The Investment Acceleration Principle Revisited by Means of a Neural Net

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November 2, 2001

Abstract

The investment acceleration principle is a heuristic for modeling investment time series out of consumption time series. The model presented herein develops a disaggregated accelerator equation whose coefficients are the weights of a Kohonen neural net that represents firms' decision-making. According to this model, investments take place when managers recognize emerging technological patterns. Furthermore, a technique borrowed from the theory of self-organizing systems is used in order to disentangle innovation-driven investments from plant-replication investments.

Keywords: Accelerator, Investment, Self-Organization, Neural Nets

1 Introduction

Investments in plants, machineries, scientific reasearch and other forms of *capital* goods are by far the most volatile, least predictable among economic time series.

*This research received financial support by *Consiglio Nazionale delle Ricerche* and *Deutsche Forschungsgemeinschaft*.

Yet this is precisely the magnitude that one would like to explicate in terms of a few determinants, since it is the engine of economic growth.

The *investment acceleration principle* is a simple heuristic for modeling investment decision-making. Basically, it asserts that investments depend on variations of demand.

In spite of its simplicity, the investment acceleration principle performs relatively well in empirical studies. A number of recent evaluations ascribe to accelerator equations a better predictive power than any competing model in such diverse situations as industrialized economies, developing economies and transition economies [5] [1] [23] [6] [26] [4] [7] [22] [20]. Not surprisingly, the investment acceleration principle is a major ingredient of many classical business cycle models such as Kaldor's [17], Samuelson's [27], Hicks's [16], Goodwin's [11] and Lucas's [21].

At a closer scrutiny, the investments acceleration principle can be broken down into the following pair of assumptions:

A1: Firms invest proportionally to variations of demand.

A2: Firms only observe demand for their own products or, at most, for products of their industrial sector.

Assumption A1 implies that firms do not make long-term strategic plans. It presumes a large degree of myopia in decision-making, as well as irrelevance of financial considerations.

Assumption A2 implies that firms that produce intermediate goods neither observe the final goods market, nor any other indicator of the general state of the economy. Thus, assumption A2 ascribes managers a sectoral myopia, in the sense that investment decisions are not influenced by consideration of trends in markets that are far down the productive chain.

In business cycle models that rely on the investment acceleration principle, assumption A1 identifies the impulses that trigger the oscillations of economic variables. In its turn, assumption A2 leads to formulate auto-regressive equations of order $p \geq 2$ that are able to generate oscillations of the macroeconomic variables. Parameter p is the number of production stages, usually set at two.

I already mentioned that, although these assumptions are very crude, accelerator equations perform better than competing models based on financial considerations. However, even accelerator equations are far from yielding reliable predictions.

In order to understand the limits of the investment acceleration principle and why neural nets could be useful in order to improve on existing models, it is important to remark that the investment acceleration principle was originally formulated in order to describe the ascending phase of busineess cycles [2] [8]. Simply, it was meant to state that with booming demand, firms increase the size of their plants.

Subsequently, this idea was extended to the descending phase of the business cycle. The very same equations have been used to say that, with contracting demand, firms decrease the size of their plants. This worked still reasonably well, although it was clear that the reaction of firms to contracting demand was of a different magnitude than the reaction to booming demand [11]. However, linking the ascending and the descending phases to one another posed some problems.

Describing the onset of a recession, i.e. the passage from an ascending to a descending phase, posed the least problems. In fact, the onset of economic crises can be ascribed to shortage of labor force, credit rationing and other macroeconomic constraints. Thus, accelerator-based business cycle models were endowed with investment ceilings that caused a decrease of demand which, in its turn, forced the accelerator to invert its functioning.

However, modeling the onset of a recovery proved to be far more difficult. In fact, the onset of economic recoveries is basically due to investments in novel fields of activity opened up by technological innovation. Thus, extending the investment acceleration principle to this part of economic dynamics requires microeconomic modelization of the cognitive process of recognition of novel investment opportunities [12].

This paper attempts to do that by reproducing firms decision-making by means of a Kohonen neural net. In this model, each neuron represents a firm with its ability to form categories out information stemming both from demand of given goods and from exogenous technological innovation. Neurons weights will correspond to the coefficients of a disaggregated accelerator equation, which can adapt its response to variations of demand according to evolution of technological possibilities.

The paper is organized in two main Sections, numbered 2 and 3, respectively, followed by a concluding Section 4. Section 2 illustrates the neural net and tests its behavior with a simulation based on exogenous information regarding novel technologies. Section 3 attempts to reproduce its findings by means of an analytical description based on the theory of self-organizing systems. Finally, Section 4 concludes.

2 The Neural Net

The minimal economic structure that we need to consider in order to apply the investment acceleration principle involves households, firms that produce final goods (hereafter labelled *final goods sector*) and firms that produce capital goods (hereafter labelled *capital goods sector*). In this way, production takes place in two stages so assumptions A1 and A2 of Section 1 can be made.

Within this framework, 'investments' are purchases of capital goods carried out by the firms that produce final goods. For simplicity, let us suppose a constant number of firms in both sectors.

Let us represent a firm's decision-making by means of:

1. A first differences operator, yielding variation of input variables (e.g. variation of demand).

2. A neuron, yielding a firm's purchase decisions based on recognition of patterns entailed in input information.

3. A summation operator, that integrates the neuron's output in order to yield the current values of variables.

Input and output variables of our neural net are information vectors conveying orders for goods and services. Notably, information flows in the opposite direction of goods.

Let vectors $\mathbf{c}$, $\mathbf{k}$, $\mathbf{l}'$ and $\mathbf{l}''$ denote households consumption, capital stock in the final goods sector, employment in the final goods sector and employment in the capital goods sector, respectively. Obviously, it must be $\mathbf{c} \geq \mathbf{0}$, $\mathbf{k} \geq \mathbf{0}$, $\mathbf{l}' \geq \mathbf{0}$, $\mathbf{l}'' \geq \mathbf{0}$.

