# Statistical Sampling to Measure Portfolio-at-Risk in Microfinance

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## Abstract

This paper describes a statistical sample design to measure portfolio-at-risk in microfinance. It applies the design to the microfinance portfolio of Banco do Nordeste in Brazil. Statistical audit sampling requires no special knowledge of statistics and is useful for due-diligence inspections by possible creditors, possible owners, or in preparation for the possible securitization of a portfolio. The sample design here stratifies by branch and by loan officer because errors in the record of arrears in the management-information system are likely to vary along these dimensions. Because errors may also vary by loan size and are more costly for large loans than for small loans, loans are sampled with probability proportional to size. This implicitly stratifies the sample by amount outstanding. Furthermore, the design samples all of the largest loans and all rescheduled loans. Given these strata, given a definition of portfolio-at-risk (for example, the outstanding balance of all loans with at least one payment at least one day overdue), given a desired upper bound on the accuracy of the estimated proportion of the portfolio-at-risk (for example, 2 percentage points), and given a desired precision for the confidence interval (for example, 90 percent), the paper tells (a) how many cases to draw; (b) how to estimate the proportion of the portfolio-at-risk; and (c) how to estimate the dollar amount of the portfolio-at-risk.

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# 1. Introduction

An audit of the measurement of risk in a microfinance portfolio checks whether arrears as reported in the electronic management-information system (MIS) match arrears as recorded in physical files. Such audits are useful to potential creditors and to the microfinance lender itself as well as to potential owners or to potential buyers of a securitized portfolio.

Audits often use sampling techniques to inform judgements about the accuracy of MIS records without the expense of an exhaustive review of all physical files of all loans. The trade-off is that sampling measures are less exact than exhaustive measures. Statistical sampling quantifies the nature of the trade-off. Compared with exhaustive samples or non-statistical samples, statistical samples require fewer resources to check physical records, but they also require more resources to prepare a sample design and to compute measures of the confidence and precision of its estimates of portfolio-at-risk.

This paper describes the design of a statistical sample to estimate the proportion of portfolio-at-risk in the microfinance portfolio of Banco do Nordeste in Brazil. Given desired levels of confidence and precision, it tells how to divide the portfolio in strata, how many cases to draw, and how to estimate the proportion of the portfolio-at-risk as well as the amount of dollars at-risk. Strata are defined along dimensions where errors are likely and/or costly: the branch, the loan officer, the loan size, and rescheduled status. Many have written about statistical sample design for audit purposes (*e.g.*, Guy *et al.*, 1997; Carmichael and Benis, 1991; Roberts, 1978; Smith, 1976 and 1974; Knight, 1974; Arkin, 1963), but no one has addressed the case most relevant for microfinance, that is, estimation of the proportion of portfolio-at-risk with multiple strata, with probability proportional to size, and with exhaustive sampling of multiple strata.

This paper lays out such a sample design. The fundamental formulae (although not their combination) come from the classic text by Cochran (1977). General formulae are presented here to facilitate adaptation for different microfinance portfolios. The general formulae are applied to the microfinance portfolio of Banco do Nordeste in Brazil. The arithmetic is tedious, but it can be programmed in a spreadsheet.

The rest of the paper presents tells how to estimate portfolio-at-risk in microfinance through statistical audit sampling. Section 2 discusses the logistics of sampling with probability proportional to size. Section 3 describes stratification, estimation, and sampling by the four strata of rescheduled loans, large loans, loan officers, and branches. Section 4 discusses trade-offs in the choice of sample size. Section 5 concludes with specific recommendations for the case of Banco do Nordeste.

# 2. Sampling with probability proportional to size

The design calls for samples not of loans outstanding but rather of dollars outstanding. This is known as *sampling with probability proportional to size* or *dollar-unit sampling* (Anderson and Teitlebaum, 1973). Loans have different amounts outstanding (sizes), and the proportion of portfolio-at-risk most directly depends not on the number of delinquent loans but rather on the number of delinquent dollars. If the sampling unit were loans, then small loans would have the same chance of being sampled as large loans even though large loans influence portfolio-at-risk much more.

## 2.1 Clustering

Of course, dollars outstanding (and physical files) are clustered in loans. Furthermore, the definition of *portfolio-at-risk* in microfinance is that all dollars outstanding in a loan are at-risk if one dollar is (Christen, 1997). Thus, the optimal design samples loans with probability proportional to the amount of dollars outstanding. Large loans are more likely to be sampled, but all dollars outstanding have the same likelihood of being sampled.

The estimation of the precision of the estimated proportion of portfolio-at-risk must account for the perfect positive correlation in risk status among all dollars in a loan. The techniques described below account for this correlation induced by the clustering of dollars in loans. Failure to do this would lead to overstatements of the precision of the estimates of portfolio-at-risk and thus to smaller-than-required sample sizes and too-tight upper bounds (Deaton, 1997).

## 2.2 Drawing the sample

Suppose that $b$ indexes branches, $l$ indexes loan officers in a given branch, and $i$ indexes loans for a given loan officer. Given the portfolio (excluding rescheduled and large loans) of a single loan officer, a simple, efficient way to draw the sample with probability proportional to size is to assign to each loan a uniformly distributed random number $\rho_{\mathrm{bli}}$ that ranges between zero and unity and then to multiply $\rho_{\mathrm{bli}}$ by the amount of dollars outstanding in the loan $Y_{\mathrm{bli}}$ to produce $\pi_{\mathrm{bli}}$:

$$\pi_{bli} = \rho_{bli} \cdot Y_{bli}. \tag{2}$$

Next, arrange the loans in descending order by $\pi_{\mathrm{bli}}$. Renumber the loans so that loan *bl1* has the highest $\pi_{\mathrm{bli}}$, loan *bl2* has the second-highest $\pi_{\mathrm{bli}}$, *etc.* Then check the physical loan files of the first $n_{\mathrm{blO}}$ cases, where $n_{\mathrm{blO}}$ is the sample size for loan officer *bl*, determined as described below. (All rescheduled and large loans will also be checked.)

## 2.3 What data to collect

To compute estimates of the proportion of portfolio-at-risk for Banco do Nordeste, the auditors will need a file with the following fields for each loan in the microfinance portfolio (not only for sampled loans):

- A unique identifier for the loan;
- A unique identifier for the borrower;
- The branch;
- The loan officer;
- Date of disbursement;
- Amount disbursed;
- Amount outstanding as of the date of the audit;
- Date that the most recent payment fell due;
- Date that the most recent payment was made (pulled from physical files);
- Days in arrears according to the MIS;
- Amount due in the most recent payment;
- Whether the loan is rescheduled or not.

# 3. Stratification and estimation

This section presents the notation and formulae to compute measures of the proportion of portfolio-at-risk and the amount of dollars at-risk for a design that uses dollar-unit sampling, that stratifies by branch and loan officer, and that exhaustively samples rescheduled and large loans.

## 3.1 The overall portfolio

Let $Y$ be the amount outstanding in the portfolio, and let $N$ be the number of loans outstanding. As of June 30, 2000, the microfinance portfolio at Banco do Nordeste had 44,581 loans, and their book value was 22,494,262 cruzeiros, or, given an exchange rate of about 1.8 cruzeiros per dollar, 12,496,812 dollars. (From now on, monetary figures are in units of dollars; the statistical formulae are invariant to this choice.) The amount outstanding per loan was about $280.

