# *The promises and perils of Agent-Based Computational Economics* ♣

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**Abstract**

In this paper I analyse the main strengths and weaknesses of agent-based computational models. I first describe how agent-based simulations can complement more traditional modelling techniques. Then, I rationalise the main theoretical critiques against the use of simulation, which point to the following problematic areas: (i) interpretation of the simulation dynamics, (ii) estimation of the simulation model, and (iii) generalisation of the results. I show that there exist solutions for all these issues. Along the way, I clarify some confounding differences in terminology between the computer science and the economic literature.

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## Introduction

From time to time, it happens that some innovating methodologies are brought in the modelling arena, causing great excitements in some researcher, and great irritation in others. Probably the last self-proclaimed 'revolution' in our discipline is the rise of Computational Economics. Among the enthusiast non-practitioners for instance stands Richard Freeman (1998, p.19). Talking about the 'War of the Models' and the need of new tools to compare different labour market institutions, he put great hopes in some of the techniques at the intersection of Evolutionary Economics, Computer Science and Cognitive Science:

> "Our empirical tools are wonderful for *ceteris paribus* problems, but many issues regarding labour institutions are *mutatis mutandis* problems. Lots of interrelated changes with no empirical counter-factuals. This implies that if we are to make progress, we need something more in our tool bag. Game theory? A language and framework, but not sufficiently specific. General equilibrium? Too general and static. Then what? (…) There are a new set of theoretic and empirical tools that seem suited for the problem of analysing labour systems and the War of the Models. The tools range from theoretical simulations of nonlinear dynamic systems to a theoretic data-mining. Complexity analyses. Neural networks. Data-mining for knowledge discovery. Landscape models. Artificial agent simulated societies. Chaos theory. Complex adaptive systems. Nonparametric statistical tools of diverse shapes and sizes. Cellular automata. The hills are alive with the sound of new tools and jargon".

True, simulations have been around in Economics for years, although mainly for prediction purposes. But they are offer more than that. As Axelrod (1997) puts it, «[u]sing simulation for prediction can help validate or improve the model upon which the simulation is based. Prediction is the use which most people think of when they consider simulation as a scientific technique. But the use of simulation for the discovery of new relationships and principles is at least important as proof or prediction». Axelrod then cites the classical Schelling's (1978) model of residential tipping as an example of how a simple simulation model may provide «an important insight into a general process»[1], and Conway's Game of Life (Poundstone, 1985) as a demonstration that extremely complex behaviour can result from very simple rules. Simulations can also be used in order to test economic theories developed with more standard modelling approaches, with particular reference to the issues of robustness and the convergence to the equilibria, as stressed by Sargent (1993), or for data analysis, where - again with Freeman - «they offer nonparametric statistical models for dealing with nonlinearities and discovering patterns in large data sets».

Much debate has going on in recent years on the virtues and sins of these techniques. In this paper I review the main arguments in favour and against the use of agent-based simulation for economic modelling. In particular, I will briefly discuss what agent-based simulation models are (section 1) and how they *can* be used for economic modelling (section 2). Then, I will turn to rationalise the discontent many economists feel about this methodology. I will thus compare agent-based models and analytical models with respect to the issues of *interpretation* of the results (section 3), *estimation* (section 4) and *generalisation* (section 5). A cross-cutting theme is the relationship between real and simulated data. Section 6 summarises and concludes. To my knowledge, the perspective adopted is an original contribution, offering a bridge between the simulation literature and the more traditional economic modelling culture.

## 1. Agent-based computational models

As Leigh Tesfatsion (2003), one of the pioneers of this technique in the field, puts it, «[a]gent-based computational economics (ACE) is the computational study of economies modelled as evolving systems of autonomous interacting agents. ACE

[1] Note that although agent-based simulation models clearly and heavily rely on the wonders of modern computers, it is not correct to characterize them as computer models. Schelling's model, for instance, requires nothing else but a sheet of paper and some buttons.

researchers rely on computational frameworks to study the evolution of decentralized market economies under controlled experimental conditions».

In a simulation agents represent the basic entities of the system, or an aggregation of basic entities. For instance, individuals, families, vacancies, firms, etc. can all be thought of as 'agents'. The interaction between different agents is regulated by a precise schedule. Aggregate behaviour is then reconstructed 'from the bottom up', i.e. by computing what emerges from the micro-behaviour of the agents. Agent-based simulations are modular in nature, and produce results that can be explored at different levels of aggregation, while always retaining their micro-foundation.