Let vectors $\Delta\mathbf{c}$, $\Delta\mathbf{k}$, $\Delta\mathbf{l}'$ and $\Delta\mathbf{l}''$ denote variations of the above vectors with respect to the previous time interval. These vectors carry information regarding demand for final goods, investments in capital goods and demand for labor in the final goods and capital goods sectors, respectively.

Furthermore, let a vector $\mathbf{e}$ represent exogenous information on innovations. Vector $\mathbf{e}$ does not represent technological details that are developed by firms themselves and that are kept strictly private unless acquired under licensing agreements. Rather, $\mathbf{e}$ represents all publicly available information about new technologies which can induce managers to invest on a specific field, eventually developing private information as a consequence of this decision. It includes basic research made available by non-profit institutions, rumors about competitors' strategies, as

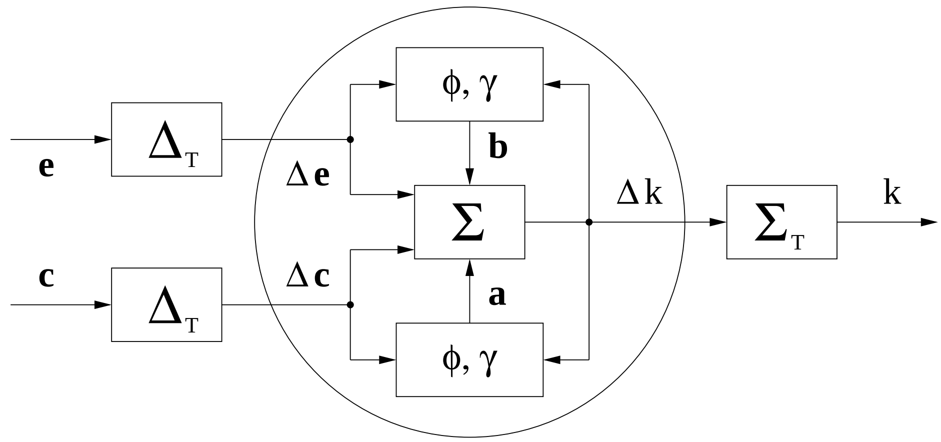

Figure 1: Decision-making by a firm in the final goods sector. The Kohonen neuron consists of a summator of inputs Δc_i and $\Delta\mathbf{e}$ with weights a_i and b_i, respectively. Both sets of weights arising from a combination of a feed-back ϕ and a feed-forward γ. Left of the neuron, first differences operators Δ performs a discrete-time derivative of demand for final goods **c** and state-of-affairs in technology **e**. Right of the neuron, a summator performs a discrete-time integration of investments $\Delta\mathbf{k}$ that yields capital stock **k**.

well as information that was intended to be private but which is actually difficult to appropriate and to trade, e.g. because of reverse engineering [3]. Just like the investment accelerator principle assumes that firms react to *variations* of demand, let us assume that firms react to variations $\Delta\mathbf{e}$ of the state of technology **e**.

Let us suppose that firms of the final goods sector react both to variation of demand and to variation of technologies, whereas firms of the capital goods sector react to variations of demand only. Then, figures (2) and (2) illustrate decision-making in a firm of the final goods sector and in a firm of the capital goods sector, respectively. Demand for labor by the firms of the final goods sector does not appear in the figure because it is supposed to be proportional to demand for capital goods according to a law that will be specified subsequently. Demand for capital goods by the firms of the capital goods sector does not appear in the figure because this sector is supposed to include all production stages from mining to capital goods production.

The feed-backs and -forwards that characterize Kohonen neurons are of the utmost importance in order to model decentralized decision-making. Possibly, a neural net where neurons weights would be set by an external operator could be

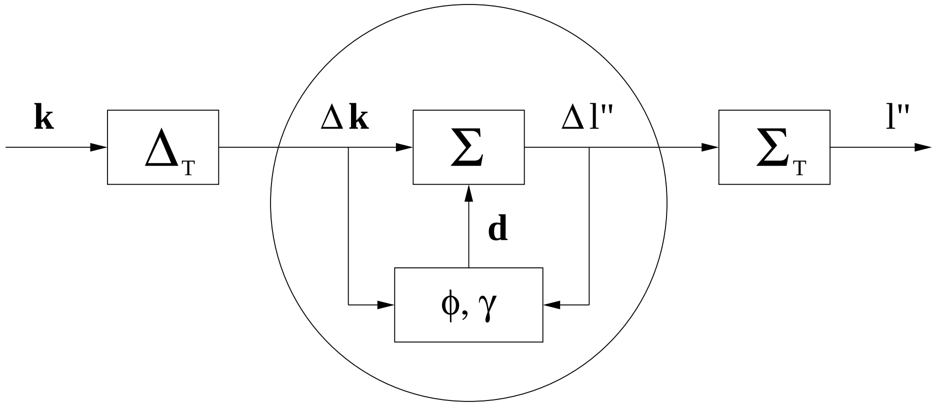

Figure 2: Decision-making by a firm in the capital goods sector. The Kohonen neuron consists of a summator of inputs Δk_i with weights d_i, arising from a feed-back and a feed-forward ϕ and γ, respectively. Left of the neuron, a first differences operator Δ performs a discrete-time derivative of capital stock $\mathbf{k}$. Right of the neuron, a summator performs a discrete-time integration of demand for labor $\Delta \mathbf{l}''$ that yields employment $\mathbf{l}''$.

used in order to represent a planned economy.

Let us arrange neurons in two layers, corresponding to firms in the final goods sector and firms in the capital goods sector, respectively. Figure (2) illustrates the ensuing neural net. Note that, since neurons are embedded in information circuits, sum and difference operators annul. Thus, in figure (2) they have been ignored altogether.