Let $y$ be the amount of dollars outstanding in loans with at least one payment at least one day overdue. According to the MIS at Banco do Nordeste, $y$ was $849,972. The value of the outstanding portfolio $Y$ is assumed correct as reported by the MIS; the sample audit checks the value of the portfolio-at-risk $y$ in the physical files.

Let $p$ be the proportion of the portfolio-at-risk:

$$p = \frac{y}{Y}. \tag{2}$$

At Banco do Nordeste, the MIS estimate of $p$ was about 6.8 percent (0.068). The main objective of the audit is to check the accuracy of the MIS estimate of $p$. The computation of the sample size required to achieved given levels of desired confidence and precision, however, depends on the true value of $p$, but, before the audit, the true proportion is unknown. The design here follows standard practice in that it takes the MIS proportion of the portfolio-at-risk as the best available pre-audit estimate.

## 3.2 Strata

The sample design stratifies by loan officer and by branch. Within a loan-officer stratum, it also stratifies loans into three types: rescheduled, large, and “other”.

Stratification by branch and loan officer serves two purposes. First, portfolio-at-risk probably varies between different branches and between different loan officers. Stratification can take advantage of this to reduce the variability of the estimates of the proportion of the overall portfolio-at-risk, although the gain in precision is probably small (Cochran, 1977; Smith, 1976). Second and more important, stratification helps to pinpoint branches or loan officers whose audited portfolio-at-risk differs greatly from the MIS portfolio-at-risk. Lenders may want to investigate such discrepancies further.

### 3.2.1 Branch

Denote the number of branches as $B$ (79 for Banco do Nordeste). The index to the branch is $b$, so the number of loans in the $b^{th}$ branch is $N_b$. Likewise, the amount of dollars outstanding in branch $b$ is $Y_b$. At Banco do Nordeste, $N_b$ ranges from 31 to 2,833 (average 564), and $Y_b$ ranges from \$7,273 to \$725,280 (average \$284,737).

### 3.2.2 Loan officer

Let $N_L$ be the number of loan officers (257 for Banco do Nordeste). The sample design assumes that a given loan officer is in only a single branch. Let the number of loan officers in branch $b$ be $N_{bL}$, and let the index to loan officers in a branch be $l$. Thus, the notation $bl$ identifies an individual loan officer. Including rescheduled and large loans, the number of loans outstanding under loan officer $l$ in branch $b$ is $N_{bl}$. At Banco do Nordeste, $N_{bl}$ ranges from 4 to 505 (average 173).

The amount of dollars outstanding (including dollars in rescheduled and large loans) for loan officer $bl$ is $Y_{bl}$, and the amount of dollars in loans found to be in arrears by the audit is $y_{bl}$. At Banco do Nordeste (including dollars in rescheduled and large loans), $Y_{bl}$ ranges from \$358 to \$149,336 (average \$48,625).

### 3.2.3 Individual loans

The index to individual loans in the portfolio of a loan officer is $i$, with a range from 1 to $N_{bl}$. The number of cases "other" loans (excluding rescheduled and large loans) sampled from the portfolio of loan officer $bl$ is $n_{bl}$. The amount of dollars outstanding in the $i^{th}$ "other" loan of the $l^{th}$ loan officer at the $b^{th}$ branch is $Y_{bli}$, and the amount of dollars found by the audit to be in arrears is $y_{bli}$. That is, $y_{bli}$ is $Y_{bli}$ if the audit test finds that loan *bli* is delinquent, and zero otherwise:

$$y_{bli} = \begin{cases} Y_{bli} \text{ if loan } bli \text{ is delinquent,} \\ 0 \text{ otherwise} \end{cases}. \quad (3)$$

### 3.2.4 Large and rescheduled loans

The design calls for exhaustive sampling of all rescheduled loans and all large loans because these strata are likely sources of costly errors. Rescheduled loans are exhaustively sampled for two reasons. First, rescheduled loans are more likely than non-rescheduled loans to become delinquent. Second, microfinance lenders, branch managers, and/or loan officers may want to obfuscate the delinquency of rescheduled loans so as to hide their mistakes in the choice to reschedule.

Large loans are also exhaustively sampled for two reasons. First, large loans are, all else constant, more likely to become delinquent because they have larger installments and offer greater implicit rewards to default (Schreiner, 1999). Second, large loans contribute disproportionately to value-at-risk.

For loan officer *bl*, let $N_{blR}$ be the number of rescheduled loans, and let $N_{blG}$ be the number of large loans. Likewise, let $Y_{blR}$ be the dollar amount outstanding in rescheduled loans, and let $Y_{blG}$ be the dollar amount outstanding in large loans. Finally, let $y_{blR}$ be the dollar amount of rescheduled loans revealed by the audit to be at-risk, and let $y_{\mathrm{blG}}$ be the dollar amount of large loans revealed by the audit to be at-risk. (Banco do Nordeste does not have rescheduled loans, but the general formulae are presented here anyway because most microfinance portfolios do have them.)

## 3.3 Proportion of portfolio-at-risk

The point estimate $p_{\mathrm{bl}}$ of the proportion of portfolio-at-risk for a loan officer has three components: rescheduled loans, large loans, and other loans.

### 3.3.1 Proportion of rescheduled portfolio-at-risk

The proportion of the rescheduled portfolio at-risk $p_{\mathrm{blR}}$ is the ratio of $y_{\mathrm{blR}}$ to $Y_{\mathrm{blR}}$:

$$p_{blR} = \frac{y_{blR}}{Y_{blR}}. \tag{4}$$

This quantity is of interest because it is one component of the overall portfolio-at-risk of a given loan officer, but it is also of independent interest; extraordinarily high values may signal that a loan officer does not choose wisely which loans to reschedule.

The variance $v_{\mathrm{blR}}$ of $p_{\mathrm{blR}}$ is (Cochran, 1977):

$$v_{blR} = p_{blR} \cdot (1 - p_{blR}) \cdot \left( \frac{N_{blR} - n_{blR}}{N_{blR} \cdot (n_{blR} - 1)} \right). \tag{5}$$

Because rescheduled loans are exhaustively sampled, $N_{\mathrm{blR}}$ equals $n_{\mathrm{blR}}$, and the variance $v_{\mathrm{blR}}$ is zero. Exhaustive sampling removes all sampling variation. Thus, confidence bounds are moot for estimates of $p_{blR}$; all rescheduled loans are sampled, so all repeated sample would produce the same estimate.

Let $s_{blR}$ stand for the standard error of the estimate of the proportion of portfolio-at-risk. It is the square root of $v_{blR}$, or zero.

To compute the proportion of all rescheduled loans at-risk, define $w_{blR}$ as the share of the overall portfolio of rescheduled loans in the portfolio of loan officer *bl*:

$$w_{blR} = \frac{Y_{blR}}{\sum_{b=1}^{B} \sum_{l=1}^{N_{bL}} Y_{blR}}. \tag{6}$$

Then the proportion of the portfolio of rescheduled loans at-risk $p_R$ is:

$$p_R = \sum_{b=1}^{B} \sum_{l=1}^{N_{bL}} w_{blR} \cdot p_{blR}. \tag{7}$$

The variance and standard error of $p_R$ is zero.