Agent-based simulation models have two distinguishing features. One is being 'agent-based', i.e. following a micro approach; the other is being simulation models, i.e. following an inductive approach to the discovery of regularities. Both features are not peculiar of the methodology. Many analytical models have micro-foundations, and many simulation models adopt an aggregate perspective, as in System Dynamics (Forrester, 1991). However, it is the conjunction of the two approaches that defines the methodology, allowing for a great flexibility in the design of the model, while avoiding all the problems connected with merely aggregate representations of the world (Kirman, 1999, 2001a). Agent-based simulation models are a third way between fully flexible but not computable and hardly testable *literary models* (which provide no more than a verbal description of the causal relationships behind a given phenomenon) on one side, and more transparent but highly simplified *analytical models* (Ostrom, 1998; Gilbert and Terna, 2000). The biggest advantage of ACE models with respect to the analytical approach is their *flexibility,* since the results are computed and need not to be solved analytically. With ACE, the researcher gains almost complete freedom about the specification of the interaction structure and the individual behaviour[2].

One of the main purposes for writing an ACE model is the desire to gain intuitions on the two-way feedback between the microstructure and the macrostructure of a phenomenon of interest (Kirman, 2001b). How is it that simple aggregate regularities may arise from individual disorder? Or that a nice structure at an individual level may lead to a complete absence of regularity in the aggregate? How is it that the complex interaction of very simple individuals may lead to surprisingly complicated aggregate dynamics? Or that sophisticated agents may be unable to organize themselves in any interesting way? Another, related, purpose «is to use ACE frameworks normatively, as computational laboratories within which alternative institutions, market designs, and organizational structures in general can be studied and tested with regard to their effects on individual behaviour and social welfare» (Tesfatsion, 2003). Both concerns relate to the importance of *interaction* in shaping social structures and individual and aggregate dynamics.

## 2. ACE in practice

Building on Robert Axtell (2000), it is possible to identify three distinct uses of agent-based computation in the social sciences, and rank them according to their auxiliary nature, with respect to analytical modelling[3]. The first use is *numerical computation of analytical models*. Note with Axtell that «[t]here are a variety of ways in which formal models resist full analysis. Indeed, it is seemingly only in *very* restrictive circumstances that one ever has a model that is completely

[2] As it is always the case, this freedom requires caution. With less need to adopt standard modelling frameworks, models become less comparable. And a greater dispersion in modelling choices leads to a greater variability in the quality of ACE works.

[3] The three categories identified below correspond only partially to Axtell's.

soluble, in the sense that everything of importance about it can be obtained solely from analytical manipulations». Situations in which resort to numerical computation may prove useful include (a) when a model is not analytically soluble for some relevant variable, (b) when a model is stochastic, and the empirical distribution of some relevant variable needs to be compared with the theoretical one, of which often few moments are known, (c) when a model is solved for the equilibrium, but the out-of-equilibrium dynamics are not known. In particular, with reference to the last point, it may happen that multiple equilibria exist, that the equilibrium or (at least some of) the equilibria are unstable, that they are realized only in the very long run. Conversely, it may happen that equilibria exist but are not computable. Axtell (2000) provides references and examples for each case. Finally, it may be the case that the equilibrium is less important than the out-of-equilibrium fluctuations or extreme events.

Clearly, agent-based simulations are not the only way to perform numerical computations of a given analytical model. However, they may prove effective and simple to implement, especially for models with micro-foundations. The second use is *testing the robustness of analytical models* with respect to departures from some of the assumptions. Assumptions may relate to the behaviour of the agents, or to the structure of the model. ACE models can easily include bounded rationality (Sargent, 1993; Leijonhufvud, 1993; Conlisk, 1996) and heterogeneity at an individual level, and investigate variations in the way agents interact with each other or with the institutional setting. One important feature of ACE is that in considering departures from the assumptions of the reference model, a number of different alternatives can be investigated, thus offering intuition towards a generalization of the model itself.

The first two uses of ACE models are *complementary* to mathematical analysis. The third use is a *substitute*, going beyond the existence of an analytical reference model. It provides *stand-alone simulation models* for (a) problems that are analytically intractable, or (b) problems for which an analytical solution bears no advantage. The latter may happen when negative results are involved, for instance. A simulation may be enough to show that some institution or norm is wrong, or does not work in the intended way. Analytical intractability may arise when more complicated assumptions are needed, or when the researcher wants to investigate the overall effect of a number of mechanisms (each possibly already analytically understood in simpler models), at work at the same time.