Besides neurons, figure (2) exhibits two boxes denoted f and g, respectively. Functions f and g represent households behavior and demand for labor by the final goods sector, respectively. These aspects of decision-making do not need to be modeled in detail in order to investigate investments acceleration. However, we must require that $f(\Delta x) = \Delta f(x)$ and $g(\Delta x) = \Delta g(x)$ in order to cancel sum and difference operators as above. Furthermore, note that since f makes consumption depend on current income, we implicitely assumed that households do not save.

The net in figure (2) exhibits two nested feed-back loops, the inner one passing through the workers of the final goods sector, the outer one through the workers of the capital goods sector, respectively. Both feed-backs make demand depend on wages and ultimately on production in their respective sectors.

For simplicity, let us suppose that the final goods sector entails N firms, that the capital goods sector entails N firms as well, that there are N different final goods

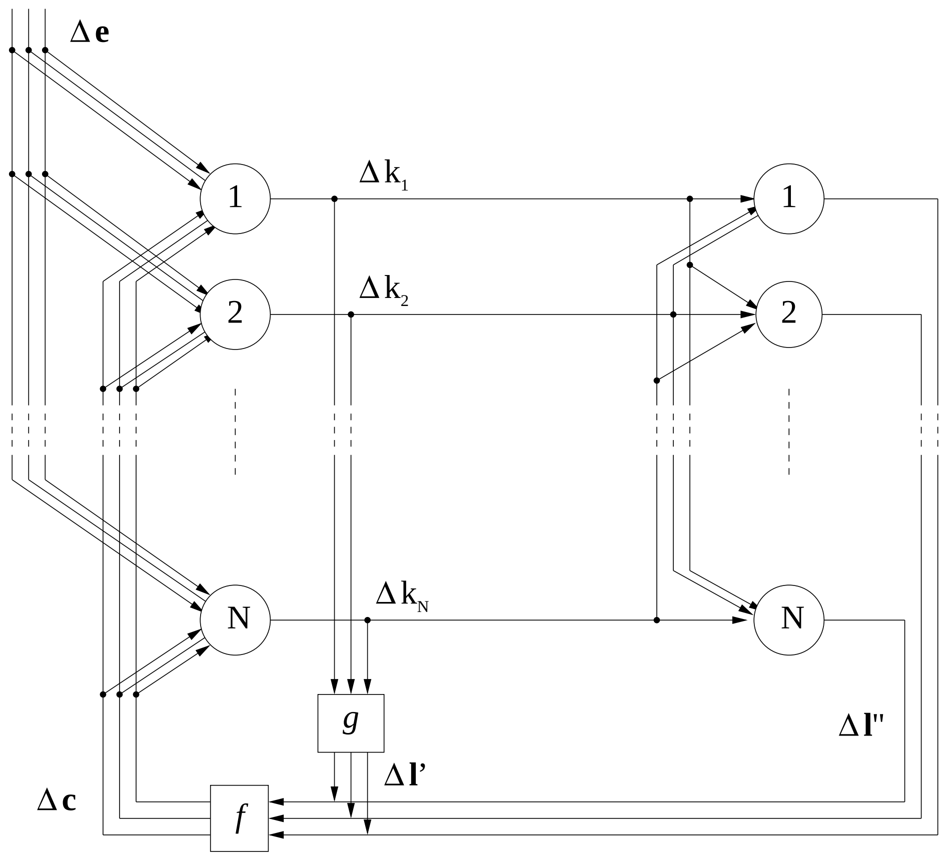

Figure 3: The structure of information flows within a productive system. Each neuron represents decision-making by one firm. The left layer represents firms in the final goods sector, the right layer represents firms in the capital goods sector, respectively. A double feed-back arises through the labor market.

but only one capital good. In this way, all vectors will be N-dimensional. Note that although the number of different goods is fixed with time, their qualitative features may vary as a consequence of technological innovation.

Let $\mathbf{A}$ entail the weights by which the neurons that represent decision-making by the final goods sector process information provided by demand for final goods. The i-th row of $N\times$ matrix $\mathbf{A}$ represents the weights by which neuron i multiplies $\Delta\mathbf{c}$.

Similarly, let $\mathbf{B}$ entail the weights by which the neurons that represent decision-making by the final goods sector process information provided by technological innovation. The i-th row of $N\times$ $\mathbf{B}$ represents the weights by which neuron i multiplies $\Delta\mathbf{e}$.

Finally, let $\mathbf{D}$ entail the weights by which the neurons that represent decision-making by the capital goods sector process information provided by demand for capital goods. The i-th row of $N\times$ matrix $\mathbf{D}$ represents the weights by which neuron i multiplies $\Delta\mathbf{k}$.

Thus, information processing is described by:

$$\Delta\mathbf{k}_t = \mathbf{A}\Delta\mathbf{c}_{t-1} + \mathbf{B}\Delta\mathbf{e}_{t-1} \tag{1}$$

$$\Delta\mathbf{l}''_t = \mathbf{D}\Delta\mathbf{k}_{t-1} \tag{2}$$

where $\Delta\mathbf{k}_t = \mathbf{k}_t - \mathbf{k}_{t-1}$, $\Delta\mathbf{c}_{t-1} = \mathbf{c}_{t-1} - \mathbf{c}_{t-2}$, $\Delta\mathbf{e}_{t-1} = \mathbf{e}_{t-1} - \mathbf{e}_{t-2}$, $\Delta\mathbf{l}''_t = \mathbf{l}''_t - \mathbf{l}''_{t-1}$, $\Delta\mathbf{k}_{t-1} = \mathbf{k}_{t-1} - \mathbf{k}_{t-2}$.