### 3.3.2 Proportion of large portfolio-at-risk

The proportion of portfolio-at-risk for large loans $p_{blG}$ is the ratio of $y_{blG}$ to $Y_{blG}$:

$$p_{blG} = \frac{y_{blG}}{Y_{blG}}. \tag{8}$$

Extraordinarily high values may signal to the microfinance organization that a given loan officer does not judge the risk of large loans well.

The variance of $p_{\mathrm{blG}}$ (denoted $v_{\mathrm{blG}}$) is zero because all large loans are sampled. The standard error $s_{\mathrm{blG}}$ is also zero. No confidence bounds are needed.

To compute the proportion of the portfolio of large loans at-risk, define $w_{\mathrm{blG}}$ as the share of the overall portfolio of large loans in the portfolio of loan officer *bl*:

$$w_{blG} = \frac{Y_{blG}}{\sum_{b=1}^{B} \sum_{l=1}^{N_{bL}} Y_{blG}}. \tag{9}$$

The proportion of the portfolio of large loans at-risk $p_{\mathrm{G}}$ is then:

$$p_G = \sum_{b=1}^{B} \sum_{l=1}^{N_{bL}} w_{blG} \cdot p_{blG}. \tag{10}$$

The variance and standard error of $p_{\mathrm{G}}$ is zero.

### 3.3.3 Proportion of other portfolio-at-risk

The estimation of the portfolio-at-risk for other loans (excluding rescheduled and large loans) is more complex because not all loans are sampled and because dollars cluster in loans. Recall that $n_{\mathrm{blO}}$ loans are drawn into the sample with probability proportional to size, that $Y_{\mathrm{bli}}$ is the amount outstanding, and that $y_{\mathrm{bli}}$ is the amount revealed by the audit to be in arrears. Then an estimate of the portfolio-at-risk for other loans $p_{\mathrm{blO}}$ is:

$$p_{blO} = \frac{\sum_{i=1}^{n_{blO}} y_{bli}}{\sum_{i=1}^{n_{blO}} Y_{bli}}. \tag{11}$$

Cochran (1977) notes that this estimate is inconsequentially biased.

The need to account for clustering produces a long, complex formula for the variance $v_{\mathrm{blO}}$ of $p_{\mathrm{blO}}$:

$$v_{blo} = \left( \frac{n_{blO} \cdot (N_{blO} - n_{blO})}{N_{blO} \cdot (n_{blO} - 1) \cdot \sum_{i=1}^{N_{blO}} Y_{bli}^2} \right) \cdot \left( \sum_{i=1}^{n_{blO}} y_{bli}^2 - 2 \cdot p_{blO} \cdot \sum_{i=1}^{n_{blO}} Y_{bli} \cdot y_{bli} + p_{blO}^2 \cdot \sum_{i=1}^{n_{blO}} Y_{bli}^2 \right). \tag{12}$$

This variance is not zero because other loans are not exhaustively sampled so $N_{\mathrm{blO}}$ exceeds $n_{\mathrm{blO}}$. The standard error $s_{\mathrm{blO}}$ is the square root of $v_{\mathrm{blO}}$.

To compute the proportion of the portfolio of all other loans at-risk, define $W_{\mathrm{blO}}$ as the share of the overall portfolio of other loans in the portfolio of loan officer *bl*:

$$w_{blO} = \frac{Y_{blO}}{\sum_{b=1}^{B} \sum_{l=1}^{N_{bL}} Y_{blO}}. \tag{13}$$

The proportion of the portfolio of other loans at-risk $p_O$ is then:

$$p_O = \sum_{b=1}^{B} \sum_{l=1}^{N_{bL}} w_{blO} \cdot p_{blO}. \tag{14}$$

The variance of $p_O$ is $v_O$:

$$v_O = \sum_{b=1}^{B} \sum_{l=1}^{N_{bL}} w_{blO}^2 \cdot v_{blO}. \tag{15}$$

The variance $v_O$ can be used to compute confidence bounds analogously to the manner described below for confidence bounds at the level of the loan officer, branch, or overall portfolio.

### 3.3.4 Proportion of portfolio-at-risk for a given loan officer

The overall proportion of portfolio-at-risk for a given loan officer combines the estimates of the proportion of portfolio-at-risk for rescheduled loans, large loans, and other loans. Define the strata weights $W_{blR}$, $W_{blG}$, and $W_{blO}$ as:

$$W_{blR} = \frac{Y_{blR}}{Y_{bl}}, \quad W_{blG} = \frac{Y_{blG}}{Y_{bl}}, \text{ and } \quad W_{blO} = \frac{Y_{blO}}{Y_{bl}}. \tag{16}$$

The estimate of the proportion of portfolio-at-risk for the entire portfolio of a given loan officer $p_{bl}$ is then (Cochran, 1977):

$$p_{bl} = W_{blR} \cdot p_{blR} + W_{blG} \cdot p_{blG} + W_{blO} \cdot p_{blO}. \tag{17}$$

As usual, the microfinance organization may wish to look closer at loan officers with very high $p_{bl}$.

The estimates of $p_{blR}$, $p_{blG}$, and $p_{blO}$ are all unbiased, so the estimate of $p_{bl}$ is also unbiased. Furthermore, these estimates are all independent because they are drawn from mutually exclusive strata. The variance $v_{bl}$ of $p_{bl}$ is therefore (Cochran, 1977):

$$v_{bl} = W_{blR}^2 \cdot v_{blR} + W_{blG}^2 \cdot v_{blG} + W_{blO}^2 \cdot v_{blO}. \tag{18}$$

The standard error $s_{bl}$ of $p_{bl}$ is the square root of the variance $v_{bl}$:

$$s_{bl} = \sqrt{v_{bl}}. \tag{19}$$

**3.3.5 Upper bounds on proportion of portfolio-at-risk for a loan officer**

What is the likelihood that the estimate of the proportion of portfolio-at-risk for a given loan officer $p_{bl}$ is not understated due to sampling variation? Given $t_\alpha$, the number of standard deviations from the mean of a Normal distribution that places a probability of $1-\alpha$ in the upper tail, define $d_{bl\alpha}$ as the distance from the point estimate $p_{bl}$ to an $\alpha$-percent upper bound (Cochran, 1977; Arkin, 1963):

$$d_{bl\alpha} = t_\alpha \cdot s_{bl}. \tag{20}$$

Now $u_{bl\alpha}$ is an $\alpha$-percent-confidence upper bound on $p_{bl}$, where:

$$u_{bl\alpha} = p_{bl} + d_{bl\alpha}. \tag{21}$$

As an example based loosely on numbers close to those likely to be relevant for Banco do Nordeste, suppose $\alpha$ is 90 percent, implying that $t_{\alpha}$ is 1.281. Suppose further that an average loan officer has $N_{\mathrm{bl}}$=173 loans in the portfolio ($n_{\mathrm{bl}}$=80 of which were sampled), that the point estimate $p_{\mathrm{bl}}$ is 0.068, and that $v_{\mathrm{bl}}$ is 0.000792, so $s_{\mathrm{bl}}$ is 0.02814. Then $d_{\mathrm{bl}\alpha}$ is 0.036. This means that the point estimate of $p_{\mathrm{bl}}$ will be less than $u_{\mathrm{bl}\alpha}$=0.068+0.036=0.104 in 90 percent of repeated samples.