However, many economists remain sceptical about this methodology. Of the two characteristics of agent-based simulations, it is not the micro approach that is questioned. Actually, for what regards the micro-foundations criticism go the other way round from ACE practitioners towards traditional analytical modelling, where «[h]eavy reliance is placed on externally imposed coordination devices such as fixed decision rules, common knowledge assumptions, representative agents, and market equilibrium constraints. Face-to-face interactions among economic agents typically play no role or appear in the form of highly stylised game interactions» (Tesfatsion, 2003). Rather, the main concern of economic orthodoxy is that simulations, as opposed to analytical modelling, "do not prove anything". This claim turns out to target three different problems of agent-based simulations. The first one is how to recover the input/output transformation function (the reduced form) implied by the simulation model, and is mainly relevant at a theoretical level in order to gain a better understanding of the behaviour of the system. Only *artificial data*, i.e. data produced by the simulation model itself by varying the parameters, are involved here. The second one is how to estimate the structural coefficients, and it is relevant at an empirical level, in order to allow comparison of the model with *real data*. The third one is how to ensure that the model will be able to make predictions also for cases that are not in the data on which the model is validated. It is directly related to the issue of the generalization of the results. The three problems are discussed in the next sections.

## 3. Recovering the reduced form

In a micro model, being it an analytical model or a simulation one, the behaviour of each agent is completely specified, but generally dependent on the behaviour of other agents. This (structural) description of the system is generally not enough. In order to gain a full understanding of how the model works, individual and aggregate behaviour must be explained in terms of the (exogenous) parameters and variables, in a reduced form. Let $y_i$ be a vector of dependent individual variables, $x_i$ a vector of independent individual variables and $Y$ a vector of endogenous aggregate variables. The micro model specifies:

[1] $$y_i = s(x_i , y_{-i} , Y ; \alpha_i)$$

with $y_{-i}$ , $x_{-i}$ being the vector of (dependent and independent) characteristics for all individuals different from $i$, and $\alpha_i$ being the coefficient vector.

By solving an analytical model, if possible, we find the only one reduced form corresponding to the structural form of the model, both at an individual and at an aggregate level:

[2] $$y_i = f(x_i , x_{-i} ; \beta)$$
$$Y = F(X ; \gamma)$$

with $X = [x_i]$ being the matrix containing the independent variables for all individuals, and $\beta$ and $\gamma$ being the coefficient vectors.

In a simulation model, these reduced forms remain unknown, and only *inductive* evidence about the input/output transformation implied by the model can be collected. In other words, simulations suffer from the problem of stating general propositions about the dynamics of the model starting only from point observations. Due to this 'original sin', simulations are considered to be less general than analytical models. Since scientific explanations are generally defined as the derivation of general laws, which are able to replicate the phenomena of interests (Hempel and Oppenheim, 1948; Hempel, 1965), simulations appear to be less 'scientific' than analytical models. Note that this is not equivalent to saying that simulations are an inductive way of doing science[4]. As Axelrod (1997) points out, «[s]imulation is a third way of doing science. Like deduction, it starts with a set of explicit assumptions. But unlike deduction, it does not prove theorems. Instead, a simulation generates data that can be analyzed inductively». Induction comes at the moment of explaining the behaviour of the model.

Why is the problem of recovering the reduced form important? After all, we have the structural form, which in general has a larger informational content. The fact is that, even if a simulation model is able to produce 'interesting' dynamics - for instance in accordance to some stylised facts - two issues are raised. First, a casual explanation of these dynamics, in terms of the simulation inputs, is often useful[5]. Second, the possibility of the system showing other 'undesired' dynamics has to be ruled out. The first issue is the problem of *interpreting the simulation results*; the second is the problem of *sampling the parameter space*. A third problem, namely the relevance of the reduced form for estimation purposes, is discussed in the next section.

When Axtell (2000) argues that «each run of such a model yields is a sufficiency theorem, [yet] a single run does not provide any information on the robustness of such theorems», he implicitly refers to the second issue. However, he doesn't

[4] actually, they are a form of *abductive* inference (Leombruni, 2002). The relationship between induction, abduction and causal explanation has been extensively investigated in the philosophy of science literature, and will not be reviewed here (see for instance Peirce, 1955).

[5] this allows to provide answers to questions like "what happens, *in the model*, to the unemployment rate if we increase the reservation wage?" or, more generally, "which are the determinants of the unemployment rate, *in the model*?"

stress that such theorems are hardly useful, if they simply connect 'point-to-point' inputs to outputs (in this sense they are far from the theorems Axelrod refers to). To have more interesting 'laws', connecting *variation* in inputs to *variation* in outputs, an *interpretation* of the underlying dynamics is needed.

### 3.1 Interpretation

Let's start with the interpretation problem, i.e. how to extract the 'laws' governing the system from the observed regularities in the simulated data. This amounts to find functions $g$ and $G$ and coefficients $\phi$ and $\varphi$ that are good proxies for the real but unknown functions $f$ and F in [2]:

[3]
$$y_i = h(x , x_{-i} \; ; \phi)$$
$$Y = H(X \; ; \varphi)$$

Note that the coefficients $\phi$ and $\varphi$ are not estimates of the 'true' reduced form coefficients $\beta$ and $\gamma$.