If we assume for simplicity that $f \equiv g \equiv \mathbf{I}$, we can write:

$$\Delta\mathbf{c}_t = \Delta\mathbf{l}_t \tag{3}$$

$$\Delta\mathbf{l}'_t = \Delta\mathbf{k}_t \tag{4}$$

By combining equations (1) and (2) with (3) and (4) and by remembering that $\mathbf{l} = \mathbf{l}' + \mathbf{l}''$, we obtain a system of equations that takes information on technologies $\Delta\mathbf{e}$ as input and yields disaggregated investments $\Delta\mathbf{k}$ as output. Eventually, by summation of the components of $\Delta\mathbf{k}$ we can obtain aggregate investments.

Business cycle models make use of accelerator equations that, apart from being aggregate, are not different from a combination of equations (1), (2), (3), (4) [9]. Upon that, business cycle models add an upper and a lower threshold to investments in order to invert the functioning of the accelerator when investments grow up to the “ceiling” or fall down to the “floor”. In these way, cycles arise.

On the contrary, we are not interested in generating a business cycle but rather in providing a better modelization of the very beginning of its ascending phase.

Thus, we shall not constrain the investments generated by equations (1), (2), (3), (4). Rather, we shall link the coefficients of these equations to the recognition of technological innovations.

Equations (1) and (2) change their coefficients according to experiences and cognitive abilities of decision-makers. Equations for describing the evolution of matrices **A**, **B**, **D** can take a number of forms [18] [19] but, typically, ϕ implements *learning* by multiplying the inputs and the output of the neuron in order to stress those weights that yielded a high output with particular combinations of inputs. On the contrary, γ implements *forgetting* by decreasing the absolute value of weights with time, for instance according to an exponential function.

Let us choose the simplest "Hebbian rule" [15] out of the many equations proposed by Kohonen [18]:

$$\Delta\mathbf{A} = \mu\Delta\mathbf{k}\Delta\mathbf{c}^T - \nu\mathbf{A} \tag{5}$$

$$\Delta\mathbf{B} = \mu\Delta\mathbf{k}\Delta\mathbf{e}^T - \nu\mathbf{B} \tag{6}$$

$$\Delta\mathbf{D} = \mu\Delta\mathbf{l}''\Delta\mathbf{k}^T - \nu\mathbf{D} \tag{7}$$

Equations (5), (6), (7) specify the variation of the coefficients of equations (1) and (2), which in combination with (3), (4) capture the essence of the investment acceleration principle. By combining accelerator equations (1), (2), (3), (4) with coefficients variation equations (5), (6), (7) one obtains investment decisions $\Delta\mathbf{k}$ out of an exogenous sequence of information on technological innovations $\Delta\mathbf{e}$.

However, decision-making can only be called *rational* if it is carried out within a set of constraints [28] [29]. Let us choose the two following rules to constrain decision-making:

1. Neurons output is not allowed to be negative. Thus, in the short term on which we are focusing, firms can neither disinvest nor can they fire their workers.

2. Credit exists, but loans cannot be indefinetly large. Since it is likely that capital stock serves as collateral, it is assumed that the output of a neuron cannot be larger than cumulative output (in order to allow starting-up business, this rule is not applied if cumulative output is zero).

At this point, by combining the above equations and rules and by feeding the model with a sequence of information vectors on technological innovations $\Delta\mathbf{e}$, one can run simulations yielding investments $\Delta\mathbf{k}$. Since we want to model the very beginning of economic recovery, when patterns of novel technological

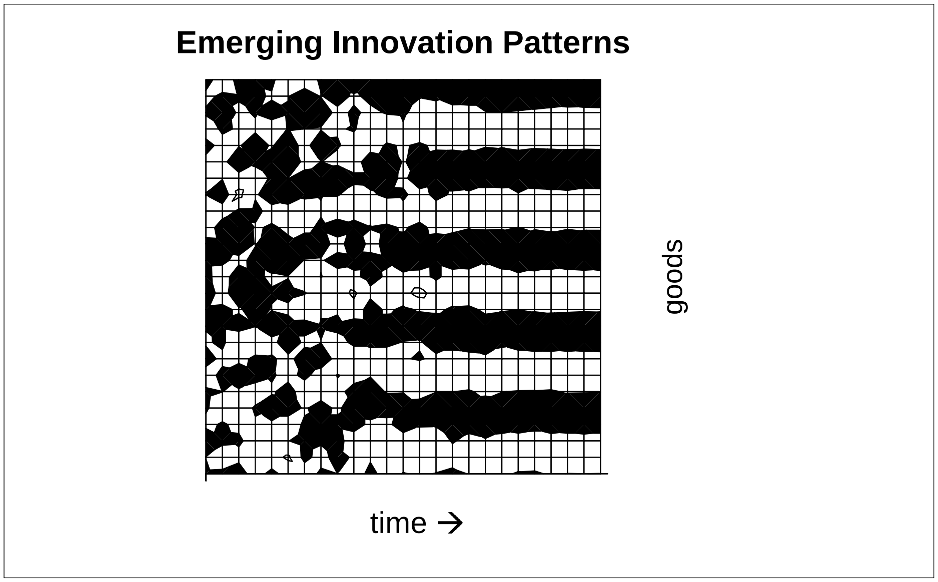

Figure 4: A sequence of a hundred vectors $\mathbf{e}$ from $t = 1$ to $t = 100$, horizontal sections at $\mathbf{e} = \mathbf{0}$. Black areas denote regions where $\mathbf{e} > \mathbf{0}$, white areas denote regions where $\mathbf{e} \leq \mathbf{0}$. The sinusoid spans 100 goods with 5 periods of 20 goods each, its amplitude increasing from $A_{min} = 0$ at $t = 1$ to $A_{max} = 2$ at $t = 100$. Upon it, a normal distribution centered around zero adds noise with variance decreasing from $V_{max} = 1$ at $t = 1$ to $V_{min} = 0$ at $t = 100$. In order to simplify the image only one out of four goods and one out of four time periods have been shown, resulting in a 25×25 grid.

possibilities emerge out of exhausted technologies, let us choose a sequence of vectors $\mathbf{e}$ such that a simple sinusoidal pattern emerges from chaos. Figure (2) illustrates a horizontal section drawn at $\mathbf{e} = \mathbf{0}$.