In audits of portfolio-at-risk in microfinance, a one-sided upper bound on the point estimate is more appropriate than a two-sided interval on both sides of the point estimate. There are two reasons for this. First, a one-sided upper bound of size $\alpha$ requires a smaller sample than a two-sided interval of size $2 \cdot \alpha$. Second, auditors (*e.g.*, Carmichael and Benis, 1991; Elliott and Rogers, 1972) and potential creditors or owners of a microfinance organization are much more concerned with the understatement of risk than with the overstatement. Of course, a focus on a one-sided upper bound does not preclude also computing a two-sided interval *ex post*.

### 3.3.6 Amount of dollars at-risk

Given an estimate of the proportion of portfolio-at-risk for a loan officer $p_{\mathrm{bl}}$, an estimate of the amount of dollars at-risk $P_{\mathrm{bl}}$ is (Cochran, 1977):

$$P_{bl} = Y_{bl} \cdot p_{bl}. \tag{22}$$

The variance of $P_{bl}$ is $V_{bl}$:

$$V_{bl} = Y_{bl}^2 \cdot v_{bl}. \tag{23}$$

The standard error of $P_{bl}$ ($S_{bl}$) is the square root of $V_{bl}$. Let $D_{bl\alpha}$ be the distance from the point estimate $P_{bl}$ to an $\alpha$-percent upper bound:

$$D_{bl\alpha} = t_\alpha \cdot S_{bl}. \tag{24}$$

The $\alpha$-percent upper bound $U_{bl\alpha}$ is then:

$$U_{bl\alpha} = P_{bl} + D_{bl\alpha}. \tag{25}$$

In the example, $P_{bl}$ is \$48,625·0.068=\$3,306. The variance of $P_{bl}$ is \$1,872,674, the product of the square of $Y_{bl}$ (\$48,626) and $v_{bl}$ (0.000792). The standard error $S_{bl}$ is \$1,368. Given a desired precision of 90 percent, $t_\alpha$ is 1.281 and $D_{bl\alpha}$ is \$1,753. A 90-percent upper bound on the amount of dollars at-risk $P_{bl}$ is $U_{bl\alpha}$, or \$3,306+\$1,753=\$5,059. In this case, the true amount of dollars at-risk does not exceed 153 percent of the figure reported by the MIS with 90-percent certainty.

## 3.4 From loan officers to branches

The formula for the proportion of portfolio-at-risk $p_b$ at the branch level (and the formulae for other related measures) are constructed from the loan-officer estimates. Recall that the amount of dollars outstanding in the portfolio of a loan officer $l$ in branch $b$ is $Y_{bl}$ and that the amount of dollars outstanding in the branch is $Y_b$. The strata weights $W_{bl}$ for loan officers in a branch are then:

$$W_{bl} = \frac{Y_{bl}}{Y_b}. \tag{26}$$

The estimated proportion of portfolio-at-risk for a branch is $p_b$:

$$p_b = \sum_{l=1}^{N_{bL}} W_{bl} \cdot p_{bl}. \tag{27}$$

The variance $v_b$ of the estimate of the proportion of portfolio-at-risk is:

$$v_b = \sum_{l=1}^{N_{bL}} W_{bl}^2 \cdot v_{bl}. \tag{28}$$

The standard error $s_b$ is the square root of $v_b$. The size of an $\alpha$-percent upper bound on $p_b$ is $d_{b\alpha}$:

$$d_{b\alpha} = t_\alpha \cdot s_b. \tag{29}$$

The $\alpha$-percent confidence bound on $p_b$ is then $u_{b\alpha}$:

$$u_{b\alpha} = p_b + d_{b\alpha}. \tag{30}$$

An estimate of the amount of dollars at-risk $P_b$ at a branch is:

$$P_b = Y_b \cdot p_b. \tag{31}$$

The variance of $P_b$ is $V_b$:

$$V_b = Y_b^2 \cdot v_b. \tag{32}$$

The standard error of $P_b$ is $S_b$, the square root of $V_b$. Let $D_{b\alpha}$ be the distance from the point estimate $P_b$ to an $\alpha$-percent upper bound:

$$D_{b\alpha} = t_\alpha \cdot S_b. \tag{33}$$

The $\alpha$-percent upper bound $U_{b\alpha}$ is then:

$$U_{b\alpha} = P_b + D_{b\alpha}. \tag{34}$$

## 3.5 From branches to the overall portfolio

The formula for the proportion of portfolio-at-risk $p$ for the overall portfolio (and the formulae for other related measures) are constructed from the branch estimates. Recall that the number of branches is $B$ and that the amount of dollars in the portfolio outstanding of a branch is $Y_b$. The strata weights $W_b$ for the branches are:

$$W_b = \frac{Y_b}{Y}. \tag{35}$$

The estimated proportion of portfolio-at-risk $p$ for the overall portfolio is:

$$p = \sum_{b=1}^{B} W_b \cdot p_b. \tag{36}$$

The variance $v$ of the estimate of the proportion of portfolio-at-risk is:

$$v = \sum_{b=1}^{B} W_b^2 \cdot v_b. \tag{37}$$

The standard error $s$ is the square root of $v$. The size of an $\alpha$-percent upper bound on $p$ is $d_\alpha$:

$$d_\alpha = t_\alpha \cdot s. \tag{38}$$

The $\alpha$-percent confidence bound on $p$ is $u_\alpha$:

$$u_{b\alpha} = p_b + d_{b\alpha}. \tag{39}$$

An estimate of the amount of dollars at-risk $P$ in the overall portfolio is:

$$P = Y \cdot p. \tag{40}$$

The variance of $P$ is $V$:

$$V = Y^2 \cdot v. \tag{41}$$

The standard error of $P$ is $S$, the square root of $V$. Let $D_\alpha$ be the distance from the point estimate $P$ to an $\alpha$-percent upper bound:

$$D_\alpha = t_\alpha \cdot S. \tag{42}$$

The $\alpha$-percent upper bound $U_\alpha$ is then:

$$U_\alpha = P + D_\alpha. \tag{43}$$

# 4. Sample size

Arkin (1963) says that the choice of the optimal sample size has five steps:

- Choose a confidence level $\alpha$ for the upper bound (for example, 90 percent);
- Choose a size $d_\alpha$ for the upper bound (for example, 2 percentage points);
- Guess the proportion of the portfolio-at-risk before sampling (for example, the MIS estimate);
- Count the population size (for example, the MIS figure);
- Compute the sample size using standard formulae.

At least three factors make the choice of the optimal sample size less straightforward than this simple list would suggest. First, budgets are limited. Sampling is useful precisely because it promises to reduce costs by reducing the number of files that must be pulled. Often, the optimal sample size costs too much.

Second, the confidence level $\alpha$ and the size of the upper bound $d_\alpha$ are not chosen in a vacuum. Rather, these choices depend on the one hand on the budget and on the other hand on the cost of a mistaken judgement from an underestimate due to sampling variation. For example, if the cost of underestimating portfolio-at-risk is very high, then a large sample is needed to narrow the confidence bounds.