The use of econometric techniques to approximate the input/output transformation function, starting from a number of - somehow designed - artificial experiments is indeed a common practice in the computer science literature. The resulting regression model is also known as metamodel, response surface, compact model, emulator, etc. (Kleijnen, 1998).

In estimating the reduced form on the artificial data, one functional form must of course be chosen. Having specified the micro-rules of the artificial world, the researcher generally knows which parameters affect the outcome variable of interest, even if sometimes, in complicated models, the causal link between inputs and outputs may be quite indirect, and thus remain at first unnoticed. Moreover, there are methodologies to reconstruct the causal structure from statistical data (see for instance Hendry and Krolzig, 2002; Glymour and Cooper, 1999). Of course, the final choice of a functional form remains to a certain extent arbitrary, and may lead to very different specifications of the aggregate laws of the system. But as long as two different specifications – say *g* and *g'*, or *G* and *G'* - provide the same description of the dynamics of the model in the relevant range of the parameters and the exogenous variables, we should not bother too much about which one is closest to the 'true' reduced form[6].

A brief example may clarify. Suppose to have a structural model of demand and supply:

[4]
$$\begin{cases} q_D = d_0 - d_1 p + d_2 Z + u \\ q_S = s_0 + s_1 p + s_2 W + v \end{cases}$$

The corresponding reduced form for equilibrium price is:

[5]
$$p = \frac{d_0 - s_0 + d_2 Z - s_2 W + u - v}{s_1 + d_1} = \gamma_0 + \gamma_1 Z + \gamma_2 W + \varepsilon$$

In a simulation model system [4] has no analytical formulation. Thus, the reduced form [5] is unknown. However, a number of artificial experiments can be designed and performed. Artificial data on inputs (*Z* and *W*) and outputs (*q* and *p*) are collected. Then, after some data mining we could come out with the following specification:

[6]
$$p = \varphi_0 + \varphi_1 Z + \varphi_2 W + \varphi_3 Z \cdot W + \varepsilon$$

[6] Here, the distinction between in-sample and out-of-sample values, and the objection that two formulations may fit equally well the first, but not the latter, is not meaningful. Any value in the relevant range can be included in the artificial experiments. The topic is discussed with reference to (structural) model selection in a later section.

Finally, after having estimated in our artificial data the parameters $\varphi$, we have our approximation of the reduced form [2]:

$$[7] \qquad p = \hat{\varphi}_0 + \hat{\varphi}_1 Z + \hat{\varphi}_2 W + \hat{\varphi}_3 Z \cdot W$$

Note that this approximation can't be used for further estimation on real data. It has no unknown coefficients. It simply describes how the simulation model behaves, for given values of the structural parameters. As such, it can be used to assess whether the simulation model is able to mimic the phenomenon of interest, by imposing the same metamodel [6] on real data, and comparing the coefficient vector $\hat{\varphi}$ estimated on the *artificial* data with the coefficient vector $\tilde{\varphi}$ estimated on the *real* data.

Now, different coefficient vectors $\hat{\varphi}$ are obtained for different values of the structural parameters vectors *d* and *s*. Intuition may suggest that we are not far from being able to estimate the structural parameters themselves, for instance by minimising the distance between $\hat{\varphi}$ and $\tilde{\varphi}$. I leave a discussion on this topic to a dedicated section, while simply anticipating that, if the model is identified, estimates for the structural parameters can be obtained. Here, we are still dealing with the goal of characterizing the unknown input/output transformation function [2]. Which values then should we choose for the structural parameters vectors *d* and *s*, in performing our simulation experiments? The most immediate answer is: their estimates! Plug them in, and keep them constant throughout all simulation runs. After all, who cares about the behaviour of the model for implausible values of the parameters? All we need is some sensitivity analysis around the estimated values of the structural coefficients. Note that the proposed approach involves a two-stage procedure, which is exactly the *opposite* of the standard practice. In dealing with analytical models, first comes the derivation of the reduced form, and then the estimation of the structural parameters.

A second approach allows for a broader description of the true input/output transformation function, but requires more computational burden. We may express the input/output transformation functions in a different way, including the structural parameters $\alpha$ in the specification:

[3']

$$y_i = g(x , x_{-i} , \alpha_i , \alpha_{-i} ; \phi')$$
$$Y = G(X , A ; \varphi')$$

with $A = [\alpha_i]$ being the matrix of all individual parameters. Then, in the artificial experiments variation in the value of the structural parameters is also included. However, even in the very simple example provided above, this may lead to a highly complicated specification for the metamodel to be estimated in the artificial data.

