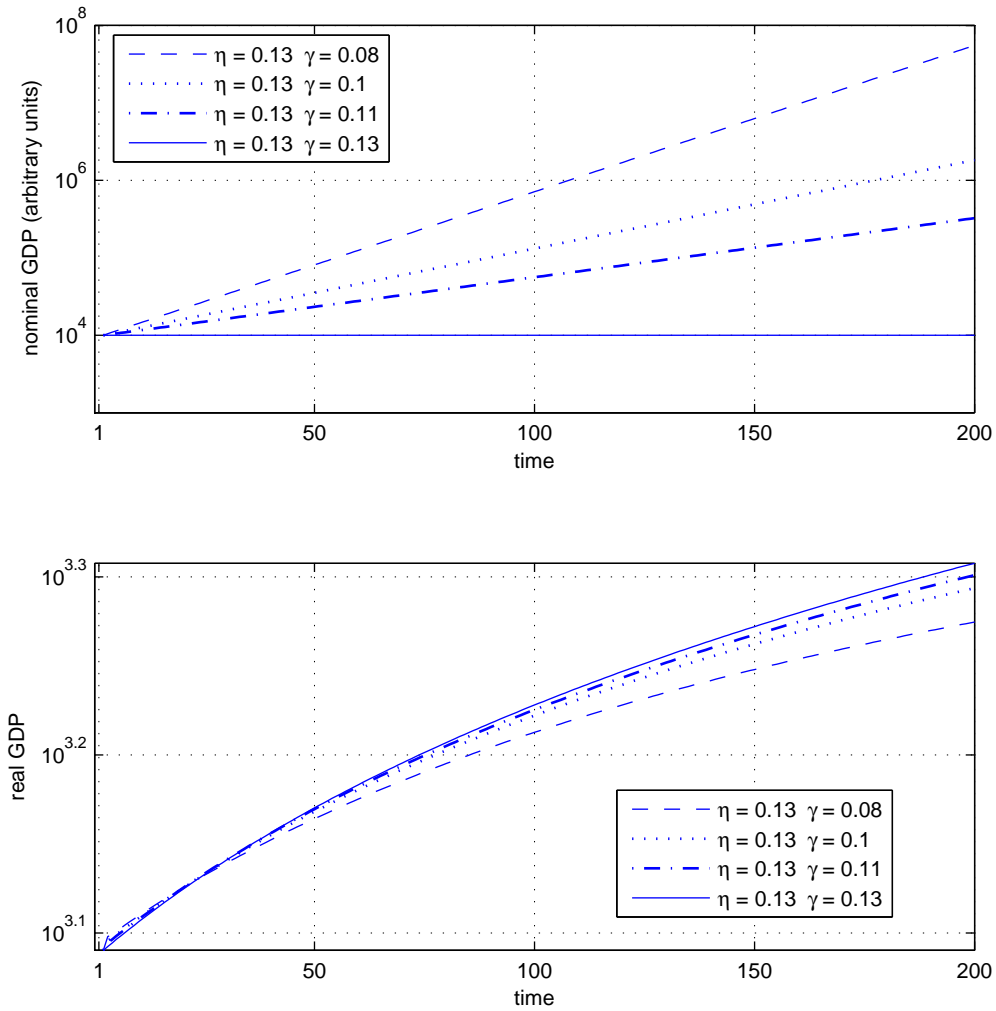


Figure 4. Time evolution of nominal GDP (top) and real GDP (bottom). η is fixed while γ varies.