Third, there are no formulae to compute optimal sample sizes in all but the simplest cases. There are formulae for simple random samples (Cochran, 1977), for single levels of stratification (Roberts, 1978), for clustered samples (Cochran, 1977), and for dollar-unit sampling (Smith, 1977), but there are no formulae for four strata, two exhaustive and two with dollar-unit sampling and clusters.

## 4.1 Conservative optimality

In practice, the choice of the (sub-optimal) sample size starts with the (optimal) sample size for the lowest strata, the loan officer. Sample sizes for higher-level strata—the branch and the overall portfolio—are aggregates of sample sizes in lower-level strata. The result is a sample size that I call *conservatively optimal*. It is *optimal* because, at the lowest strata, it is computed from standard formulae for a given upper bound and level of confidence. It is also *conservative* because, in higher strata, it exceeds the requirement for a given upper bound and confidence level. This happens because the required sample size increases with the population size at a decreasing rate. (In the design proposed here, the number of cases in higher strata are even more conservative because the lower strata include exhaustive samples of rescheduled and large loans and because of the use of dollar-unit sampling.)

### 4.1.1 An example

For a simple random sample from the portfolio of loan officer *bl*, computing the optimal sample size $n_{\text{blO}}$ requires an MIS estimate $p^*_{\text{blO}}$ of the proportion of the portfolio-at-risk for non-rescheduled, non-large loans, a desired size of the upper bound $d_{\text{blO}}$, the factor $t_{blO}$ based on a desired confidence level $\alpha_{blO}$, and the number of loans $N_{blO}$ (Cochran, 1977; Arkin, 1963):

$$n_{blO} = \frac{p^*_{blO} \cdot (1 - p^*_{blO})}{\left(\frac{d_{blO}}{t_{blO}}\right)^2 + \frac{p^*_{blO} \cdot (1 - p^*_{blO})}{N_{blO}}}. \quad (44)$$

For example, suppose that branch $b$ at Banco do Nordeste has two loan officers, one with an $N_{b1O}$ of 173 (the portfolio average) and one with an $N_{b2O}$ of 300. For both, $p^*_{blO}$ is 0.068. With a 2-percentage-point upper bound with 90-percent confidence, $d_{blO}$ is 0.02 and $t_{blO}$ is 1.281. For the first loan officer, $n_{b1O}$ is 103.87, or, after conservatively rounding upward, 104. For the second loan officer, $n_{b2O}$ is 140. Thus, the optimal sample size increases from 104 to 140 as the population size increases from 173 to 300. The share of the portfolio sampled, however, decreases (from 60 percent to 47 percent) as the population size increases.

Now suppose that the sample estimates of the proportion of portfolio-at-risk ($p_{b1O}$ and $p_{b2O}$) revealed by the audit are 0.06 and 0.08. Suppose further that equation 12 gives $v_{b1O}$ as 0.0002184 and $v_{b2O}$ as 0.0002824. Ignoring rescheduled and large loans and supposing that all loans are the same size, the strata weights $W_{b1}$ and $W_{b2}$ are 0.3658 and 0.6342. The proportion of portfolio-at-risk for the branch $p_b$ is then $0.3658 \cdot 0.06 + 0.6342 \cdot 0.08 = 0.073$ (equation 17). The variance $v_b$ is $0.3658^2 \cdot 0.0002184 + 0.6342^2 \cdot 0.0002824 = 0.0001428$ (equation 18), and the standard error

$s_b$ is 0.01195. The size of a 90-percent upper bound on $p_b$ is 1.281·0.01195=0.015 (equation 20), so the upper bound itself is 0.073+0.015=0.88 (equation 21).

Thus, sampling to achieve a 2-percentage-point upper bound in the loan-officer strata leads to a 1.5-percentage-point upper bound in the branch strata. This is the essence of conservative optimality; sample sizes optimal for given upper bounds in the lowest strata lead to smaller upper bounds in higher strata.

#### 4.1.2 Rescheduled and large loans

All rescheduled loans are sampled. The sample size for large loans depends on how large is *large*. The choice may derive from an arbitrarily set number of loans and/or from natural breaks in the loan-size distribution. For Banco do Nordeste, the set of large loans will probably include 300 to 1,000 cases.

### 4.2 Factors that affect sample size

#### 4.2.1 Algebraic

The formula for the optimal number of cases for a simple random sample (equation 44) highlights several factors that affect sample size. First, the required sample size increases with the guessed proportion of the portfolio-at-risk. Second, the required sample size increases with the size of the population, but at a decreasing rate. Third, more precision requires bigger samples, both because the required sample size increases as the width of the upper bound decreases and because the required sample size increases as the desired level of confidence increases.

Of these three factors, the guess for the proportion of the portfolio-at-risk is derived from the MIS and is beyond the control of the auditor. The size and confidence level of the upper bound are more malleable, and these are normally chosen based on budget constraints and on the likely cost of mistakenly understating portfolio-at-risk.

### 4.2.2 Non-algebraic

At least four other factors also affect the required sample size, given the desired size and confidence in the upper bound. First, stratification can increase or decrease the required sample size. Unless the proportion of portfolio-at-risk varies greatly among strata (and it probably does not in microfinance portfolios), stratification by branch and loan officer increase the required sample size. Although the required sample size increases with the size of the population, it increases at a decreasing rate. Thus, the sum of the sample sizes for lower-level strata exceeds the sample size that would be required for a higher-level stratum on its own. For example, without strata and without clusters, the proportion of portfolio-at-risk for the microfinance portfolio at Banco do Nordeste could be estimated with an 90-percent upper bound of 2 percentage points with only about 259 cases (equation 44, assuming $p^*$ is 0.068 and $N$ is 44,581). Stratified at the loan-officer level and assuming that each of the 257 loan officers has a $p^*_{\mathrm{bl}}$ of 0.068 and an $N_{\mathrm{bl}}$ of 173 (the portfolio averages), the total required number of cases is

257·104=26,728. (Of course, the upper bound on the estimated proportion of the overall portfolio-at-risk with this many cases is very tight.) In short, strata can be expensive in terms of sample size. To get samples within budget may require either fewer strata or wider upper bounds and/or lower confidence levels in lower-level strata. The choice depends on the objectives of the audit, the budget constraints, and the cost of understating risk. In particular, precise estimates of the risk of the overall portfolio do not require very large sample sizes.

Second, dollar-unit sampling decreases the required sample size. This technique preserves the virtues of random sampling but concentrates the effort of pulling physical files on larger loans, thus boosting the share of dollars outstanding checked in a given number of pulled files. On the other hand, dollars are clustered in loans—that is, all dollars in a loan have the same at-risk status—and this dampens the reduction in sample size due to dollar-unit sampling. On net, however, dollar-unit sampling decreases the sample size required for a given upper bound and level of confidence. Unfortunately, the need to account for clustering means that it is impossible to solve for the optimal sample size in the lowest-level stratum $n_{\mathrm{blO}}$ in the formula for the variance $v_{\mathrm{blO}}$ (equation 12). The use of equation 44 as a second-best practical solution conservatively overstates the required sample size.