Moreover, when the structural parameters $\alpha$ are included but the structural model is not identified, there may be a number of vectors $\phi'$ and $\varphi'$ for each given specification $g$ and $G$, that approximate equally well the real input/output transformation implied by the model.

**3.2 Robustness**

The second problem concerns the possibility that the artificial data may not be representative of all outcomes the model can produce. In other words, it may happen that as soon as we move to different values of the parameters, the behaviour of the reduced form functions $f$ and $F$ will change dramatically, for example exhibiting singularities. The metamodels $g$ and $G$ will then become a poor description of the simulated world. While analytical results are conditional on the specific hypothesis made about the *model* only, simulation results are conditional both on the specific hypothesis of the model and the specific values of the *parameters* used in the simulation runs.

At a theoretical level, this critique can be contrasted with two observations. First, if it applies to what we know about the *artificial* world defined by the simulation model, it also applies to what we know about the *real* world. The real data generating process being itself unknown, stylised facts could in principle turn wrong, at some point in time. From an epistemological point of view, our belief that the sun will rise tomorrow remains a probabilistic assessment. Second, we should not worry too much about the behaviour of a model for particular 'evil' combinations of the parameters, as long as these combinations remain extremely rare[7]. If the design of the experiments is sufficiently accurate, the problem of how 'local' is the estimated local data generating process becomes marginal. «While the curse of dimensionality places a practical upper bound on the size of the parameter space that can be checked for robustness, it is also the case that vast performance increases in computer hardware are rapidly converting what was once perhaps a fatal difficulty into a manageable one» (Axtell, 2000).

## 4. Estimating the structural coefficients

A second critique of agent-based simulation models is the claim that they often cannot be identified in the data. Again, this critique need to be further specified. In particular, two different problems seem to be involved. A first issue deals with

[7] There is one relevant exception when rare events are the focus of the investigation, for instance as in risk management (Segre-Tossani, 2003). Here, simulations may prove extremely useful, by dispensing from making assumptions - such as the gaussian distribution of some relevant parameters - which may be necessary in order to derive analytical results but have unpleasant properties – like too thin tails. In a simulation, the reproduction of such rare events is limited only by the computational burden imposed to the computer. However, techniques can be used in order to artificially increase the likelihood of their occurrence. Particular combinations for the ranges of the relevant parameter can often be guessed, and oversampled in the artificial experiments [Kleijnen, 1998].

the possibility to recover the structural coefficients from the real data, assuming the model is indeed identified. A second issue deals with the claim that ACE models tend to be underidentified, due to the fact that they are less parsimonious than analytical models (after all, this is their main advantage, ACE guys say) and thus they often lack a sufficient number of exclusion restrictions. I now turn to contrast both statements.

### 4.1 Estimation of identified models

Let's start with the first one. In an analytical model the reduced form coefficients $\beta$ and $\gamma$ of eq. [2] can be estimated in the real data. If the model is identified, there is a one-to-one relationship between the structural and the reduced form coefficients. Thus, estimates for the structural coefficients $\alpha$ can be recovered. In a simulation model this can't be done. However, as already stressed, we could compare the outcome of the simulation with the real data, and change the structural coefficients values until the distance between the simulation output and the real data is minimised. In the simulation literature, this is called *calibration*. More precisely, we could choose some moments of the simulated data, and compare them with the moments of the true data:

[8] $$\Delta = L\left(m(Y;\alpha) - m(Y_{real})\right)$$

where $\Delta$ is the distance to be minimised with respect to $\alpha$, $L$ is a loss function weighting in some way the different moments, and $Y_{real}$ are the real data on the outcome variable of interest. Known as *method of simulated moments*, this approach has been introduced in the econometric literature by McFadden (1989) and Pakes and Pollard (1989), and has since then found a number of applications for structural models estimation[8]. Again, its main limitation lies in the computational burden it imposes, when the simulated models take a long time to

[8] For a general exposition of simulation-based estimation, and a review of applications in the empirical literature, see Stern (1997) and Liesenfeld and Breitung (1999)

run. However, the increasing power of modern computers, and the use of particular techniques to reduce the number of times the model has to be solved (Ackerberg, 2001; Stern, 2000) greatly reduce the impact of this computational limit.

### 4.2 Underidentification

A further critique says that the richer specifications of simulation models often lead to underidentification, due to the lack of exclusion restrictions. This claim seems to suggest that analytical models are characterised by lean specifications only to avoid the problem of underidentification, and not because of symbolic tractability.