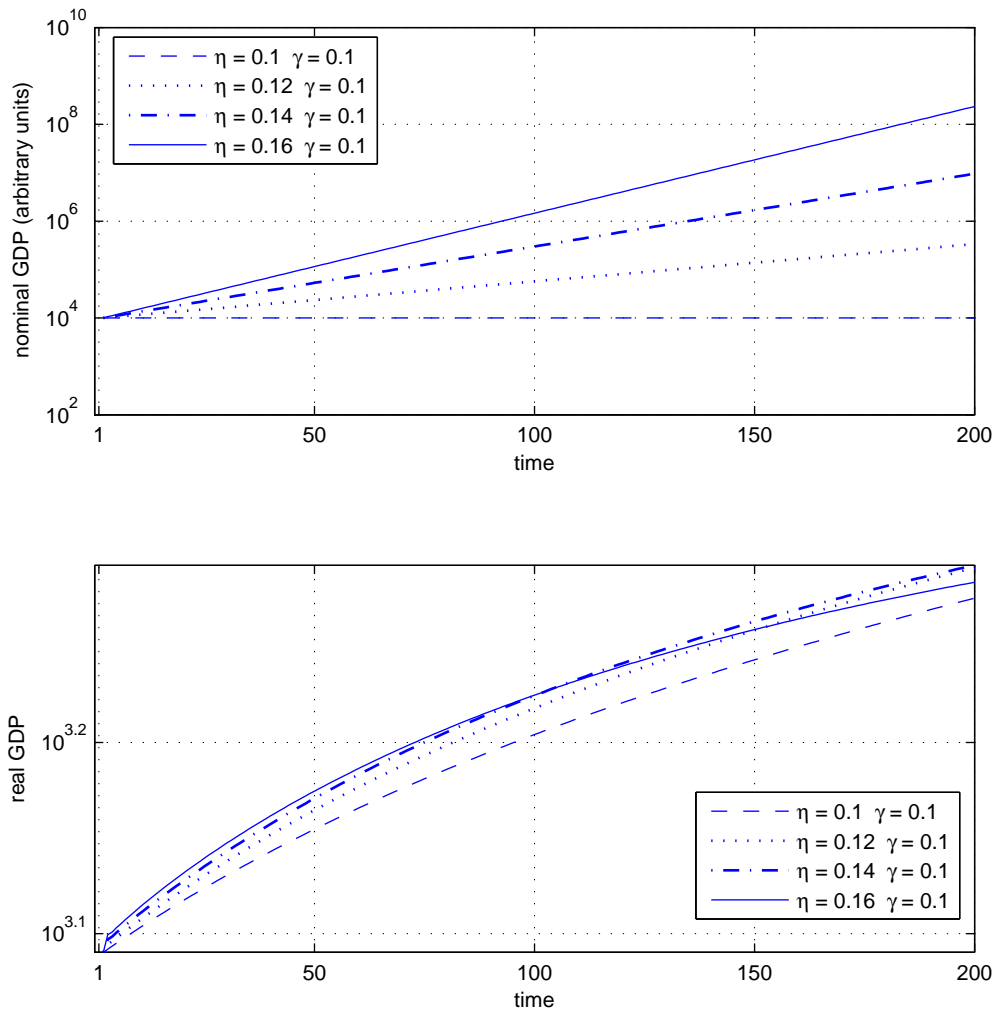


Figure 5. Time evolution of nominal GDP (top) and real GDP (bottom). γ is fixed while η varies.

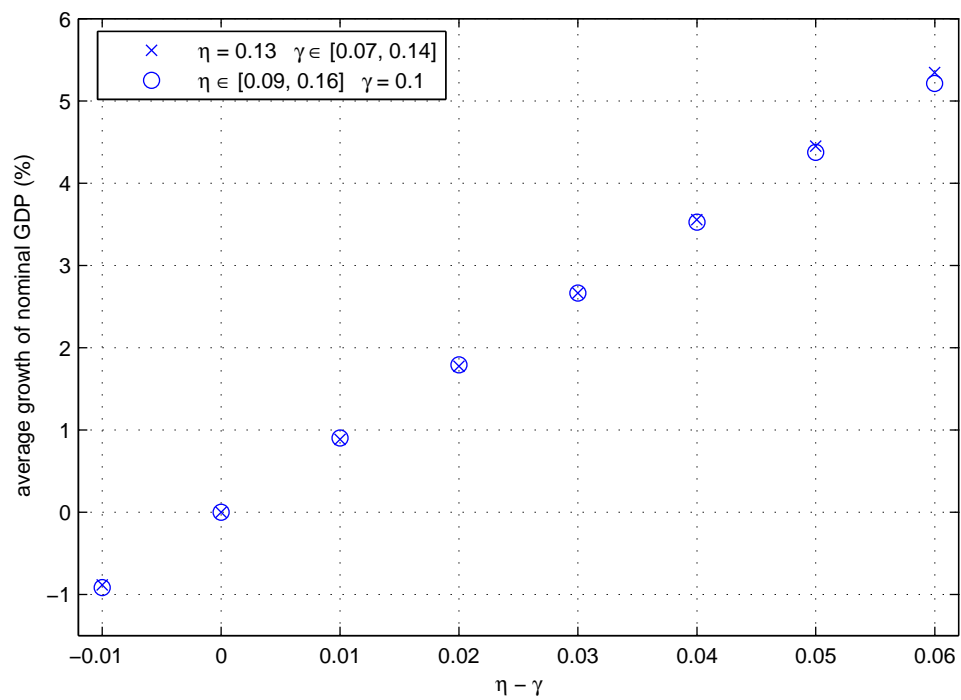


Figure 6. Nominal GDP growth as a function of $\eta - \gamma$.

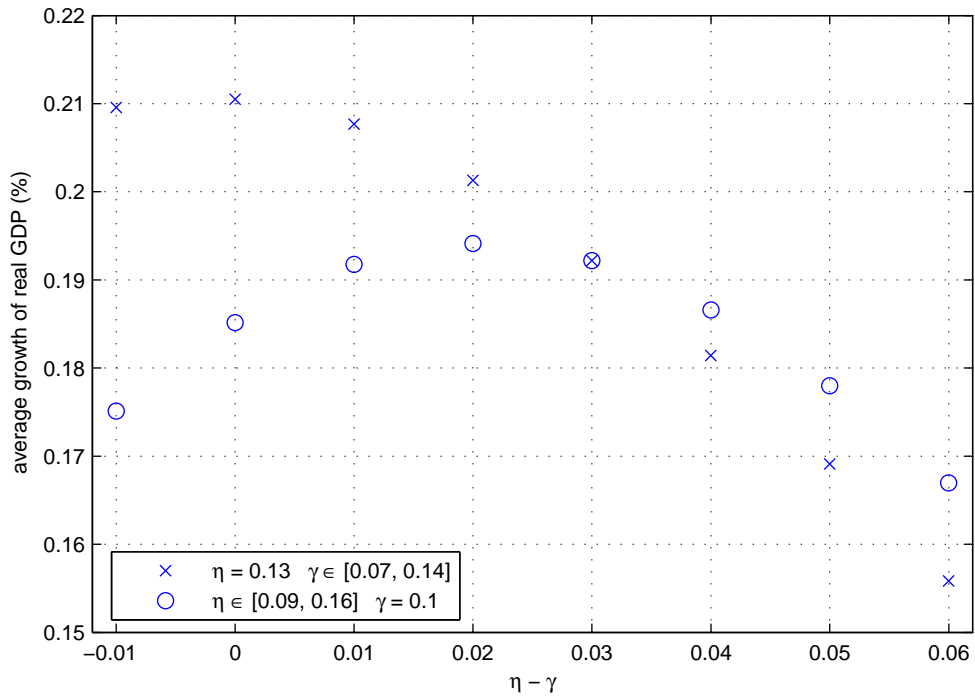


Figure 7. Average growth of real GDP as a function of $\eta - \gamma$.

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