Third, exhaustive sampling of restructured and large loans increases sample size. Still, exhaustive sampling makes sense if mistakes in the MIS record of arrears may differ systematically between rescheduled loans, large loans, and other loans. Dollar-unit sampling obviates at least some of the gains from exhaustive sampling of large loans because many of these loans would have been sampled anyway.

Fourth, for a given level of confidence $\alpha$, two-sided intervals require more cases than one-sided bounds. One-sided bounds make sense for microfinance audits because understatements of arrears are much more dangerous than overstatements.

In summary, most audits of microfinance portfolios probably will use one-sided confidence bounds, dollar-unit sampling, and exhaustive sampling of rescheduled and large loans. The main adjustments in specific audits are the desired upper bound and confidence level and/or the use of stratification by branch or loan officer. The next section discusses trade-offs among these choices.

### 4.3 Trade-offs in choices of sample size

As pointed out above, a simple random sample of the overall portfolio with about 259 cases could serve to estimate the proportion of portfolio-at-risk with 90 percent confidence of not understating the true proportion by more than 2 percentage points. The addition of branch and/or loan-officer strata boost the required number of cases by one or two orders of magnitude. The size of the upper bound and the level of confidence also strongly influence sample size.

### 4.3.1 Level of confidence

In practice, the level of confidence for the upper bound $\alpha$ is almost always 90 percent. The shift to 95 percent increases sample size a lot (shown below) but adds little strength to the measure of confidence. More important, few people are satisfied with 95-percent confidence but unsatisfied with 90 percent. Despite devastating critiques (McCloskey and Ziliak ,1996; McCloskey, 1985; Cowger, 1984), convention dictates that the lowest credible level of confidence is 90 percent. Although confidence levels of 80 percent or even 70 percent are good enough for government work (in the absence of other knowledge, even 51-percent confidence is useful), no one questions whether 90 percent is adequate, but many insist that 80 percent is inadequate.

### 4.3.2 Size of upper bound

The upper bound must be narrow so that the estimated proportion of portfolio-at-risk is not useless, but it must also be wide so as to accommodate budget constraints on sample size. Increases in the upper bound increase sample size at an increasing rate (Figures 1 and 2 and Tables 1 and 2; Cochran, 1977).

The size of the upper bound depends on the proportion of portfolio-at-risk reported by the MIS and the level of risk considered dangerous. If the MIS proportion is very low (say, 1 percent) and a portfolio-at-risk of 5 percent is not dangerous, then an upper bound that spans four percentage points might not be too wide even though it implies that the sample audit cannot rule out a risk four times as great as that reported by the MIS. On the other hand, a microfinance lender whose MIS reports a portfolio-at-risk of 5 percent might not be considered creditworthy if the audit sample cannot rule out a portfolio-at-risk of 9 percent (almost double what the MIS reports).

In some cases, it may be useful to compare the upper bound on portfolio-at-risk with annual profits or with annual provisions for loan losses. The goal of such a comparison is to get a rough feel for the possible cost of mistakenly understating risk.

Given that the true proportion of the portfolio-at-risk in microfinance if often between 2 and 10 percent, confidence bounds of 2 or 3 percentage points will often be appropriate. Smaller bounds may require too many cases, and larger bounds may be too fuzzy to inform judgements.

#### 4.3.3 Stratification by branch and loan-officer

Trade-offs between levels of confidence, size of bounds, and number of strata depend on the specifics of a given microfinance portfolio. This section and the next discuss these trade-offs in terms of Banco do Nordeste. The specific tables and figures here do not apply to other microfinance portfolios, although of course the main concepts and the approach to the investigation of trade-offs are valid in general.

##### 4.3.3.1 Calculations

Given stratification by loan officer, Figure 1 and Table 1 show how the required sample size varies with the level of confidence $\alpha$ and with the size of the upper bound $d_{\mathrm{bl}\alpha}$. Sample sizes were computed by applying equation 44 to the MIS-reported portfolio-at-risk $p^*_{bl}$ for each of 273 loan officers and then summing to get the overall sample size $n$. Each $p^*_{bl}$ was computed from the loan-officer-specific portfolio outstanding $Y_{bl}$ and from the MIS-reported portfolio-at-risk $y_{bl}$. Both $Y_{bl}$ and $y_{bl}$ include loans in arrears past 90 days, as well as current loans and loan with arrears between 1 and 90 days.

Optimal sample sizes for loan officers with very low MIS-reported arrears can be very small, even reaching zero for those with no reported arrears. To ensure that some loans are sampled for each loan officer, any $p^*_{bl}$ below 0.03 is set at 0.03. All the loans of some loan officers are sampled because non-exhaustive samples do not achieve the desired precision. As discussed above, the sample-size calculation does not adjust for the exhaustive sampling of rescheduled and large loans nor for dollar-unit sampling. Thus, the figures in Table 1 are conservative in that upper bounds derived from the audit will probably, for a given level of confidence, be a bit tighter than required.

**4.3.3.2 Results**

Stratification by loan officer requires large sample sizes. For example, 90-percent confidence with an upper bound of 2 percentage points for the estimated portfolio-at-risk for each loan officer requires a sample of almost 20,000 cases, or about 44 percent of the overall portfolio (Figure 1 and Table 1).

To shrink the upper bound by 1 percentage point (given 90-percent confidence) would require an additional 13,000 cases. To expand the upper bound by 1 percentage point would reduce the sample by about 7,000 cases, and to expand it by two percentage points would cut an additional 4,000 cases. The desired size of the upper bound has a strong influence on the required sample size.

The desired level of confidence also has a strong influence. For example, with the upper bound fixed at 2 percentage points, a 95-percent confidence level requires almost 25,000 cases, a 90-percent level requires almost 20,000, 80 percent requires about 12,000 cases, and 70 percent requires about 6,000 cases.

#### 4.3.4 Stratification by branch

The sample-size calculations for stratification by branch are identical to those for stratification by loan officer, but equation 44 is applied to 79 branches rather than to 273 loan officers. The results are in Figure 2 and Table 2. This sample design ensures a given level of precision for the measurement of risk at the level of branches, but it does not ensure anything for risk measurements at the level of loan officers.

Compared with stratification by loan officer, stratification by branch requires fewer cases—the curves in Figure 2 are below and to the right of those in Figure 1. The number of loans outstanding in the portfolio of a branch $N_{\mathrm{b}}$ exceeds the number of loans outstanding in the portfolios of any of its individual loan officers $N_{\mathrm{bl}}$, so the required sample at a branch $n_{\mathrm{b}}$ is larger than the required sample for any of its individual loan officers $n_{\mathrm{bl}}$. The sample size $n_{\mathrm{b}}$ for the branch on its own, however, is less than the sum of the sample sizes $n_{\mathrm{bl}}$ for the loan officers at the branch.

Changes in the desired size of the upper bound have little effect as long as the bound is less than 2 percentage points. Thus, a bound of 2 percentage points might be an efficient use of sample size.

Trade-offs between levels of confidence are large. With the size of the upper bound fixed at 2 percentage points, 95-percent confidence requires about 15,000 cases, 90-percent confidence requires about 11,000 cases, 80-percent confidence requires about 6,000 cases, and 70-percent confidence requires about 3,000 cases. Even in the absence of loan-officer strata, the sharp trade-off between sample size and confidence remains.