Moreover, underidentification should not be the fear number one in writing a model. Rather, the inability of a model to provide a good description of the underlying phenomenon is a much greater limit. «Economic variables are considered by econometricians as mutually dependent, but the degree of simultaneity is recognized only to the extent that it does not prevent the structural coefficients from being identified. But is there any logical reason why the degree of simultaneity must always stop short of causing real troubles? The answer given in the literature is that economic theory or a priori information often requires us to exclude from a given structural relationship a sufficient number of variables so that it become overidentified. […] [Q]uite to the contrary, economic theory requires the inclusion of a much larger number of variables than those included in the existing models of economic structures. The complexity of modern economic societies makes it much more likely that the true structural relationships are underidentified rather than overidentified» (Liu, 1960). Simulation allows complicating models. This must be considered positively, not negatively, since a better description of the phenomena of interest becomes possible. The risk of underidentification is often simply unavoidable: analytical models that claim to be immune are sometimes only poor models. «When a reasonable structural relationship could be obtained either by dropping variables from, or adding variables to, an over-simplified relationship, the complexity of the modern economy ensures that the 'enlarged' estimate is a closer approximation to reality than the two simpler ones. Obviously, the high intercorrelations of the large number of explanatory variables so included would almost certainly result in large standard errors or even wrong signs for some of the estimated structural coefficients. When this happens, the only legitimate conclusion we can draw is that the complexity of economic reality and the nature of basic data are such that the structural coefficients are really indeterminate. The temptation to omit relevant variables until a seemingly reasonable and significant 'structural estimate' is obtained must be resisted» (*ibidem*).

## 5. The challenge of complexity

In the above-mentioned work, Liu referred to the 'complexity' of economic systems rather loosely. Now, it is somehow ironic that the goal of defining complexity seems to be itself 'complex'. A large literature has investigated what complexity means, and which characteristics complex system share[9]. Most definitions relate it to the property of a system «which makes it difficult to formulate its overall behaviour even when given almost complete information about its atomic components and their inter-relations» (Edmonds, 1999). The difficulty stems from the non-linear relationships between the components of the system, and between the microstructure and the macrostructure. A related feature is that the system seems to be able to respond in more than one way to its environment. As Allen (2001) puts it, «The 'choice' in response arises from the fact that non-linear processes within the system can potentially amplify microscopic heterogeneity hidden within it». ACE enthusiasts often link the future of the methodology to the claim that it is best suited for the analysis of non-linear complex systems. Since the world is intrinsically non-linear and complex, the

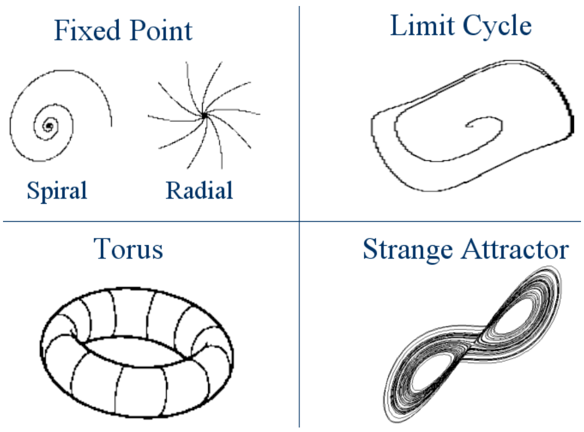
[9] for a recent survey, seeRichardson and Cilliers, 2001; it may also be interesting to look at Gell-Mann, 1995, in vol.1, no.1 of *Complexity*

argument goes, ACE should take the centre stage in the modelling arena (Goldspink, 2002). Now, it is true that non-linear models often resist mathematical analysis, while their implementation in an agent-based simulation setting brings little additional cost, with respect to linear models. But non-linear models of complex systems generally give raise to a number of problems, which may pose severe limits to the possibility of generalising the results. The first one is the already mentioned *sensitive dependence on initial conditions*, i.e. the well-known 'butterfly effect' (the possibility that a butterfly's wing in brazil may set off a tornado in Texas, in Lorenz original words). A second problem is *equifinality* (Von Bertalanffy, 1969; Richardson, 2002), i.e. the existence of many structural models characterised by the same fit with the real data, but different out-of-sample properties. A third problem is *overfitting*, i.e. the risk of having a model that is too complex, and may fit the noise, in addition to the signal in the data. The 'butterfly effect' challenges the possibility of fitting a reduced metamodel on the artificial data, and of estimating the parameters of the structural model. Equifinality on the other hand challenges the very possibility of choosing an appropriate specification for the structural model. Overfitting is a much narrower problem, simply increasing the risk of making a wrong specification choice.

### 5.1 Sensitive dependence on initial conditions

Complex systems, even when totally deterministic, often appear as "seas of disorder with islands of structure" (Gleick, 1987). Even supposing to have a model that perfectly reproduces the complex system under investigation, is it possible to 'reduce' it into an input-output transformation function?. In particular, if the system shows sensitive dependence on initial conditions, the exercise to write down a reduced form metamodel may become entirely worthless. It may simply be impossible to connect inputs to outputs, since any arbitrary small difference in inputs may lead to large deviations in outputs. However, it may be possible to describe the 'islands of structure' of the system, for instance by looking at its invariant sets[10] and attractors[11]. The metamodel could then describe where the system cannot go, or where it will probably go, instead of trying to predict where it will actually go.