Vector $\Delta\mathbf{e}$ is obtained out of $\mathbf{e}$ taking a vector of zeros as initial condition. Since managers are likely to attach comparable importance to information on innovation and information stemming from demand, vectors $\Delta\mathbf{e}$ and $\Delta\mathbf{c}$ should be of similar size. Thus, at each simulation step and before feeding $\Delta\mathbf{e}$ into the model, the following operations are carried out:

1. Components of $\Delta\mathbf{e}$ that eventually exceed the the $[-A_{max}, A_{max}]$ interval, are cut.

2. The interval spanned by $\Delta\mathbf{e}$ is adjusted to the interval spanned by $\Delta\mathbf{c}$ by multiplying $\Delta\mathbf{e}$ by $(max(\Delta\mathbf{c}) - min(\Delta\mathbf{c}))/2A_{max}$.

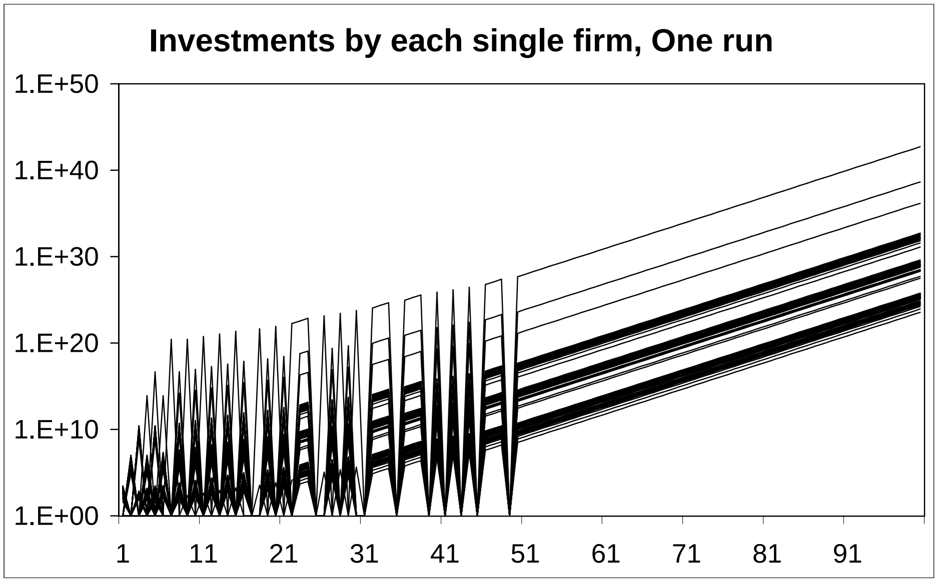

Figure 5: Investments by each single industrial sector. Each curve represents a different Δk_i for $i = 1, 2, \ldots 100$ during one single simulation. In order to depict zero values of investments, a one has been added to all values.

3. The median of $\Delta\mathbf{e}$ is shifted to that of $\Delta\mathbf{c}$.

The resulting investments by each single industrial sector are illustrated in figure (2). Each curve represents the output of a neuron in the left layer of figure (2). Initial conditions, keeping in mind that we are describing the onset of a recovery, are obviously $\mathbf{c} = \mathbf{0}$, $\mathbf{k} = \mathbf{0}$, $\mathbf{l}' = \mathbf{0}$, $\mathbf{l}'' = \mathbf{0}$ and $\Delta\mathbf{c} = \mathbf{0}$, $\Delta\mathbf{k} = \mathbf{0}$, $\Delta\mathbf{l}' = \mathbf{0}$, $\Delta\mathbf{l}'' = \mathbf{0}$. Learning and forgetting parameters have been set at $\mu = 0.1$ and $\nu = 0.1$, respectively.

Since investments grow exponentially, logarithmic pictures have been used. In order to depict zero values, a one has been added to all components of $\Delta\mathbf{k}$.

The most notable feature of the graph illustrated in figure (2) is that at about a half of running time, precisely when according to figure (2) a technological pattern is emerging, firms stop to invest erratically and step onto an exponential growth path.

Other simulations yielded similar results. Figure (2) illustrates aggregate investments produced by ten different simulations, as well as their average. Note that according to *all* simulations investments start to grow exponentially at some point between the 50^{th} and the 60^{th} time step.

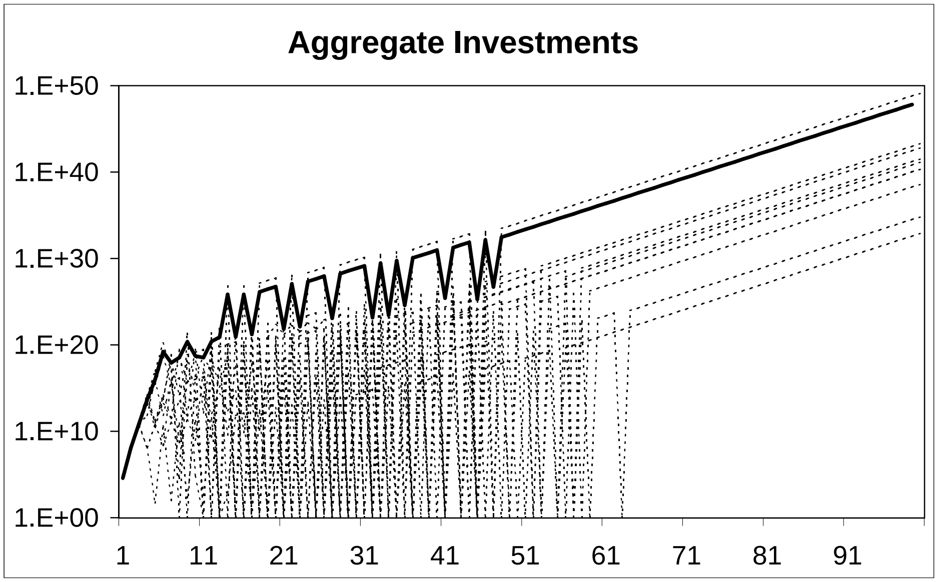

Figure 6: Aggregate investments produced by ten different simulations (dashed lines) and their average (thick line). In order to depict zero values of investments, a one has been added to all values.