# 5. Conclusion

Statistical audit sampling can help potential lenders, potential owners, and potential buyers of securitized portfolios to cross-check the accuracy of MIS measures of portfolio-at-risk. The sample design described here accounts for many of the unique features of microfinance portfolios. It samples with probability proportional to loan size, and it exhaustively samples rescheduled loans and large loans. The design also stratifies by branch and/or by loan officer. Although the math is tedious, it is straightforward to implement on a spreadsheet. Increased precision of measurement and reduced cost make statistical sampling a worthwhile audit tool.

## 5.1 Options for Banco do Nordeste

With no stratification by branch or loan officer, 90-percent certainty that the true proportion of the portfolio-at-risk does not exceed the estimated proportion by more than two percentage points requires a sample of 259 cases. With stratification by branch, it requires about 11,000 cases; with stratification by loan officer, it requires about 20,000 cases. Exhaustive samples of rescheduled and large loans add about 500 to 1,000 cases to these figures.

If stratification by branch is attractive because the budget will not support more than 10,000 cases, a good option is to stratify by loan officer anyway but to reduce the desired confidence and/or to increase the size of the bound. Any level of precision with the branch as the basic strata can be achieved with the loan officer as the basic strata without losing the gains to stratifying by loan officer. For example, rather than a confidence level $\alpha$ of 90 percent with an upper-bound size $d$ of 0.0233 at the branch level (with a required $n$ of 8,746), the auditors could set $\alpha$ at 80 percent and $d$ at 0.0233 at the loan-officer level (with a required $n$ of 8,142). For most branches—although probably not for all—this technique will achieve levels of precision for $\alpha$ and $d$ very close to the desired levels while producing greater overall precision and greater knowledge of the accuracy of the measurement of arrears at the level of the loan officer. Even with 1,000 rescheduled and large loans, $n$ is still less than 10,000 and quantitative statements about the accuracy of the MIS can be made for all loan officers, for all branches, and for the overall portfolio.

The estimates of the required sample sizes here are conservative. In practice, stratification and dollar-unit sampling will lead to the achievement of the desired bounds and levels of confidence with a smaller $n$. Before drawing the sample, however, it is impossible to know exactly how much smaller $n$ could be.

In a pinch, a sampling plan based on 5,000 cases—for example, that with $\alpha$ as 80 percent and $d$ as 0.0367 under loan-officer stratification—and an additional 1,000 rescheduled and large loans might achieve 90-percent confidence for an upper bound of 2 percentage points at the branch level. At the loan-officer level, of course, confidence would be lower, and bounds wider. At the level of the overall portfolio, any random sample with more than 300 cases will achieve this level of precision.

### 5.2 A dynamic sampling plan

A final, sophisticated option is to compute measures of precision as sampling takes place, stopping once a desired level is reached. Given a spreadsheet to implement the formulae in this paper and a level of confidence, upper bounds could be checked at the end of each day's audit. Files would be pulled until precision was satisfactory for all loan officers, for all branches, and/or for the overall portfolio. Furthermore, sampling for specific branches or loan officers could be stopped as soon as desired bounds and confidence were reached within those strata.

With such daily checks, cases would have to be pulled in a specific, costly manner. In particular, auditors would have to pull—assuming stratification by loan officer—one case for each loan officer before they drew a second case for any loan officer. (A better but perhaps more difficult-to-implement plan would maintain a constant ratio of cases drawn to $n_{\text{bl}}$ across all loan officers.) Furthermore, cases would have to be pulled in a specific order. For each loan officer, the loan with the highest $\pi_{\text{bli}}$ would have to be drawn first, then—after the loan with the highest $\pi_{\text{bli}}$ had been drawn for all other loan officers—the loan with the second-highest $\pi_{\text{bli}}$ would be drawn, and so on. Auditors could not, for example, pull all the files sampled for a given loan officer at once nor do anything else that might reduce costs but that might also process cases in some order other than that indicated by $\pi_{bli}$.

Logistics issues may also preclude a plan with daily checks. For example, the microfinance lender may store physical files in the branches that disbursed the loans. If the number of branches exceeds the number of auditors, then auditors, once they pull one file for each loan officer in a given branch, would have to travel to the next branch. This quickly could become more costly than sampling a fixed number of cases.