Figure 1: Types of attractors




A second problem with the 'butterfly effect' appears when one tries to estimate the structural parameters of the model. As suggested above, this may be done by comparing the moments of its invariant distributions with the real data. However, since numerical simulations always require some degree of approximation, a question arises whether these moments remain sufficiently close to those of the 'real' model. Fortunately, there exist generalised laws of large numbers that guarantee that the moments computed from simulations of numerical solutions converge to the true moments as the approximation errors of the numerical solution converge to zero, for non-linear dynamic systems. Some of them require technical conditions that (a) are difficult to check and (b) impose severe limitations to the model structure (Doob, 1953, ch. 5). However, recent advances

[10] An invariant set, A, is a set such that any trajectory, x that starts in A stays in A for all time

[11] An attractor is a closed set A with the following properties: (i) A is an invariant set; (ii) A attracts all trajectories that start sufficiently close to it; (iii) there is no proper subset of A that satisfies conditions i and ii

(Santos and Peralta-Alva, 2003) have established generalised laws that rely on much simpler conditions, such as the compactness of the domain and continuity of the equilibrium solution.

### 5.2 Equifinality

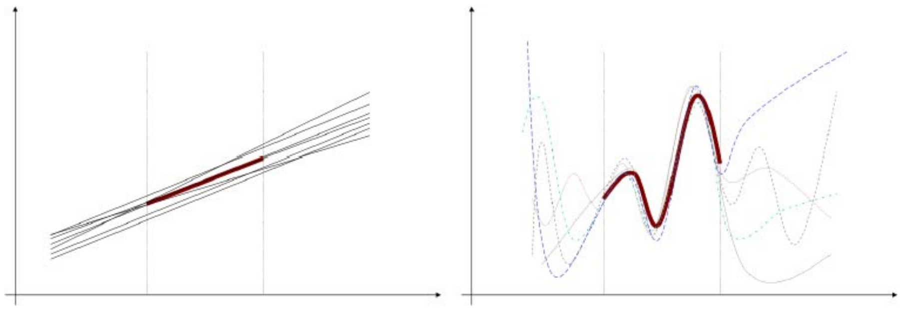
«[C]omputers offer a solution to the problem of incorporating heterogeneous actors and environments, and nonlinear relationships (or effects). Still, the worry is that the entire family of such solutions may be trivial, since an infinite number of such models could be constructed» (Lansing, 2002). The problem with equifinality is that these different non-linear models, equally validated by the real data, are in general characterised by totally different out-of-sample dynamics. Thus, they cannot be considered equivalent in order to explain the phenomenon of interest, and exploit this knowledge for interpreting new events[12]. The problem is ubiquitous, whenever the system we're interested in modelling is nonlinear. This is in sharp contrast with linear models of linear systems, which exhibit only limited deviations for limited departures from the validation set.

The figure below shows this difference. The thick red line between the two vertical lines represents real data, taken from an underlying true data generating process. The thinner lines represent output from other models that might be developed in order to explain the observed data. In case (a), where the underlying true model is linear, it is easy to see that the other models deviate only slightly and gradually from the true one, for out-of-sample data. «Furthermore, we only need to validate our models against limited data to ensure that they are valid for all qualitatively similar contexts (in fact we would only require two data points!). [...] So, the knowledge contained in our models can be easily transferred to other contexts – assuming that the world and our models are linear» (Richardson, 2002). In case (b), depicting non-linear competing models of an underlying non-linear

[12] For a recent discussion about observational equivalence see Hendry (2002)

phenomenon, extrapolation from the observed data becomes strongly questionable.

Figure 2: Linear models of a linear universe versus nonlinear models of a nonlinear universe (source: Richardson, 2002)



(a) (b)

However, some considerations may contribute to a more optimistic picture. First of all, the problem applies both to ACE and analytical models. Should we refrain from analysing non-linear systems altogether? Clearly not. What is needed is simply *caution* in interpreting the results, together with the adoption of a number of different modelling strategies. Building non-linear models in order to reproduce some complex behaviour is not totally uninformative. At least, they prove that the assumptions of the models are sufficient to replicate that behaviour[13]. Other sets of assumptions may prove unable to do it, thus adding to these *possibility* results equally important *impossibility* ones.