Clearly, this happens because matrices **A**, **B**, **D** change their weights. Figure (2) illustrates three indicators of the variation of components of **A**, **B** and **D**, respectively. It is clear that recognition of the innovation pattern terminates a period of turbulence and sets the variation of **A**, **B** and **D** on an exponential path that reflect the exponential growth of investments.

The above curves have been obtained out of a model that combines variations of economic magnitudes such as demand, investments and information on technologies, with variations of cognitive parameters represented by **A**, **B**, **D**. On the contrary, the ensuing section makes an attempt to disentangle these two effects.

3 Analytical Description

Even if we assumed that $f \equiv g \equiv \mathbf{I}$, any combination of equations (1), (2), (3), (4) is non-linear. In fact, in these equations variables $\Delta\mathbf{c}$, $\Delta\mathbf{k}$, $\Delta\mathbf{l}$, $\Delta\mathbf{e}$ multiply variable coefficients **A**, **D**, **B**.

However, analytical treatment is possible on the ground that, since mental categories must vary at a much slower pace than the information that they classify,

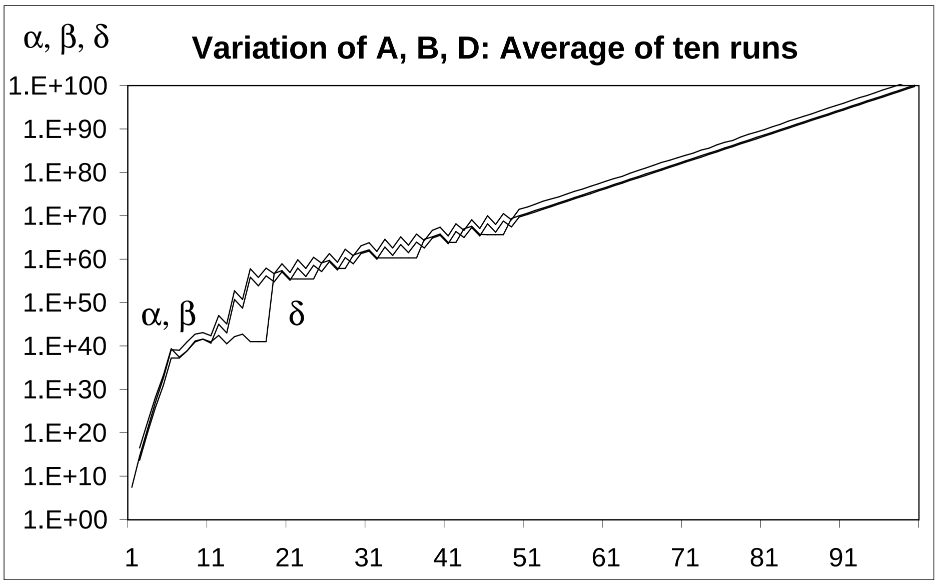

Figure 7: Variation of matrices **A**, **B**, **D** illustrated by indicators α, β, δ, respectively. Indicators α, β, δ are defined as the sum of the absolute variations of all elements of **A**, **B**, **D**, respectively. Curves illustrate the averages of ten simulations.

the corresponding weight matrices **A**, **D**, **B** must vary at a much slower pace than information vectors $\Delta\mathbf{c}$, $\Delta\mathbf{k}$, $\Delta\mathbf{l}$, $\Delta\mathbf{e}$. Furthermore, since classifying information in a mental category translates into a vector falling into a basin of attraction, fast information vectors are likely to be *stable* as well.

Since $\Delta\mathbf{e}$ is an exogenous variable and since $\Delta\mathbf{l}$ can be expressed in terms of $\Delta\mathbf{c}$ and $\Delta\mathbf{k}$, let us focus on fast variables $\Delta\mathbf{c}$, $\Delta\mathbf{k}$ and slow variables **A**, **D** only. For the above reasons, fast variables $\Delta\mathbf{c}$ and $\Delta\mathbf{k}$ are stable within the basins of attraction specified by the slow, unstable variables **A** and **D**. In fact, matrices **A** and **D** represent mental categories by means of potential functions in the space of $\Delta\mathbf{c}$ and $\Delta\mathbf{k}$, respectively.

In their turn, slow variables **A** and **D** evolve according to the the equilibrium values taken by fast variables $\Delta\mathbf{c}$ and $\Delta\mathbf{k}$, respetively. In fact, innovation produces novel patterns that must be classified by means of novel categories. Consequently, slow variables **A** and **D** are likely to change as a consequence of the movements of $\Delta\mathbf{c}$ and $\Delta\mathbf{k}$, respectively. Slow, unstable variables **A** and **D** are also called *order parameters* since they specify the configurations taken by $\Delta\mathbf{c}$ and $\Delta\mathbf{k}$, respectively.

The above dichotomy between fast, stable variables and slow, unstable variables is typical of self-organizing systems [24] [13] [25] [14]. This kind of non-linear system can be studied analytically by separating:

1. A regime where fast variables $\Delta\mathbf{c}$ and $\Delta\mathbf{k}$ are infinetly fast so they reach instantaneously their equilibrium values. In this situation, it is easy to study the evolution of slow variables **A** and **D**.