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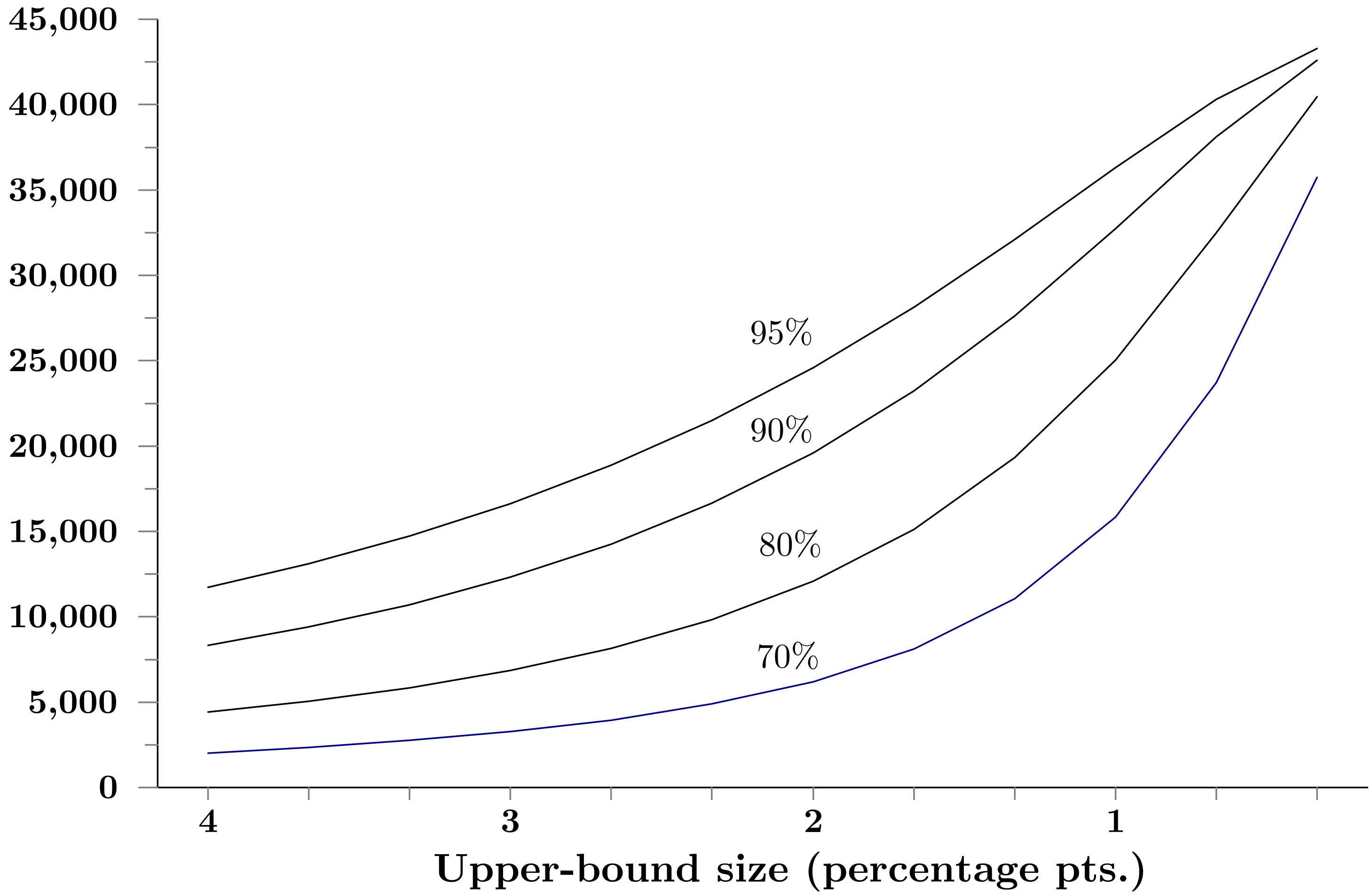
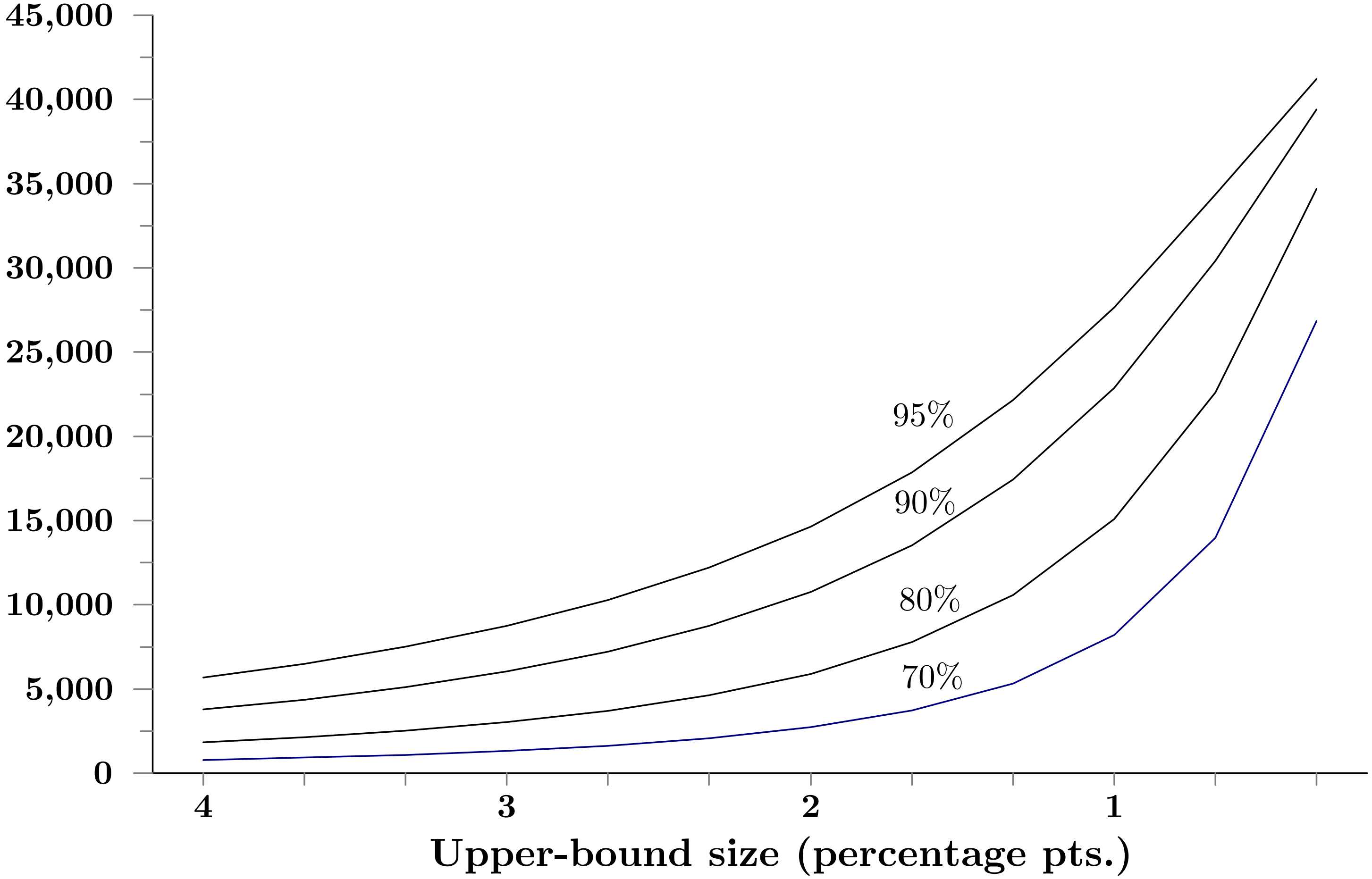
## Figure 1: Required sample sizes with stratification by loan officer



# Figure 2: Required sample sizes with stratification by branch



## Table 1: Required sample sizes with stratification by loan officer

| | Size of upper bound | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Confidence** | 0.0400 | 0.0367 | 0.0333 | 0.0300 | 0.0267 | 0.0233 | 0.0200 | 0.0167 | 0.0133 | 0.0100 | 0.0067 | 0.0033 |
| 95 percent | 11,725 | 13,097 | 14,727 | 16,622 | 18,869 | 21,496 | 24,578 | 28,130 | 32,109 | 36,321 | 40,302 | 43,287 |
| 90 percent | 8,317 | 9,406 | 10,712 | 12,319 | 14,260 | 16,643 | 19,601 | 23,232 | 27,623 | 32,727 | 38,105 | 42,598 |
| 80 percent | 4,411 | 5,051 | 5,846 | 6,854 | 8,142 | 9,839 | 12,088 | 15,129 | 19,325 | 25,050 | 32,500 | 40,465 |
| 70 percent | 2,019 | 2,348 | 2,757 | 3,281 | 3,939 | 4,887 | 6,198 | 8,116 | 11,072 | 15,832 | 23,712 | 35,755 |

Source: Calculations by the author.

## Table 2: Required sample sizes with stratification by branch

| | Size of upper bound | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Confidence** | 0.0400 | 0.0367 | 0.0333 | 0.0300 | 0.0267 | 0.0233 | 0.0200 | 0.0167 | 0.0133 | 0.0100 | 0.0067 | 0.0033 |
| 95 percent | 5,693 | 6,505 | 7,509 | 8,734 | 10,268 | 12,206 | 14,634 | 17,868 | 22,162 | 27,644 | 34,359 | 41,221 |
| 90 percent | 3,773 | 4,362 | 5,114 | 6,041 | 7,214 | 8,746 | 10,753 | 13,527 | 17,438 | 22,884 | 30,411 | 39,406 |
| 80 percent | 1,833 | 2,137 | 2,536 | 3,035 | 3,710 | 4,620 | 5,905 | 7,771 | 10,593 | 15,096 | 22,618 | 34,686 |
| 70 percent | 778 | 917 | 1,084 | 1,314 | 1,627 | 2,060 | 2,745 | 3,742 | 5,317 | 8,213 | 13,972 | 26,835 |

Source: Calculations by the author.