### 5.3 Overfitting

As I have already mentioned, one common critique of traditional analytical models is that they sometimes offer too a poor description of a phenomenon that

[13] More precisely, the hypothesis are INUS conditions, i.e. *Insufficient conditions, Necessary to an Unnecessary but Sufficient condition* (Leombruni, 2002, citing Mackie, 1962).

may be much more rich and complex[14]. In other words, they fail to replicate the *signal* that is in the observed data. However, when choosing to enrich the specification of the models one can go too far, up to a point in which they become too complex, and may replicate not only the signal, but also the *noise* of the real data. The random disturbances in the data used for calibration are then included in the model as being meaningful. Overfitting has the same implications of equifinality: it can easily lead to totally wrong predictions. The model has no general validity; it explains only the data on which it was calibrated. Complex models contain more information on observed data, but less information on future data.

Fortunately, there are a number of techniques to control for the problem of overfitting. The simplest one is *cross validation*. The observed data are split between a calibration set and a validation set. Different specifications for the model are estimated on the calibration set. Then, the final choice is made by comparing the fit on the validation set. The approach can be generalised by considering *k* partitions of the data. The goodness of fit for each specification is computed *k* times, taking a different fold to be the validation set, and the remaining *k*-1 to be the calibration set, each time. The backdrop of cross-validation is that the resulting model may be worse than what we could get using

[14] Clearly, this critique does not apply to models whose aim is simply to show the implications of specific mechanisms, many of which may be contemporarily at work and contribute to the observed behaviour of some real world phenomenon of interest. Two examples that «nearly all economists would be glad to have in their list of publications» are cited by Sugden (2000): Akerlof's "The Market for Lemons", which showed the existence of adverse selection due to asymmetrical information, and Schelling's "Dynamic Models of Segregation", which showed that even a mild preference for living adjacent to one own group may lead to very strict homogeneous spatial segregation. As he says, the relevance of these models is to a certain extent independent of the need to confront their implications with real data.

the whole data set. Other methods involve considering penalties for richer specifications. Overall, a combination of good validation and adherence to the KISS ("keep it simple, stupid") principle should preserve from the risk of overfit.

## 6. Conclusions

In this paper I have rationalised the main theoretical critiques that can be moved to the use of agent-based computational models. They are sometimes summed-up in the claim that "simulations do not prove anything". Discerning the different components of this overall judgement is a first achievement of this work. They point to the following problematic areas: (i) interpretation of the simulation dynamics, (ii) estimation of the simulation model, and (iii) generalisation of the results.

Interpretation of the results has to do with recovering the input/output transformation function implicit in the system. In analytical models, this can be done by deriving the unique reduced form corresponding to the structural form of the model. When the structural form is not explicited algebraically, as in simulation models, this is not possible. However, a reduced form can be estimated in the artificial data resulting from a number of experiments with the simulation model itself. A related problem is the robustness of the estimated reduced form, with respect to changes in the values of the inputs. This is often referred to as "the curse of dimensionality", but its relevance has been shown to be limited, due to the increasing performances of modern computers, at least if the number of structural parameters does not become too large.

The second critique involves the possibility of estimating the structural parameters of a simulation model. Since only a proxy for the reduced form can be obtained, differently from analytical models it is not possible to exploit the one-to-one relationship between the structural and the reduced form parameters, in identified models. But other econometric techniques, like the method of simulated moments, can be successfully employed for structural estimation in simulation models. A related claim is that agent-based models suffer from underidentification, since they allow for richer specifications. After noting that allowing for richer specifications does not mean requiring them, it is argued that the risk of underidentification is often simply unavoidable: analytical models that claim to be immune are sometimes only poor models.

Finally, it is recognized that the use of non-linear models for the analysis of complex systems may lead to a problem with respect to the possibility of generalizing the results, i.e. applying the model either in out-of-sample data or to similar yet different contexts. This is due to a characteristic of non-linear systems called equifinality: a number of different structural specifications may lead to the same fit in the data, although exhibiting totally different behavior with untested inputs. Although theoretically relevant, this problem has not prevented the development of many successful models of non-linear systems, both in Economics and in many other related fields. The modeler should be aware of the problem, and take care in analyzing the results of a simulation model, together with comparing the outcome of different modelling strategies. Understanding that a set of assumptions is enough to generate some dynamics of interest, or that other assumptions are not able to do it, is often a valuable result to start with.

Overall, I believe that this discussion on the limits of agent-based simulation models has shown that the methodology is indeed sound and can be valuably employed for the analysis of many economic issues. Moreover, this paper should have clarified some confounding differences in terminology between the computer science and the economic literature. In particular, *metamodels* have been interpreted as estimated reduced forms (input/output transformation functions) of the simulation models on artificial data, while *calibration* has been linked to the estimation of the structural parameters in the real data, without recurring to a reduced form of the model.

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