2. A regime where slow variables **A** and **D** are infinetly slow, i.e. thy do not change at all. In this situation, it is easy to study the evolution of fast variables $\Delta\mathbf{c}$ and $\Delta\mathbf{k}$.

In our context, regime 1 is approximated by the very beginning of economic recoveries. In fact, in this phase investments are small, easy to carry out and, possibly, fast enough to be close to their equilibrium level. At the same time, a lot of technological novelties force managers to change their mental categories, represented by slow and unstable order parameters.

On the contrary, regime 2 is approximated by the growing phase of the business cycle, when technologies vary less rapidly, the accelerator coefficients are stable and large-scale investments take place by replication of existing plants. In this phase, order parameters can be taken to be constant while investments grow exponentially according to traditional, fixed-coefficient accelerator equations.

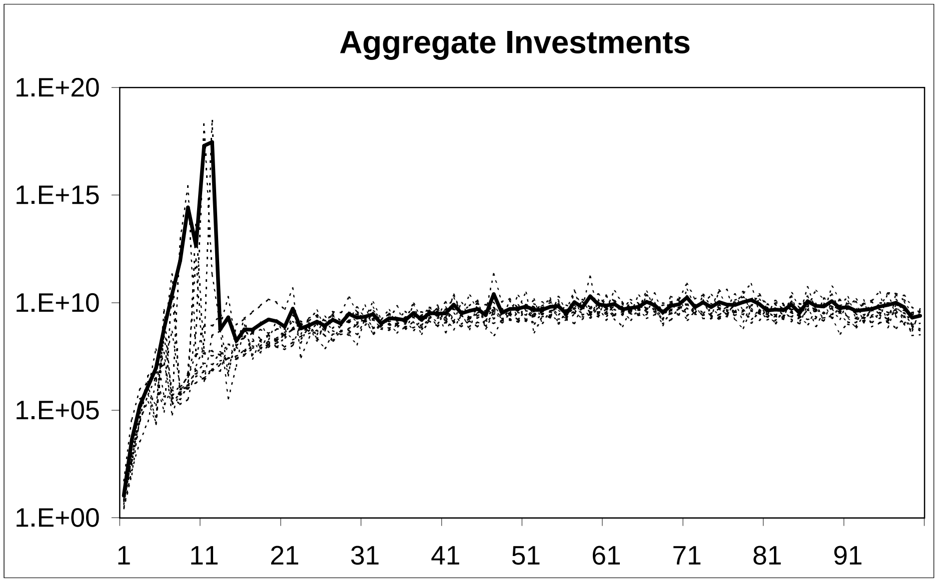

Figure 8: Aggregate investments calculated by assuming that information vectors $\Delta\mathbf{c}$ and $\Delta\mathbf{k}$ vary with infinite speed. The figure illustrates ten single runs (dashed lines) as well as their average (thick line).

When an economy is in regime 1, neurons processing time can be neglected. Thus, upon examination of figure 2 and by assuming $f \equiv g \equiv \mathbf{I}$ we can write $\Delta\mathbf{l}' = \Delta\mathbf{k}$, $\Delta\mathbf{l}'' = \mathbf{D}\Delta\mathbf{k}$, $\Delta\mathbf{c} = (\mathbf{I}+\mathbf{D})\Delta\mathbf{k}$, $\Delta\mathbf{k} = \mathbf{A}\Delta\mathbf{c} + \mathbf{B}\Delta\mathbf{e}$. Combination of these expressions yields:

$$\Delta\mathbf{k} = \frac{\mathbf{B}}{\mathbf{I}-\mathbf{A}-\mathbf{AD}}\Delta\mathbf{e} \tag{8}$$

Thus, we can envision an alternative algorithm in order to derive a sequence of $\{\Delta\mathbf{k}\}$ from a sequence of $\{\Delta\mathbf{e}\}$. First, for each $\Delta\mathbf{e}$ equation (8) yields a vector $\{\Delta\mathbf{k}\}$ that may not respect conditions of economic feasibility. Secondly, application of rules (1 and (2) listed in section 2 yields a feasible $\{\Delta\mathbf{k}\}$. Finally, coefficient matrices $\mathbf{A}$ and $\mathbf{D}$ are updated by means of equations 5 and 7.

Figure (3) illustrates aggregate investments calculated in this way over ten simulation runs, as well as their average. It is evident that, after a brief transitory, investments take on a constant value.

Figure (3) makes clear that, contrary to the model expounded in Section 2, investments approach a constant, low level. This is hardly surprising, since assuming infinite speed of information takes away the very rationale of the investment acceleration principle, namely that production time lags and the structure

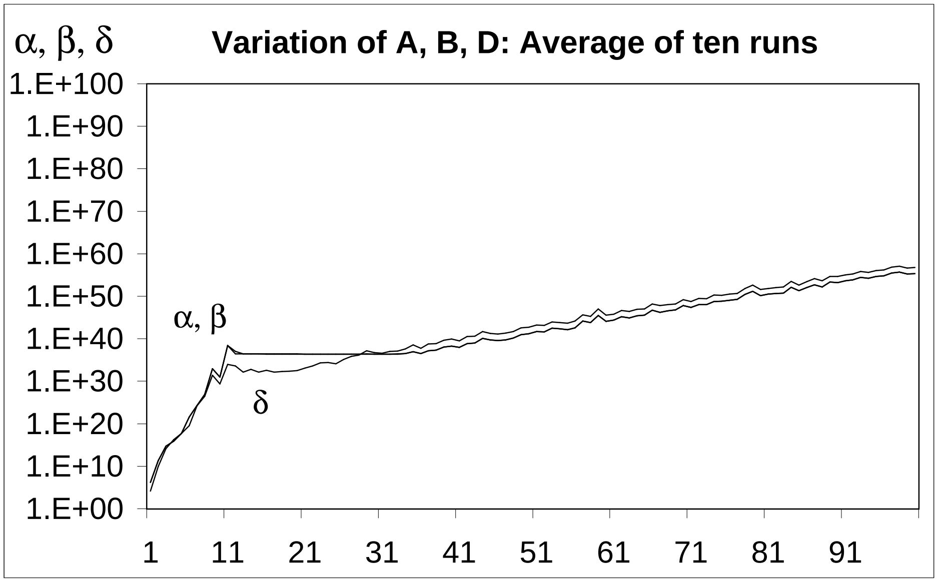

Figure 9: Variation of matrices **A**, **B**, **D** illustrated by indicators α, β, δ, respectively. Indicators α, β, δ are defined as the sum of the absolute variations of all elements of **A**, **B**, **D**, respectively. Curves illustrate the averages of ten simulations.

of information flows between industrial sectors operating at different production stages may cause excessive productive capacity. However, the purpose of this infinite-speed model is only that of attaining values of **A** and **D** that can be used in a standard, fixed-coefficients accelerator equation.

Figure (3) illustrates the variation of **A**, **B** and **D** by means of the same indicators that had been used in Section 2. According to this figure, since the 30^{th} time step these matrices begin to vary according to slow but regular exponential path.

Figures(3) makes clear that matrices **A**, **D**, **B** on average approach a stable path as soon as the technological pattern begins to appear at the 30^{th} time step. Matrices **A**, **D**, **B** are still growing because equations (5), (6) and (7) make their variation proportional to the investments level, but a stable variation like this could be easily inserted in an investments acceleration model that does not employ a neural net. The only difference with a fixed-coefficients model would be a more steeply exponential growth path.

Thus, let us use the values attained by **A** and **D** at the 30^{th} step to run a traditional, fixed-coefficients accelerator equation that receives no exogenous input $\Delta\mathbf{e}$. In this equation, the initial level of investments will be the one attained when

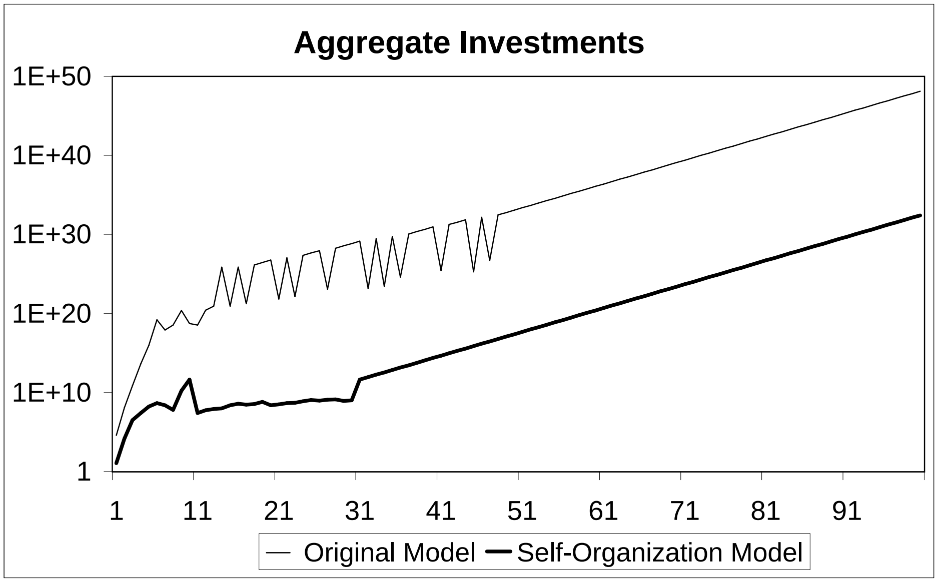

Figure 10: Aggregate investments according to the original model (thin line) and aggregate investments according to the two-regimes model (thick line). Both curves depict the averages of ten simulation runs.

A and **D** were frozen. Note that, by doing this, we are entering regime 2.

By combining equation (1) deprived of the $\mathbf{D}\Delta\mathbf{e}_{t-1}$ term with (2), (3), (4) one obtains:

$$\Delta\mathbf{k}_t = \mathbf{A} f(g(\Delta\mathbf{k}_{t-1}) + \mathbf{D}\Delta\mathbf{k}_{t-2}) \tag{9}$$

which, albeit disaggregated, is a traditional, fixed-coefficients accelerator equation. Just as in the case of regime 1, each time step we have to check whether neuron outputs are yielding feasible values by means of rules 1 and 2.

Figure (3) compares aggregate investments obtained by the original model expounded in Section 2 with the two-regimes model explained in this Section. Only the averages of ten simulations are shown in the picture.

Figure (3) allows us to compare a model where the two processes of understanding technological novelties and undertaking investments take place at the same time, with a model that separates these two concepts. It makes clear that traditional, fixed-coefficients accelerator equations are only able to account for investments that are carried out according to a given technological paradigm, once this emerged. In an earlier phase where it is still unclear which will be the winning technologies of the future, a neural net adds interesting insights.

4 Concluding Remarks

Although feed-forward neural nets find a lot of applications in handling financial data, use of a Kohonen neural net in order to reproduce decision-making by economic agents is entirely new to economics. On the contrary, this paper aimed to open a new field of research by establishing a theoretical bridge between an economic model and a tool of artificial intelligence.

This has been interesting in itself, because it allowed at least a partial modeling of a very important but very vague concept like that of “corporate culture”. However, the model presented herein does not have immediate practical implications.

Nonetheless, applications could be envisioned in a number of fields. First of all, a Kohonen neural net could be used in order to better understand the empirical dynamics of investments. A first attempt in this sense gave encouraging results [10].

Secondly, and possibly most importantly, Kohonen neural nets might be used on POS data in order to guide investments into novel fields. This is an entirely unexplored field of research, since until now POS data have been only used in order to forecast seasonal variations of demand at selling points or, at most, demand at a new location.

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